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Flow in the Boundary Layer of Visco Elastic Liquid around a Body of Revolution

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Abstract
The flow of a visco elastic liquid in the boundary layer around a body of revolution in a particular reference to a circular cylinder is considered in this paper. Here the body oscillates harmonically on the liquid at rest. It is observed that the dividing stream lines
move away from the wall of the cylinder. The thickness of the inner vertex system increases and the intensity of the secondary flow near the solid boundary also increases. The intensity of the secondary flow is grater near the solid boundary.

Key words: visco elastic, Boundary layer, Unsteady flow, circular cylinder

Introduction

The flow of fluids such as polymer melts, blood, dyes and certain oils occur in a wide range of practical applications. These visco elastic fluids have attracted the attention of a large variety of researchers.

The growth of unsteady boundary layer where the body oscillates harmonically with time in a liquid at rest are of great physical importance. Schlichting (1932) obtained the solution of the two dimensional non-steady boundary layer equations for a viscous liquid where the free stream oscillates harmonically with time. The same type of flow of a fluid between two concentric cylinders which could be entirely in response to the fluctuations in the velocity of either inner or outer cylinder has been considered by Uchida (1975). Chang and Schowatter (1975) considered the flow of a visco-elastic liquid of oscillating cylinder. Boundary layer flow in a visco-elastic liquid oscillating cylinder has been considered by change (1977). Rath and Jena (1979) studied the flow of viscous fluid generated in response of fluctuation in the axial velocity of the outer cylinder. Biswal, Mishra and Pratihari (1985) studied the above problem in case of visco-elastic liquid. Escudier, Oliveira and Pinho (2002) have studied the flow of non-Newtonian liquid through annuli including the effects of eccentricity and inner cylinder rotation. Morand and Campos (2004) studied the flow in a boundary layer with high peclet number. Cruz and Pinho (2007) also studied the flow of visco-elastic fluids in fully developed pipe and channel. Feiz-Dizazi et al (2008) studied flow field of a non-Newtonian fluid in the anulous of rotating concentric cylinders. Similar types of hydro magnetic flow in a visco elastic fluid was discussed by Abbas et al(2008).Two dimensional unsteady flow of power law fluids over a cylinder was discussed by Patnana et al (2009). Similar types of flows were studied by Richler and Iaccarino (2010), Salleh et al (2010) and Prhashanana and Chhabra (2011). Recently a mixed convection boundary layer flow from a horizontal circular cylinder was discussed by Tham et al (2012). Similar types of flows were considered by Mossaz et al (2012), Lashagari et al (2012), Swati Mukhopadhyay (2013) and R. Cortell (2014) got very interesting results.

In this paper, the flow of a visco-elastic liquid in the boundary layer around a body of revolution in particular reference to a circular cylinder has been studied when the body oscillates harmonically on the liquid at rest.

Boundary Layer Equations

Here we need to solve the equations of the boundary layer over a body of revolution when the stream is parallel to its axis. In Cartesian frame of reference, we can write the modified Navier-Stokes equations in the form

$$p\left[\frac{\partial u_i}{\partial t}+u_k\frac{\partial u_i}{\partial X_k}\right]=-\frac{\partial p}{\partial X_i}+\mu\frac{\partial^2 u_i}{\partial X_k\partial X_k}-k[\frac{\partial}{\partial t}\left(\frac{\partial^2 u_i}{\partial X_k\partial X_k}\right)$$

$$+u_m\frac{\partial^3 u_i}{\partial X_m\partial X_k\partial X_k}-\frac{\partial u_i}{\partial X_m}.\frac{\partial^2 u_i}{\partial X_k\partial X_k}-2\frac{\partial^2 u_i}{\partial X_k\partial X_k}\frac{\partial u_m}{\partial X_k} \quad (1)$$

