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Economic Order Quantity Model With Taguchi's Cost Of Poor Quality

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ABSTRACT

This paper proposes an inventory model with Taguchi's cost of poor quality in a fuzzy situation by employing the type of fuzzy numbers which are triangular. The objective is to determine the optimal order lot size to maximize the total profit. We propose two fuzzy inventory model in which first model with fuzzy defective rate and second model with fuzzy defective rate and fuzzy annual demand is presented. For each case, we employ the signed distance, a ranking method for fuzzy numbers, to find the estimate of total profit per unit time in the fuzzy sense, and then derive the corresponding optimal lot size. Numerical examples are provided to illustrate the results of proposed models.

Keywords: Inventory, Fuzzy Set, Signed Distance, EOQ, Taguchi Cost, Quality.

1. INTRODUCTION

In the classical economic order quantity models, the items received are implicitly assumed to be with perfect quality. However it may not always be the case due to imperfect production process, natural disasters, damage or breakage in transit or for many other reasons, the lot sizes received may contain some defective items. Specifically we note that in Rosenblatt and Lee's study (1986) they assumed that the defective items could be reworked at a cost and found that the presence of defective motivates smaller lot sizes.

Uncertainties of product quality and demand are inherent in real inventory problems. While in a recent article, Salameh and Jaber (2000) assumed that the defective items could be sold as a single batch at a discounted price prior to receiving the next shipment, and found that the economic lotsize quantity tends to increase as the average percentage of imperfect quality items increase. Later, Goyal and Cardenas-Barron (2002) reconsidered the task done and presented a simple approach for determining the optimal lotsize.

The defective rate in lot sizes received is a fixed constant, while others assumed it as a random variable with a known probability distribution to reflect the uncertainty (1996) of imperfect quality. For example, Park (1987) and Vujosevic et al extended the classical EOQ model by introducing the fuzziness of ordering cost and holding cost.

Huang-ChiChang (2004), Chio-Huei Ho, Hungu-Chi Chang (2011), Chen and Wang (1996) fuzzified the demand, ordering cost, inventory cost and backorder cost into trapezoidal fuzzy numbers in EOQ model with backorder. Roy and Maiti (1997) presented a fuzzy EOQ model with demand dependent unit cost under limited storage capacity. Chang et al (1998) presented a fuzzy model for inventory with backorder where the backorder quantity was fuzzified as the triangular fuzzy number

Lee and Yao (1998) and Lin and Yao (2000) discussed the production inventory problems where Lee and Yao (2000) fuzzified the demand quantity and production quantity per day, and Lin and Yao (2000) fuzzified the production quantity per cycle, treating all as the triangular fuzzy numbers. Yao et al (2000) proposed the EOQ model in the fuzzy sense, where both order quantity and total demand were fuzzified as the triangular fuzzy numbers. Recently, Tsou (2007) extended the classical economic order quantity model to include Taguchi's cost of poor quality. In order to do this he considered that when the quality of inventory is outside the specification limits, defective products are shaped. All the products within the specification limits are sold with a discount of Taguchi's cost of poor quality. Also the quality of inventory was assumed to follow a normal distribution function.

This paper is organized as follows. In section 2, a brief review of the work done by Tsou (2007) model is given. In section 3, some definitions and properties about fuzzy sets related to this study are introduced. Section 4 presents two fuzzy models as described earlier. Section 5 provides numerical examples to illustrate the results of the proposed models. Section 6 summarises the work done in this paper.

2. A BRIEF REVIEW OF THE WORK DONE BY TSOU

To propose an EOQ model with Taguchi's cost of poor quality, Tsou () first formulated the total cost per cycle TC(y) and total revenue per cycle TR(y), then the total profit per cycle TP(y) = TR(y)-TC(y).

