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Estimation of performance measures related to time to recruitment in a two grade manpower system with two sources for depletion, correlated inter-decisions and three components for thresholds

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Abstract- The problem of time to recruitment is analysed for a two grade manpower system in which attrition takes place due to two independent sources of depletion using a univariate policy of recruitment when inter-policy decision times form a sequence of exchangeable and constantly correlated exponential random variables and the threshold for the cumulative loss of manpower in each grade has three components. Analytical results for the variance of the time to recruitment and other related performance measures are derived for three different forms of the breakdown threshold.

Keywords- Two grade manpower system, two independent sources of depletion, correlated inter-policy decisions, thresholds with three components, univariate policy of recruitment and performance measures of the time to recruitment.

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1. Introduction

The study on manpower planning begins with Bartholomew[1]. Young and Almond[2], Bartholomew and Morries[3], Young and Vassiliou[4], Grinold[5] have further contributed to this study in different directions. Numerous stochastic models for the estimation of time to recruitment in a single and multi graded organization with **only one source of depletion**(usually policy decisions) using shock model approach have been constructed and studied by Elangovan[6], Saavithri[7], Esther Clara[8], Muthaiyan[9], Parthasarathy[10], Sendhamizh Selvi[11], Vidhya[12], to name a few. Elangovan et.al [13] have initiated the study on recruitment problem for a single grade manpower system with **two sources of depletion** and obtained the variance of time to recruitment using univariate CUM policy of recruitment when the loss of man power in the organization due to the two sources of depletion, inter-policy decision times, inter-transfer decision times, and the breakdown threshold for the cumulative loss of man power in the organization are independent and identically distributed exponential random variables. Usha et.al[14] have studied the work in[13] when the inter-policy decision times are exchangeable and constantly correlated exponential random variables. In [15,16] Dhivya and Srinivasan have extended the work in [13] for a **two grade manpower system** according as the inter-policy decisions and inter-transfer decisions form the same or different ordinary renewal process respectively. In [17, 18], Dhivya and Srinivasan have studied their work in [15, 16] when thresholds for the cumulative loss of manpower in the two grades have three components. In [19], Dhivya and Srinivasan have studied their work in [15] when inter-policy decision times are exchangeable and constantly correlated exponential random variables. The objective of the present paper is to study the problem of time to recruitment in [19] when the thresholds for the cumulative loss of manpower in the two grades have three components.

2. Model Description

Consider an organization taking decisions at random epoch (0,∞) and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower if a person quits. It is assumed that the loss of manpower is linear and cumulative. For i=1,2,3…, let $X_{Ai}$and X_{Bi} be the continuous random variables representing the amount of depletion of manpower(loss of man hours) in grades A and B respectively caused due to the ith policy decision. It is assumed that X_{Ai} and X_{Bi} are independent for each i and each form a sequence of independent and identically distributed random variables with distributions $G_A(.)$ and $G_B(.)$ and probability density functions $g_A(.)$ and $g_B(.)$ respectively. Let $\bar{X}_{Am}$ and $\bar{X}_{Bm}$ be the total depletion of manpower in the first m policy decisions in grades A and B respectively. Let $\bar{X}_m$ be the cumulative depletion of manpower in the organization due to thefirst m policy decisions. For j=1,2,3…, let Y_{Aj} and Y_{Bj} be the continuous random variables representing the amount of depletion of manpower in grades A and B respectively caused due to the jth transfer decision. It is assumed that $Y_{Aj}$and Y_{Bj} are independent for each j and each form a sequence of independent and identically distributed random variables with probability density functions $h_A(.)$ and $h_B(.)$ respectively.Let $\bar{Y}_{An}$ and $\bar{Y}_{Bn}$ be the total depletion of manpower in the first n transfer decisions in grades A and B respectively. Let $\bar{Y}_n$ be the cumulative depletion of manpower in the organization due to the first n transfer decisions. For each i and j, X_{Ai}, X_{Bi},$Y_{Aj}$and$Y_{Bj}$are statistically independent. Let $\bar{g}_A(.)$,$\bar{g}_B(.)$, $\bar{h}_A(.)$and $\bar{h}_B(.)$ be the Laplace transforms of