We now make the usual boundary layer assumptions of the viscous flow theory. The same assumptions are also applicable in visco-elastic case within the boundary layer, $u,\frac{\partial u}{\partial X},\frac{\partial^2 u}{\partial X^2},\frac{\partial p}{\partial X}$ are assumed to be 0(1) and y to be 0(δ) where δ is the thickness of the boundary layer near a solid boundary y = 0. From the equation of continuity it can be easily seen that v = 0 (δ). In order that the viscous, visco-elastic and inertia terms in the equations of motion shall be of the same order of magnitude, it is necessary that

$$v = \frac{\mu}{p} = 0\left(\delta^2\right), k^* = \frac{k}{p} = 0\left(\delta^2\right)$$

Under the above conditions, the boundary layer equations are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial X} + v\frac{\partial u}{\partial y} = -\frac{1}{p}\frac{\partial p}{\partial X} + v\frac{\partial^2 u}{\partial y^2} - k^*$$

$$[\frac{\partial^3 u}{\partial t\,\partial y^2} + u\frac{\partial^3 u}{\partial X\partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial X}.\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}.\frac{\partial^2 u}{\partial X\partial y}] \tag{2}$$

$$\frac{\partial p}{\partial y} = 0(\delta) \tag{3}$$

Solutions of Boundary Layer Equations

Here we solve the boundary layer equation (2) and (3) subject to the boundary conditions

$$\left.\begin{array}{l} \mathrm{y} = 0, \mathrm{u} = \mathrm{v} = 0 \\ \mathrm{y} \to \infty, \mathrm{u} \to \mathrm{U(x, t)} \end{array}\right\} \tag{4}$$

Where U (x,t) denotes the potential flow about the body of revolution, the pressure in the in viscid flow is given by

$$-\frac{\partial p}{\partial x} = p\left[\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x}\right] \tag{5}$$

Introducing the expression above for $\frac{\partial p}{\partial x}$ into the equation (2) we have

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + v\frac{\partial^2 U}{\partial y^2}$$

$$-k^*\left[\frac{\partial^3 u}{\partial t\partial y^2} + u\frac{\partial^3 u}{\partial x\partial y^2} + \frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}.\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}.\frac{\partial^2 u}{\partial x\partial y}\right] \tag{6}$$

In order to solve the equation (6) we apply the general approximation as follows

$$\left.\begin{array}{l} u(x,y,t) = u_o(x,y,t) + u_1(x,y,t) \\ v(x,y,t) = v_o(x,y,t) + v_1(x,y,t) \end{array}\right\}$$

Introducing the above in equation (6) we get

$$\frac{\partial u_o}{\partial t} - v\frac{\partial^2 u_o}{\partial y^2} + k^*\frac{\partial^3 u_o}{\partial t\partial y^2} = \frac{\partial U}{\partial t} \tag{7}$$

And

$$\frac{\partial u_i}{\partial t} - v\frac{\partial^2 u_i}{\partial y^2} + k^*\frac{\partial^3 u_i}{\partial t\partial y^2} = U\frac{\partial U}{\partial x} - u_o\frac{\partial u_o}{\partial x} - v_o\frac{\partial u_o}{\partial y}$$

$$-k^*\left[\frac{\partial u_o}{\partial y} - v\frac{\partial^2 u_o}{\partial y^2} + u_o\frac{\partial^3 u_o}{\partial x\,\partial y^2} + \frac{\partial u_o}{\partial y}\frac{\partial^2 v_o}{\partial y^2} + v_o\frac{\partial^3 u_o}{\partial y^3}\right] \tag{8}$$

Boundary conditions (4) give

$$\left.\begin{array}{l} y = 0, u_o = 0 = u_1 \\ y \to \infty, u_o = U(x,t), u_1 = 0 \end{array}\right\} \tag{9}$$