Notations

$C \rightarrow$ purchasing price
$y \rightarrow$ the lotsize
$K \rightarrow$ ordering cost
$d \rightarrow$ screening cost
$h \rightarrow$ holding cost
$P \rightarrow$ defect rate

T → cycle length
z → screening rate
S → the unit selling price of an item within the specification limits
USL → upper specification limits
LSL → lower specification limits
D(x) → quality distribution function normally distributed with mean μ_t and standard deviation σ.
L(x) → Taguchi's cost of pure quality
N → demand rate

Mathematical Model

$$TC(y) = Cy + K + dy + hx\left(\frac{y(1-P)T}{2} + \frac{Py^2}{Z}\right)$$

$$TR(y) = y\, y\int_{LSL}^{USL} D(x)(S - L(x))\, dx$$

$$= ys\int_{LSL}^{USL} D(x)\, dx - y\int_{LSL}^{USL} D(x)\, L(x))\, dx$$

The total profit per cycle
TPY(y)=

$$NS - \frac{N}{1-P}\int_{LSL}^{USL}\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu_t)^2}{2\sigma^2}}\ x\ G(x-\mu_t)^2\ dx$$

$$-\left[\frac{NC}{1-P} + \frac{NK}{y(1-P)} + \frac{Nd}{1-P} + \frac{yh(1-P)}{2} + \frac{PhNy}{z(1-P)}\right]$$

Where 1-p = q
TPY(y)=

$$NS - \frac{N}{q}\int_{LSL}^{USL} D(x)\, dx - \left[\frac{NC}{q} + \frac{NK}{yq} + \frac{Nd}{q} + \frac{yhq}{2} + \frac{PhNy}{zq}\right] \quad (2.1)$$

The optimal lot size was derived as

$$\frac{\partial\ TPY(y)}{\partial\ y} = 0$$

$$y = \sqrt{\frac{NK}{\frac{hq^2}{2} + \frac{PhN}{z}}} \quad (2.2)$$

3. PRELIMINARIES

Before presenting the fuzzy inventory models, we introduce some definitions and properties about fuzzy numbers with relevant operations. Much of these can be found in [15-17].

Definition 1. For $0 \le \alpha \le 1$, the fuzzy set $\tilde{a}_z$ defined on R = ($-\infty$, ∞) is called an α-level fuzzy point if the membership function of $\tilde{a}_z$ is given by

$$\mu_{\tilde{a}_z}(x) = \begin{cases}\alpha, & x = a,\\ 0, & x \ne a.\end{cases} \quad (3.1)$$

Definition 2. The fuzzy set $\tilde{A}$ = (a, b, c), where a < b < c and defined on R, is called the triangular fuzzy number, if the membership function of $\tilde{A}$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases}(x-a)/(b-a), & a \le x \le b,\\ (c-x)/(c-b), & b \le x \le c,\\ 0, & \text{otherwise.}\end{cases} \quad (3.2)$$

Remark 1. (i) When $\alpha = 1$, the membership function of the 1-level fuzzy point $\tilde{a}_1$ becomes the characteristic function, ie., $\mu_{\tilde{a}_1}(x) = 1$ if x = a and $\mu_{\tilde{a}_1}(x) = 0$ if $x \ne a$. In this case, the real number $a \in R$ is the same as the fuzzy point $\tilde{a}_1$ except for their representations.
(ii) If c = b = a, then the triangular fuzzy number $\tilde{A}$ = (a, b, c) is identical to the 1-level fuzzy point $\tilde{a}_1$.

Definition 3. For $0 \le \alpha \le 1$, the fuzzy set $[a_\alpha, b_\alpha]$ defined on R is called an α-level fuzzy interval if the membership function of $[a_\alpha, b_\alpha]$ given by

$$\mu_{\alpha B(\alpha)}(x) = \begin{cases}\alpha, & a \le x \le b\\ 0, & \text{otherwise.}\end{cases} \quad (3.3)$$

Definition 4. Let $\tilde{B}$ be a fuzzy set on R, and $0 \le \alpha \le 1$. The α-cut B(a) of $\tilde{B}$ consists of points x such that $\mu_{\tilde{B}}(x) \ge \alpha$, that is, $B(\alpha) = \{x | \mu_{\tilde{B}}(x) \ge \alpha\}$.