$g_A(.)$, $g_B(.)$, $h_A(.)$ and $h_B(.)$ respectively. Let Z be the breakdown threshold level for the cumulative loss of manpower in the organization. For grade A, let Z_{A1} be the normal exponential threshold of depletion of manpower with mean $\frac{1}{\theta_{A1}}$ ($\theta_{A1} > 0$), Z_{A2} be the exponential threshold of frequent breaks of existing workers with mean $\frac{1}{\theta_{A2}}(\theta_{A2} > 0)$ and Z_{A3} be the threshold of backup or reservation of manpower sources with mean $\frac{1}{\theta_{A3}}(\theta_{A3} > 0)$. For grade B, let $Z_{B1,}Z_{B2}$ *and* Z_{B3} be the normal exponential threshold of depletion of manpower, exponential threshold of frequent breaks of existing workers and exponential threshold of backup or reservation of manpower sources with means $\frac{1}{\theta_{B1}}, \frac{1}{\theta_{B2}}$ and $\frac{1}{\theta_{B3}}$ $(\theta_{B1}, \theta_{B2}\, and\, \theta_{B3} > 0)$ respectively. Let $k(.)$ be the probability density function of Z. Let the inter-policy decision times be exchangeable and constantly correlated exponential random variables with distribution $F(.)$, probability density function $f(.)$ and parameter a, Let $F_m(.)$ be the distribution of the waiting time upto m policy decision times. Let the inter-transfer decision times be independent and identically distributed exponential random variables with distribution $W(.)$, probability density function $w(.)$ and mean $\frac{1}{\mu_2}(\mu_2 > 0)$. It is assumed that the two sources of depletion are independent. Let $W_n(.)$ be the n-fold convolution of $W(.)$ with itself. The univariate CUM policy of recruitment employed in this paper is stated as follows:

Recruitment is done whenever the cumulative loss of man hours in the organization exceeds the breakdown threshold Z.

Let T be the random variable denoting the time to recruitment with distribution L(.),probability density function l(.), Laplace transform $\bar{l}(s)$, mean E(T) and variance V(T). Let $N_P(T)$ and $N_{Trans.}(T)$ be the number of policy decisions and transfer decisions taken until the time to recruitment T respectively. Let $\bar{X}_{N_P(T)}$ and $\bar{Y}_{N_{Trans.}(T)}$ be the respective total loss of manpower in $N_P(T)$ and $N_{Trans.}(T)$ decisions until the time to recruitment T.

3. Main Results

$$P(T > t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left\{\begin{array}{c} Probability\ that\ there\ are\ exactly \\ m\ policy\ decisions\ and\ n\ transfer \\ decisons\ in (0,t] \\ and\ the\ total\ loss\ of\ manhours\ due\ to \\ m\ policy\ decisions\ and\ n\ transfer \\ decisions\ does\ not\ exceed\ the \\ breakdown\ threshold\ Z \end{array}\right\}$$

From renewal theory [20],

$$P(T > t) = \sum_{m=0}^{\infty}[F_m(t) - F_{m+1}(t)]\sum_{n=0}^{\infty}[W_n(t) - W_{n+1}(t)]P(\bar{X}_m + \bar{Y}_n \le Z) \quad (1)$$

where $F_0(t) = W_0(t) = 1$.

Invoking to the law of total probability, we get

$$P(\bar{X}_m + \bar{Y}_n \le Z) = \int_0^{\infty} P\ (\bar{X}_m + \bar{Y}_n < z)\ k(z)\ dz \qquad (2)$$

We now obtain some performance measures related to time to recruitment for different forms of Z.

Case (i)

$$\boldsymbol{Z = \min(Z_{A1} + Z_{A2} + Z_{A3}, Z_{B1} + Z_{B2} + Z_{B3}).}$$

Since $P(Z > z) = P(Z_{A1} + Z_{A2} + Z_{A3} > z)P(Z_{B1} + Z_{B2} + Z_{B3} > z)$, from the hypothesis and on simplification it can be shown that

$$\begin{aligned} k(z) = {} & C_1(\theta_{A1} + \theta_{B1})e^{-(\theta_{A1}+\theta_{B1})z} + C_2(\theta_{A2} + \theta_{B2})e^{-(\theta_{A2}+\theta_{B2})z} + \\ & C_3(\theta_{A3} + \theta_{B3})e^{-(\theta_{A3}+\theta_{B3})z} - C_4(\theta_{A1} + \theta_{B2})e^{-(\theta_{A1}+\theta_{B2})z} + \\ & C_5(\theta_{A1} + \theta_{B3})e^{-(\theta_{A1}+\theta_{B3})z} - C_6(\theta_{A2} + \theta_{B1})e^{-(\theta_{A2}+\theta_{B1})z} - \\ & C_7(\theta_{A2} + \theta_{B3})e^{-(\theta_{A2}+\theta_{B3})z} + C_8(\theta_{A3} + \theta_{B1})e^{-(\theta_{A3}+\theta_{B1})z} - \\ & C_9(\theta_{A3} + \theta_{B2})e^{-(\theta_{A3}+\theta_{B2})z} \end{aligned} \qquad (3)$$