Now we take the potential flow about the body of revolution to be

$$U(x,t) = U_o(x)\cos(nt) \tag{10}$$

In complex notation, we take it as

$$U(x,t) = U_o(x)\exp(int) \tag{11}$$

\Here only the real parts of the complex quantities in question have physical meaning attached to them. We introduce the dimensionless quantity

$$\eta = y(n/v)^{1/2} \tag{12}$$

And assume that the first approximation to the stream function ψ_o is of the form

$$\psi_o(x,y,t) = (v/n)^{1/2}U_o(x), r(x)F(\eta)e^{\mathrm{int}} \tag{13}$$

Hence in view of the equation of continuity, we have

$$ru_o = U_o rF^I(\eta)e^{\mathrm{int}} \tag{14}$$

And

$$rv_o = -(v/n)^{1/2}\left[r\frac{dU_o}{dx} + U_o\frac{dr}{dx}\right]F(\eta)e^{\mathrm{int}} \tag{15}$$

From the equation (7) and (14), we get

$$iF^{I} - (1 - i\alpha) F^{III} = i \quad (16)$$

Where $\alpha = k^* n/v$

It is clear that α is small enough so that its second and higher powers can be neglected.
The differential equation (16) is to be solved under the boundary conditions

$$\left.\begin{array}{l} \eta = 0 : F(\eta) = F^{I}(\eta) = 0 \\ \eta \to \infty : F^{I}(\eta) = 1 \end{array}\right\} \quad (17)$$

Solving (16) with the conditions (17), we get

$$F^{I}(\eta) = 1 - e^{-p\eta} \quad (18)$$

Where

$$p = \frac{[\beta(\alpha)+\alpha]^{1/2} + i[\beta(\alpha)-\alpha]^{1/2}}{\sqrt{2\beta(\alpha)}}$$

$$= p_1 + i\, p_2 \quad (19)$$

And $\beta(\alpha) = \sqrt{1+\alpha^2}$

From (14) and (18), we get

$$u_o(x,y,t) = U_o(x)\left[1 - e^{-p\eta}\right] e^{\text{int}} \quad (20)$$

Changing to real notation, we obtain

$$u_o(x,y,t) = U_o(x)\left[\cos(nt) - e^{-p_1\eta}\cos(p_2\eta - nt)\right]$$

Here the viscous solution is obtained as a particular case when $\alpha = 0$.
We express the second approximation to the stream function ψ_1 in the form

$$\psi_1(x,y,t) = (v/n)^{1/2}\left[\frac{rU_o}{n}\frac{dU_o}{dx}\left\{F_1(\eta)e^{2\text{int}} + F_2(\eta)\right\} + \frac{U_o^2}{n}\frac{dr}{dx}\left\{F_3(\eta)e^{2\text{int}} + F_4(\eta)\right\}\right] \quad \text{-- (21)}$$

Hence

$$u_1(x,y,t) = \frac{U_o}{n}\frac{dU_o}{dx}\left\{F_1^{I}(\eta)e^{2\text{int}} + F_2^{I}(\eta)\right\} + \frac{U_o^2}{rn}\frac{dr}{dx}\left\{F_3^{I}(\eta)e^{2\text{int}} + F_4^{I}(\eta)\right\} \quad (22)$$

Real part of equations (11), (14), (15) and (22) can be written in the form

$$U(x,t) = (U_o/2)\left[e^{\text{int}} + e^{-\text{int}}\right] \quad (23)$$

$$u_o = (U_o/2)\left[F^{I}e^{\text{int}} + \bar{F}^{I}e^{-\text{int}}\right] \quad (24)$$

$$v_o = -\frac{1}{2}(v/n)^{1/2}\left[\frac{dU_o}{dx} + \frac{U_o}{r}\frac{dr}{dx}\right]\left[F\, e^{\text{int}} + \bar{F}\, e^{-\text{int}}\right] \quad (25)$$

$$u_1 = \frac{U_o}{n}\frac{dU_o}{dx}\left[\frac{1}{2}\left\{F_1^{I}\, e^{2\text{int}} + \bar{F}_1^{I}\, e^{-2\text{int}}\right\} + F_2^{1}\right] + \frac{U_o^2}{nr}\frac{dr}{dx}\left[\frac{1}{2}\left\{F_3^{I}\, e^{2\text{int}} + \bar{F}_3^{I}\, e^{-2\text{int}}\right\} + F_4^{1}\right] \quad (26)$$