Decomposition Principle. Let $\tilde{B}$ be a fuzzy set on R and $0 \le \alpha \le 1$. Suppose the α-cut of $\tilde{B}$ to be closed interval $[B_L(\alpha), B_U(\alpha)]$, that is, $B(\alpha) = [B_L(\alpha), B_U(\alpha)]$. Then, we have (see, [16])

$$\tilde{B} = \bigcup_{0 \le \alpha \le 1} \alpha B(\alpha) \quad (3.4)$$

Or

$$\mu_{\tilde{B}}(x) = \bigvee_{0 \le \alpha \le 1} \alpha C_{B(\alpha)}(x), \quad (3.5)$$

Where
(i) $\alpha B(\alpha)$ is a fuzzy set with membership function

$$\mu_{\alpha B(\alpha)}(x) = \begin{cases}\alpha, & a \le x \le b\\ 0, & \text{otherwise.}\end{cases}$$

(ii) $C_{B(\alpha)}(x)$ is a characteristic function $B(\alpha)$, that is

$$C_{B(\alpha)}(x) = \begin{cases}1, & x \in B(\alpha)\\ 0, & x \notin B(\alpha)\end{cases}$$

Remark 2. From the Decomposition Principle and (5), we obtain

$$\tilde{B} = \bigcup_{0 \le \alpha \le 1} \alpha B(\alpha) = \bigcup_{0 \le \alpha \le 1} [B_L(\alpha)_\alpha, B_U(\alpha)_\alpha] \quad (3.6)$$

or

$$\mu_{\tilde{B}}(x) = \underset{0\le\alpha\le1}{V} \alpha C_{B(\alpha)}(x) =$$
$$\underset{0\le\alpha\le1}{V} \mu_{[B_L(\alpha)_\alpha,\ B_U(\alpha)_\alpha]}(x) \tag{3.7}$$

For any a, b, c, d, k ∈ R, a < b, and c < d, the interval operations are as follows [16]:

(i) [a, b](+)[c, d] = [a + c, b + d]

(ii) [a, b](-)[c, d] = [a-d, b-c] (3.8)

(iii) $k(.)[a, b] = \begin{cases} ka, kb, & k > 0, \\ kb, ka, & k < 0, \end{cases}$

Further, for a > 0 and c > 0,

(i) [a, b](.)[c, d] = [ac, bd]

$[a, b](\div)[c, d] = \left[\dfrac{a}{d}, \dfrac{b}{c}\right]$

Next, as in Yao and Wu [15], we introduce the concept of the signed distance of fuzzy set. We first consider the signed distance on R.

Definition 5. For any a and 0 ∈ R, define the signed distance from a to 0 as $d_0(a, 0) = a$. If a > 0, the distance from a to 0 is $a = d_0(a, 0)$; if a < 0, the distance from a to 0 is $-a = -d_0(a, 0)$. Hence, $d_0(a, 0) = a$ is called the signed distance from a to 0.

Let Ω be the family of all fuzzy sets $\tilde{B}$ defined on R with which the α-cut $B(\alpha) = [B_L(\alpha), B_U(\alpha)]$ exists for every α ∈ [0, 1], and both $B_L(\alpha)$ and $B_U(\alpha)$ are continuous functions on α ∈ [0, 1]. Then, for any $\tilde{B} \in \Omega$, from (8) we have

$$\tilde{B} = \bigcup_{0\le\alpha\le1} [B_L(\alpha)_\alpha,\ B_U(\alpha)_\alpha] \tag{3.9}$$

From Definition 5, the signed distance of two end points, $B_L(\alpha)$ and $B_U(\alpha)$, of the α-cut $B(\alpha) = [B_L(\alpha), B_U(\alpha)]$ of $\tilde{B}$ to the origin 0 is $d_0(B_L(\alpha), 0) = B_L(\alpha)$ and $d_0(B_U(\alpha), 0) = B_U(\alpha)$, respectively. The average, $(B_L(\alpha) + B_U(\alpha))/2$.

In addition, for every α ∈ [0, 1], there is a one-to-one mapping between the α-level fuzzy interval $[B_L(\alpha), B_U(\alpha)]$ and the real interval $[B_L(\alpha), B_U(\alpha)]$, that is, the following correspondence one-to-one mapping.

$$[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha] \leftrightarrow [B_L(\alpha), B_U(\alpha)] \tag{3.10}$$

Also, the 1-level fuzzy point $\tilde{0}_1$ is mapping to the real number 0. Hence, the signed distance of $[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha]$ to $\tilde{0}_1$ can be defined as $d([B_L(\alpha)_\alpha, B_U(\alpha)_\alpha], \tilde{0}_1) = d([B_L(\alpha), B_U(\alpha)], 0) = ((B_L(\alpha) + B_U(\alpha))/2)$. Moreover, $\tilde{B} \in \Omega$, since the above function is continuous on $0 \le \alpha \le 1$, we can use the integration to obtain the mean value of the signed distance as follows

$$\int_0^1 d([B_L(\alpha)_\alpha, B_U(\alpha)_\alpha], \tilde{0}_1)\, dx = \frac{1}{2}\int_0^1 (B_L(\alpha) + B_U(\alpha))\, dx. \tag{3.11}$$

Then, from (11) and (13), we have the following definition.