Where the bar over a symbol denotes the corresponding conjugate complex quantity.
Substituting (23) – (26) in (8) and equating the coefficients of like terms we get the set of differential equations.

$$2iF_1^{I} - (1-2i\alpha)F_1^{III} = \frac{1}{2}\left[1 - F^{I^2} + FF^{II}\right] - \alpha\left[F^{I}F^{III} - \frac{1}{2}\left(F^{II^2} + FF^{iv}\right)\right] \quad (27)$$

$$-F_2^{III} = \frac{1}{2} - \frac{1}{2}F^{I}\bar{F}^{I} + \frac{1}{4}\left(F\bar{F}^{II} + \bar{F}F^{II}\right) - \alpha\left[\frac{1}{2}\left(F^{I}\bar{F}^{III} + \bar{F}^{I}F^{III}\right) - \frac{1}{2}F^{II}\bar{F}^{II} - \frac{1}{4}\left(F\bar{F}^{iv} + \bar{F}F^{iv}\right)\right] \quad (28)$$

$$2iF_3^{I} - (1-2i\alpha)F_3^{III} = \frac{1}{2}FF^{II} - \frac{1}{2}\alpha\left(FF^{II^2} + FF^{iv}\right) \quad (29)$$

$$-F_4^{III} = \frac{1}{4}\left(F\bar{F}^{II} + \bar{F}F^{II}\right) + \frac{1}{2}\alpha\left[F^{II}\bar{F}^{II} + \frac{1}{2}\left(F\bar{F}^{iv} + \bar{F}F^{iv}\right)\right] \quad (30)$$

The corresponding boundary conditions are as follows

$$\left.\begin{aligned} \eta = 0 : F_1 = F_1^I = 0, F_2 = F_2^I = 0 \\ F_3 = F_3^I = 0, F_4 = F_4^I = 0 \end{aligned}\right\} \quad (31)$$

$$\left.\begin{aligned} \eta \to \infty : F_1^I \to 0, F_3^I \to 0 \\ F_2^I = \text{finite}, F_4^I = \text{finite} \end{aligned}\right\} \quad (32)$$

Solving the equations (27) – (30) subject to the conditions (31) and (32), we get

$$F_1^I = \left(\frac{k}{p^2 - m^2}\right)\left[\left\{1 + 2p^2/\left(p^2 - m\right)\right\}\left\{e^{-(n_1 + in_2)\eta} - e^{-p\eta}\right\} - \eta\, p e^{-p\eta}\right] \quad (33)$$

$$F_2^I = M_1\, e^{-2p_1\eta} + M_2\, e^{-p_1\eta}\cos(p_2\eta) + M_3\, e^{-p_1\eta} + \sin(p_2\eta) + M_4\eta\, e^{-p_1\eta}\cos(p_2\eta) + M_5\eta\, e^{-p_1\eta}\sin(p_2\eta) - M_6 \quad (34)$$

$$F_3^I = \left[\frac{\left(1 - \alpha p^2\right)\left(p^2 + m\right)}{\left(p^2 - m\right)^2} + \frac{1 - 2\alpha p^2}{4p^2 - m}\right]\frac{e^{-(n_1 - in_2)\eta}}{2(1 - 2i\alpha)} - \frac{\left(1 - \alpha p^2\right)e^{-p\eta}}{2(1 - 2i\alpha)\left(p^2 - m\right)}\left[p\eta + \frac{p^2 + m}{p^2 - m}\right] - \frac{\left(1 - \alpha p^2\right)e^{-2p\eta}}{2(1 - 2i\alpha)\left(4p^2 - m\right)} \quad (35)$$

$$F_4^I = N_1 e^{-2p_1\eta} + N_2 e^{-p_1\eta}\cos(p_2\eta) + N_3 e^{-p_1\eta}\sin(p_2\eta) + N_4\eta e^{-p_1\eta}\cos(p_2\eta) + N_5\eta e^{-p_1\eta}\sin(p_2\eta) - N_6 \quad (36)$$

Where

$$k = \frac{1 + \alpha p^2}{2(1 - 2i\alpha)}, m = m_1 + im_2 = \frac{-4\alpha + 2i}{1 + 4\alpha^2}$$

$$n_1 + in_2 = \frac{\{\beta(2\alpha) + 2\alpha\}^{1/2} + i\{\beta(2\alpha) - 2\alpha\}^{1/2}}{\beta(2\alpha)}$$

$$M_1 = \frac{p_2^2 M_7}{4p_1^2}$$

$$M_2 = 2p_1p_2M_8 - \left(p_1^2 - p_2^2\right)M_9 + \frac{2p_1\left(3p_2^2 - p_1^2\right)M_{10} - 2p_2\left(3p_1^2 - p_2^2\right)M_{11}}{p_1^4\left(p_1^2 - 3p_2^2\right) + p_2^4\left(p_2^2 - 3p_1^2\right)}$$

$$M_3 = 2p_1p_2M_9 + \left(p_1^2 - p_2^2\right)M_8 + \frac{2p_2\left(3p_1^2 - p_2^2\right)M_{10} + 2p_1\left(3p_2^2 - p_1^2\right)M_{11}}{p_1^4\left(p_1^2 + 3p_2^2\right) + p_2^4\left(p_2^2 + 3p_1^2\right)}$$

$$M_4 = \frac{2p_1p_2M_{11} + \left(p_1^2 - p_2^2\right)M_{10}}{\left(p_1^2 + p_2^2\right)^2}$$

$$M_5 = \frac{2p_1p_2M_{10} - \left(p_1^2 - p_2^2\right)M_{11}}{\left(p_1^2 + p_2^2\right)^2}$$

$$M_6 = M_1 + M_2$$

$$M_7 = \frac{1 + 2\alpha\left(p_1^2 - p_2^2\right)}{p_1^2 + p_2^2}$$

$$M_8 = \frac{p_1p_2\left(1 - 4\alpha p_2^2\right)}{\left(p_1^2 + p_2^2\right)^3}$$

$$M_9 = \frac{\left(p_1^2 + 3p_2^2\right) + \alpha\left(p_1^4 + 6p_1^2p_2^2 - 3p_2^4\right)}{2\left(p_1^2 + p_2^2\right)^3}$$

$$M_{10} = \frac{1}{2}p_1\left[1 + \alpha\left(p_1^2 - 3p_2^2\right)\right],$$

$$M_{11} = \frac{1}{2}p_2\left[1 + \alpha\left(3p_1^2 - p_2^2\right)\right]$$

$$N_1 = \frac{-\left(p_1^2 - p_2^2\right)M_7}{8p_1^2}$$

$$N_2 = \frac{\left(p_1^2 - p_2^2\right)N_7 + 2p_1p_2N_8}{\left(p_1^2 + p_2^2\right)^2} + 2p_1\left(3p_2^2 - p_1^2\right)M_{10}$$

$$+\frac{2p_1\left(3p_2^2 - p_1^2\right)M_{10} + 2p_2\left(p_2^2 - 3p_1^2\right)M_{11}}{p_1^4\left(p_1^2 + 3p_2^2\right) + p_2^4\left(p_2^2 + 3p_1^2\right)}$$

$$N_3 = \frac{\left(p_1^2 - p_2^2\right)N_8 - 2p_1p_2N_7}{\left(p_1^2 + p_2^2\right)^2} +$$

$$\frac{2p_2\left(3p_1^2 - p_2^2\right)M_{10} + 2p_1\left(3p_2^2 - p_1^2\right)M_{11}}{p_1^4\left(p_1^2 + 3p_2^2\right) + p_2^4\left(p_2^2 + 3p_1^2\right)}$$