Definition 6. For $\tilde{B} \in \Omega$, define the signed distance of $\tilde{B}$ to $\tilde{0}_1$ (ie. y-axis) as

$$d(\tilde{B}, \tilde{0}_1) =$$
$$\int_0^1 d([B_L(\alpha)_\alpha, B_U(\alpha)_\alpha], \tilde{0}_1)\, dx = \frac{1}{2}\int_0^1 (B_L(\alpha) + B_U(\alpha))\, dx. \tag{3.12}$$

According to Definition 6, we obtain the following property.

Property 1. For the triangular fuzzy number $\tilde{A}$ = (a, b, c), the α-cut of $\tilde{A}$ is $[A_L(\alpha), A_U(\alpha)]$, α ∈ [0, 1], where $A_L(\alpha) = a + (b-a)\alpha$ and $A_U(\alpha) = c-(c-b)\alpha$. The signed distance of $\tilde{A}$ to $\tilde{0}_1$ is

$$d\left(\tilde{A}, \tilde{0}_1\right) = \frac{1}{4}(a + 2b + c) \tag{3.13}$$

Furthermore, for two fuzzy sets $\tilde{B}, \tilde{G} \in \Omega$, where $\tilde{B} = \bigcup_{0\le\alpha\le1} [B_L(\alpha)_\alpha, B_U(\alpha)_\alpha]$ and $\tilde{G} = \bigcup_{0\le\alpha\le1} [G_L(\alpha)_\alpha, G_U(\alpha)_\alpha]$, and k ∈ R, using (10) and (12), we have

$$\text{(i)}\ \tilde{B}\,(+)\,\tilde{G} = \bigcup_{0\le\alpha\le1} [B_L(\alpha)_\alpha + G_L(\alpha)_\alpha,\ B_U(\alpha)_\alpha + G_U(\alpha)_\alpha],$$

$$\text{(ii)}\ \tilde{B}\,(-)\,\tilde{G} = \bigcup_{0\le\alpha\le1} [B_L(\alpha)_\alpha - G_L(\alpha)_\alpha,\ B_U(\alpha)_\alpha - G_U(\alpha)_\alpha], \tag{3.14}$$

$$\text{(iii)}\ \tilde{k}_1\,(\cdot)\,\tilde{B} = \begin{cases} \bigcup_{0\le\alpha\le1} [(kB_L(\alpha))_\alpha,\ (kB_U(\alpha))_\alpha], & k > 0, \\ \bigcup_{0\le\alpha\le1} [(kB_U(\alpha))_\alpha,\ (kB_L(\alpha))_\alpha], & k > 0, \\ \tilde{0}_1, & k = 0. \end{cases}$$

From the above and Definition 6, we obtain the following property.

Property 2. For two fuzzy sets $\tilde{B}, \tilde{G} \in \Omega$ and k ∈ R,

$$d(\tilde{B}, \tilde{0}_1) = \int_0^1 d([B_L(\alpha)_\alpha, B_U(\alpha)_\alpha], \tilde{0}_1)\, dx = \frac{1}{2}\int_0^1 (B_L(\alpha) + B_U(\alpha))\, dx.$$

(i) $d(\tilde{B}\,(+)\,\tilde{G}, \tilde{0}_1) = d(\tilde{B}, \tilde{0}_1) + d(\tilde{G}, \tilde{0}_1)$

(ii) $d(\tilde{B}\,(-)\,\tilde{G}, \tilde{0}_1) = d(\tilde{B}, \tilde{0}_1) - d(\tilde{G}, \tilde{0}_1)$ (3.15)