$$N_4 = M_4, N_5 = M_5, N_6 = N_1 + N_2$$

$$N_7 = \frac{\left(p_1^2 - p_2^2\right) + \alpha\left(p_1^4 - 6p_1^2p_2^2 + p_2^4\right)}{2\left(p_1^2 + p_2^2\right)^2},$$

$$N_8 = p_1p_2M_7$$

It is observed that the second approximation contains a steady state term which does not vanish out side the boundary layer.That is at a large distance from the body.
The velocity field, correct to second approximation can be written as

$$u = \mathrm{Re}\left[U_oF^Ie^{\mathrm{int}} + \frac{U_o}{n}\frac{dU_o}{dx}\left\{F_1^Ie^{2\mathrm{int}} + F_2^I\right\} + \frac{U_O^2}{n}\frac{dr}{dx}\left\{F_3^Ie^{2\mathrm{int}} + F_4^I\right\}\right] \quad (37)$$

Taking r (x) = constant, the body of revolution becomes a circular cylinder. In that case the circular oscillates in a stream of velocity U_∞.

The potential flow in this case is given by

$$U(x,t) = U_\infty \sin(\pi x/R)e^{\mathrm{int}} \quad (38)$$

Where R/π is the radius of the cylinder. In this case equation (37) is reduced to

$$u = \mathrm{Re}\left[U_oF^Ie^{\mathrm{int}} + \frac{U_o}{n}\frac{dU_o}{dx}\left\{F_1^Ie^{2\mathrm{int}} + F_2^I\right\}\right] \quad (39)$$

In order to consider the steady streaming in the potential flow, let us consider the average velocity component $\bar{u}$ which is defined as

$$\bar{u} = \frac{n}{2\pi}\int_o^{2x/n} u\ dt \quad (40)$$

From (39) and (40), we get,

$$\bar{u} = \frac{U_o}{n}\frac{dU_o}{dx}\ F_1^I(\eta) \quad (41)$$

Where from (11) and (38), we get

$$U_o(x) = U_\infty \sin(\pi x/R) \quad (42)$$

The stream function corresponding to (41) is given by

$$\bar{\psi} = \sqrt{v/n}\ U_o\frac{dU_o}{dx}.\frac{1}{n}F_2(\eta) \quad (43)$$

From (42) and (43), we have

$$\bar{\psi} = \left(U_\infty^2/2n\right)(\pi/R)\sqrt{v/n}\ \sin(2\pi x/R)F_2(\eta)$$

Using the dimensionless parameter $x^1 = x/R$, the non-dimensional stream function will be given by

$$\psi = \frac{\bar{\psi}\,2nR}{\pi\sqrt{v/n}\ U_\infty^2} = \sin\left(2\pi x^1\right)F_2(\eta) \quad (44)$$

Integrating (35) subject to the condition in (31) we get

$$F_2(\eta) = M_{12} - M_{13}\eta - M_{14}e^{-2p_1\eta} - M_{15}e^{-p_1\eta}\cos(p_2\eta) - M_{16}e^{-p_1\eta}\sin(p_2\eta) - M_{17}\eta e^{(-p_1\eta)}\cos(p_2\eta) - M_{18}\eta e^{-p_1\eta}\sin(p_2\eta) \quad (45)$$

Where

$$M_{12} = M_{14} + M_{15},\ M_{13} = M_6, M_{14} = M_1/2p_1$$

$$M_{15} = \frac{M_2 p_1 + M_3 p_2}{p_1^2 + p_2^2} + \frac{M_4\left(p_1^2 - p_2^2\right) + 2p_1 p_2 M_5}{\left(p_1^2 + p_2^2\right)^2}$$

$$M_{16} = \frac{M_3 p_1 - M_2 p_2}{p_1^2 + p_2^2} + \frac{M_5\left(p_1^2 - p_2^2\right) - 2p_1 p_2 M_4}{\left(p_1^2 + p_2^2\right)^2}$$

$$M_{17} = \frac{M_4 p_1 + M_5 p_2}{p_1^2 + p_2^2}$$

$$M_{18} = \frac{M_5 p_1 - M_4 p_2}{p_1^2 + p_2^2}$$

Neglecting terms containing second and higher powers of α, it can also be seen that

$$F_2^1(\infty) = -\frac{3}{4} + 4\alpha$$

Which, in Newtonian case becomes $-\frac{3}{4}$ when $\alpha = 0$

Discussions of the Results

In this paper we have studied the unsteady flow of visco-elastic liquid in the boundary layer around a circular cylinder, when r (x) = constant, the solution obtained is not a uniformly valid solution except for certain value of the elastic parameter α which is α = 3/16 (in the first order of approximation). This is because the solution does not satisfy the boundary conditions at infinity due to the boundary layer approximation.

From the equation (44), it is seen for $0 < x^1 < 0.5, \psi > 0 \text{ or } < 0$ according as $F_2(\eta) > 0 \text{ or } < 0$. Also when $F_2(\eta) = 0$, then $\psi = 0$. From the figure (1) it can be seen that $F_2(\eta) = 0$ at $\eta = 2.70, 3.14, 3.65$ for $\alpha = 0.0,\ 0.02$ and 0.04 respectively. We conclude that the dividing stream lines move away from the wall of the cylinder.

The effect of elasticity of the liquid on F_2^1 and F_4^1 have been presented in figure (2). An examination of this figure shows that the effect of the elasticity of the liquid is to increase both F_2^1 and F_4^1. It means that the thickness of the inner vertex system increases and the intensity of the secondary flow near the solid boundary increases. In a thin liquid layer near the rigid body F_2^1 first increases and then decreases. It means that the intensity of the secondary flow is greater near the solid boundary.

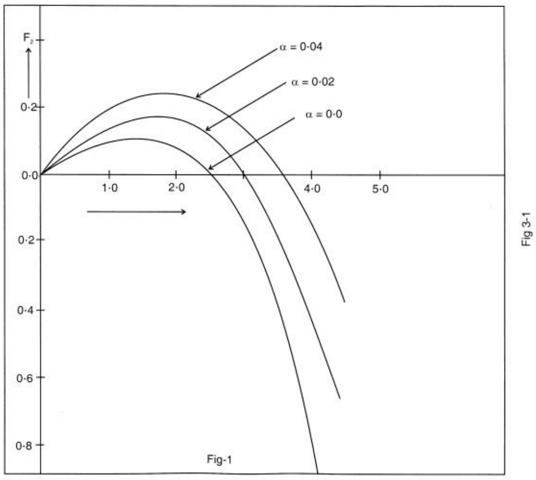

Figure1:
$F_2(\eta) = 0$ at $\eta = 2.70,\ 3.14,\ 3.65$ for $\alpha = 0.0$, 0.02 and 0.04

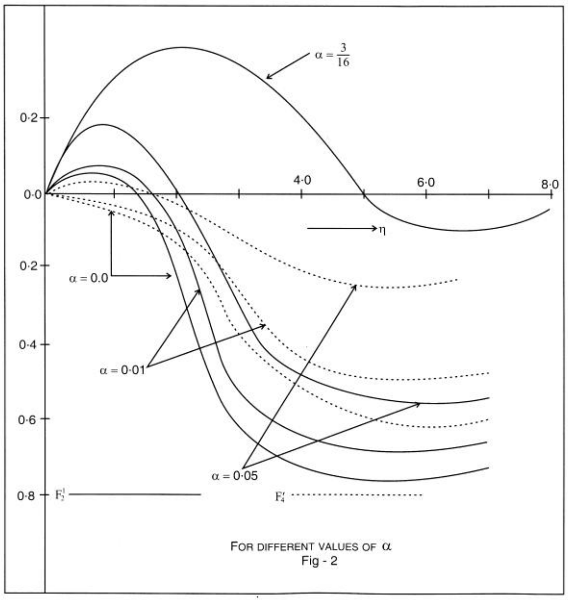

Figure 2:

The effect of elasticity of the liquid on F_2^1 and F_4^1 for different values of α.

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