(iii) $d(\tilde{k}_1(\cdot)\tilde{B}, \tilde{0}_1) = kd(\tilde{B}, \tilde{0}_1)$

4. FUZZY EOQ MODELS FOR ITEMS WITH TAGUCHI'S COST OF POOR QUALITY

4.1. Model with fuzzy defective rate

In this subsection we modify the crisp defective rate model by incorporating the fuzziness of the defective rate P. For convenience, we let q = 1-p. Then we fuzzify q to be a triangular fuzzy number $\tilde{q} = (q-\Delta_1, q, q + \Delta_2)$ where $0 < \Delta_1 < q$ and $0 < \Delta_2 \le 1-q$, Δ_1 and Δ_2 are determined by the decision makers. In this case, the total profit per unit time is a fuzzy value also, which is expressed as

$$\widetilde{TP}Y(y) = NS - \frac{N}{\tilde{q}}\int_{LSL}^{USL} D(x)\,dx - \left[\frac{NC}{\tilde{q}} + \frac{NK}{y\tilde{q}} + \frac{Nd}{\tilde{q}} + \frac{yh\tilde{q}}{2} + \frac{PhNy}{z\tilde{q}}\right]$$

$$= NS - N\left[\int_{LSL}^{USL} D(x)\,dx + C + \frac{K}{y} + d + \frac{Phy}{z}\right]\frac{1}{\tilde{q}} - \frac{yh\tilde{q}}{2} \quad (4.1.1)$$

Now we defuzzify $\widetilde{TP}Y(y)$ using the signed distance method. The signed distance of $\widetilde{TP}Y(y)$ to c is given by

$$d(\widetilde{TP}Y(y),\ \tilde{0}_1) =$$

$$NS - \left[N\left[\int_{LSL}^{USL} D(x)\,dx + C + \frac{K}{y} + d + \frac{Phy}{z}\right] d\left(\frac{1}{\tilde{q}},\ \tilde{0}_1\right)\right] - \frac{yh}{2} d(\tilde{q},\ \tilde{0}_1) \quad (4.1.2)$$

Where $d(\tilde{q},\ \tilde{0}_1)$ and $d\left(\frac{1}{\tilde{q}},\ \tilde{0}_1\right)$ are measured as follows.

The signed distance of fuzzy number $\tilde{q}$ to $\tilde{0}_1$ is

$$d(\tilde{q},\ \tilde{0}_1) = \frac{1}{4}\left[(q - \Delta_1) + 2q + (q + \Delta_2)\right]$$

$$= q + \frac{1}{4}(\Delta_1 - \Delta_2) \quad (4.1.3)$$

Also, the left and right end points of the α-cut ($0 \le \alpha \le 1$) of $\tilde{q}$ are $q_L(\alpha) = (q - \Delta_2) + \Delta_1\alpha > 0$ and $q_U(\alpha) = (q + \Delta_2) - \Delta_2\alpha > 0$ Respectively. Since $0 < q_L(\alpha) < q_U(\alpha)$.

The left and right end points of the α-cut ($0 \le \alpha \le 1$) of $\frac{1}{\tilde{q}}$ are

$$d\left(\frac{1}{\tilde{q}},\ \tilde{0}_1\right) = \frac{1}{2}\int_0^1 \left[\left(\frac{1}{q}\right)_L \alpha + \left(\frac{1}{q}\right)_U \alpha\right]$$

$$= \frac{1}{2}\left(\frac{1}{\Delta_1} \text{In} \frac{q}{q - \Delta_1} - \frac{1}{\Delta_2} \text{In} \frac{q}{q + \Delta_2}\right)$$

Which is positive since $\Delta_1 > 0$, $\Delta_2 > 0$. In $(q/q-\Delta_1) > 0$ and In $(q/q+\Delta_2) < 0$.

We have

$$TPY^*(y) = d(\widetilde{TP}Y(y),\ \tilde{0}_1)$$

$$= NS - \left[N\left[\int_{LSL}^{USL} D(x)\,dx + C + \frac{K}{y} + d + \frac{Phy}{z}\right]\right.$$

$$\left.\frac{1}{2}\left(\frac{1}{\Delta_1}\text{In}\frac{q}{q - \Delta_1} - \frac{1}{\Delta_2}\text{In}\frac{q}{q + \Delta_2}\right)\right] - \frac{yh}{2}\left(q + \frac{1}{4}\left(\Delta_2 - \Delta_1\right)\right) \quad (4.1.4)$$

$TPY^*(y)$ is regarded as the estimate of the total profit per unit time in the fuzzy sense. The objective of the problem is to determine the optimal order lotsize y^{**}, such that $TPY^*(y)$ has a maximum value. We take the first derivative of $TPY^*(y)$ with respect to y, and obtain

$\frac{\partial TPY^*(y)}{\partial y} = 0.$ Because $\frac{\partial^2 TPY^*(y)}{\partial y^2} < 0.$

$TPY^*(y)$ is uncare in y and hence solving fuzzy we obtain the optimal lotsize.

$$y^* = \sqrt{\frac{NS - \left(\frac{1}{\Delta_1}\text{In}\frac{q}{q - \Delta_1} - \frac{1}{\Delta_2}\text{In}\frac{q}{q + \Delta_2}\right)}{h\left(q + \frac{\Delta_2 - \Delta_1}{4}\right) + \frac{PhN}{z}\left(\frac{1}{\Delta_1}\log\frac{q}{q - \Delta_1} - \frac{1}{\Delta_2}\log\frac{q}{q + \Delta_2}\right)}} \quad (4.1.5)$$

4.2. Model with fuzzy defective rate and fuzzy annual demand

This subsection further incorporates the fuzziness of demand per year into the previous fuzzy model. The crisp annual demand N is fuzzified as the triangular fuzzy number $\tilde{N}$ = ($N-\Delta_3$, N, N + Δ_4) where Δ_3 and Δ_4 are determined by the decision makers and should satisfy the conditions $0 < \Delta_3 < N$ and $0 < \Delta_4$. For this case we express the fuzzy total profit per unit time as

$$\widetilde{TP}Y(y) = \tilde{N}S - \left[\tilde{N}\left[\int_{LSL}^{USL} D(x)\,dx + C + \frac{K}{y} + d + \frac{Phy}{z}\right]\frac{1}{\tilde{q}}\right] - \frac{yh\tilde{q}}{2}$$

The signed distance of $\widetilde{TP}Y(y)$ to $\tilde{0}_1$ is given by

$$d(\widetilde{TP}Y(y),\ \tilde{0}_1) =$$

$$Sd(\tilde{N}, \tilde{0}_1) - \left[\left[\int_{LSL}^{USL} D(x)\,dx + C + \frac{K}{y} + d + \frac{Phy}{z}\right] d\left(\frac{\tilde{N}}{\tilde{q}},\ \tilde{0}_1\right)\right] - \frac{yh}{2} d(\tilde{q},\ \tilde{0}_1)$$

Where, $d(\tilde{q},\ \tilde{0}_1) = q + \frac{\Delta_2 - \Delta_1}{4}$ and

$$d(\tilde{N},\ \tilde{0}_1) = N + \frac{\Delta_4 - \Delta_3}{4} \text{ d}$$

Next we calculate the signed distance $d\left(\frac{\tilde{N}}{\tilde{q}},\ \tilde{0}_1\right)$.

The left and right end points of the α-cut ($0 \le \alpha \le 1$) of $\tilde{q}$ are

$$\left(\frac{N}{q}\right)_L(\alpha) = \frac{N_L(\alpha)}{q_U(\alpha)} = \frac{(N - \Delta_3) + \Delta_3\alpha}{(q + \Delta_2) - \Delta_2\alpha} \text{ and}$$

$$\left(\frac{N}{q}\right)_U(\alpha) = \frac{N_U(\alpha)}{q_L(\alpha)} = \frac{(N + \Delta_4) - \Delta_4\alpha}{(q - \Delta_1) + \Delta_1\alpha}$$

The signed distance of $\frac{\tilde{N}}{\tilde{q}}$ to $\tilde{0}_1$ can be derived as

$$d\left(\frac{\tilde{N}}{\tilde{q}},\ \tilde{0}_1\right) = \frac{1}{2}\int_0^1 \left[\left(\frac{N}{q}\right)_L \alpha + \left(\frac{N}{q}\right)_U \alpha\right] d\alpha$$

$$= \frac{1}{2}\left[\frac{q\Delta_4 + N\Delta_1}{\Delta_1^2}\text{In}\frac{q}{q - \Delta_1} + \frac{q\Delta_3 + N\Delta_2}{\Delta_2^2}\text{In}\frac{q + \Delta_2}{q} - \frac{\Delta_3}{\Delta_2}\right]$$

Which is positive, since the left and right end points of the α-cut of $\frac{\tilde{N}}{\tilde{q}}$ are positive continuous functions on $0 \le \alpha \le 1$.

We obtain

$TPY^*(y) = d(\widetilde{TP}Y(y), \tilde{0}_1)$

$$= S\left[N + \frac{\Delta_4 - \Delta_3}{4}\right] - \left[\int_{LSL}^{USL} D(x)\,dx + C + \frac{K}{y} + d + \frac{Phy}{z}\right]$$

$$\frac{1}{2}\left[\frac{q\Delta_4 + N\Delta_1}{\Delta_1^2} \ln \frac{q}{q - \Delta_1} + \frac{q\Delta_3 + N\Delta_2}{\Delta_2^2} \ln \frac{q + \Delta_2}{q} - \frac{\Delta_3}{\Delta_2}\right] - \frac{yh}{2}\left(q + \frac{\Delta_2 - \Delta_1}{4}\right)$$

$TPY^*(y)$ is regarded as the estimate of total profit per unit time in the fuzzy sense we determine the optimal lotsize y^{**} to maximize the total profit function $TPY^*(y)$ & $TPY^*(y)$ is concave in y. They by solving the first order necessary condition

(ie) $\frac{\partial TPY^*(y)}{\partial y} = 0.$

We obtain the optimal lotsize

$$y^{**} = \sqrt{\frac{K\,\delta}{\frac{PhN}{z}\delta + h\left(q + \frac{\Delta_2 - \Delta_1}{4}\right)}} \quad (4.2.3)$$

Where

$$\delta = \frac{q\Delta_4 + N\Delta_1}{\Delta_1^2} \ln \frac{q}{q - \Delta_1} - \frac{\Delta_4}{\Delta_1} + \frac{q\Delta_3 + N\Delta_2}{\Delta_2^2} \ln \frac{q + \Delta_2}{q} - \frac{\Delta_3}{\Delta_2}$$

5. NUMERICAL EXAMPLES

To illustrate the results of the proposed models (4.1 and 4.2), we consider an inventory system with the data.

Purchasing Price C = $ 5
Selling Price S = $ 12
Ordering Cost K = $ 100 / order
Screening Cost d = $ 1 / unit
Holding Cost h = $ 4 / unit / year
Screening Rate z = 2 units / min = 2 x 365 x 24 x 60 units / year.
Demand rate N = 20000 units / year.

D(x) is a normal distribution with mean 0.5 cm LSL = 4.8 cm, USL = 5.2 cm.

P = 0.0455 q = 0.9545

$y^* = 1049$ units

TPY (1049) = 90932 dollars / year

$\Delta_1 = 0.0005$ $\Delta_2 = 0.0200$

$\tilde{q} = (0.954, 0.9545, 0.9745)$ with $0 < \Delta_1 < q$ & $0 < \Delta_2 < 1-q$

Model 1:

$d(\tilde{q}, \tilde{0}_1) = 0.9594$

$d\left(\frac{1}{\tilde{q}}, \tilde{0}_1\right) = 1.0425$

$y^* = 1041.41$ units (by 4.1.5)

Model 2:

$d(\tilde{q}, \tilde{0}_1) = 0.9594$

$d(\tilde{N}, \tilde{0}_1) = 19962.5$

$d\left(\frac{\tilde{N}}{\tilde{q}}, \tilde{0}_1\right) = 127602.57$

$\Delta_1 = 0.0005$ $\Delta_2 = 0.015$ $\Delta_3 = 200$ $\Delta_4 = 50$

$0 < \Delta_1 < q$, $0 < \Delta_2 < 1-q$, $0 < \Delta_3 < D$, $0 < \Delta_4$

$y^{**} = 1042$ units (by 4.2.3)

$TCY^{**}(y) = 90643.24$ Dollars / year (4.2.2)

CONCLUSION

This paper proposed two fuzzy models for an inventory problem with imperfect-quality items. In the first model, the defective rate is represented by a fuzzy number, while the annual demand is treated as a fixed constant. In the second model, defective rate and annual demand are represented by a fuzzy number. For each fuzzy model, a method of defuzzification, namely the signed distance, is employed to find the estimate of total profit per unit time in the fuzzy sense, and the then the corresponding optimal order lotsize is derived to minimize the total profit. Numerical examples are carried out to investigate the behavior of our proposed models and the results are compared with those obtained from the crisp model.

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