where

$$C_1 = \frac{\theta_{A2}\theta_{A3}\theta_{B2}\theta_{B3}}{(\theta_{A2} - \theta_{A1})(\theta_{A3} - \theta_{A1})(\theta_{B2} - \theta_{B1})(\theta_{B3} - \theta_{B1})},$$
$$C_2 = \frac{\theta_{A1}\theta_{A3}\theta_{B1}\theta_{B3}}{(\theta_{A2} - \theta_{A1})(\theta_{A3} - \theta_{A2})(\theta_{B2} - \theta_{B1})(\theta_{B3} - \theta_{B2})},$$
$$C_3 = \frac{\theta_{A1}\theta_{A2}\theta_{B1}\theta_{B2}}{(\theta_{A3} - \theta_{A2})(\theta_{A3} - \theta_{A1})(\theta_{B3} - \theta_{B2})(\theta_{B3} - \theta_{B1})},$$
$$C_4 = \frac{\theta_{A2}\theta_{A3}\theta_{B1}\theta_{B3}}{(\theta_{A2} - \theta_{A1})(\theta_{A3} - \theta_{A1})(\theta_{B2} - \theta_{B1})(\theta_{B3} - \theta_{B2})},$$
$$C_5 = \frac{\theta_{A2}\theta_{A3}\theta_{B1}\theta_{B2}}{(\theta_{A2} - \theta_{A1})(\theta_{A3} - \theta_{A1})(\theta_{B3} - \theta_{B2})(\theta_{B3} - \theta_{B1})},$$
$$C_6 = \frac{\theta_{A1}\theta_{A3}\theta_{B2}\theta_{B3}}{(\theta_{A2} - \theta_{A1})(\theta_{A3} - \theta_{A2})(\theta_{B2} - \theta_{B1})(\theta_{B3} - \theta_{B1})},$$
$$C_7 = \frac{\theta_{A1}\theta_{A3}\theta_{B1}\theta_{B2}}{(\theta_{A2} - \theta_{A1})(\theta_{A3} - \theta_{A2})(\theta_{B3} - \theta_{B2})(\theta_{B3} - \theta_{B1})},$$
$$C_8 = \frac{\theta_{A1}\theta_{A2}\theta_{B2}\theta_{B3}}{(\theta_{A3} - \theta_{A2})(\theta_{A3} - \theta_{A1})(\theta_{B2} - \theta_{B1})(\theta_{B3} - \theta_{B1})},$$
$$C_9 = \frac{\theta_{A1}\theta_{A2}\theta_{B1}\theta_{B3}}{(\theta_{A3} - \theta_{A2})(\theta_{A3} - \theta_{A1})(\theta_{B2} - \theta_{B1})(\theta_{B3} - \theta_{B2})} \qquad (4)$$

From (1), (2) and (3) we get

$$\begin{aligned} P(T > t) = {} & C_1D_{\theta_{A1},\theta_{B1}}(t) + C_2D_{\theta_{A2},\theta_{B2}}(t) + C_3D_{\theta_{A3},\theta_{B3}}(t) - C_4D_{\theta_{A1},\theta_{B2}}(t) \\ & + C_5D_{\theta_{A1},\theta_{B3}}(t) - C_6D_{\theta_{A2},\theta_{B1}}(t) - C_7D_{\theta_{A2},\theta_{B3}}(t) + C_8D_{\theta_{A3},\theta_{B1}}(t) \\ & - C_9D_{\theta_{A3},\theta_{B2}}(t) \end{aligned}$$

where

$$\begin{aligned} D_{\alpha,\beta}(t) = {} & \sum_{m=0}^{\infty}[F_m(t) - F_{m+1}(t)][\bar{g}_A(\alpha + \beta)\bar{g}_B(\alpha + \beta)]^m \\ & X\sum_{n=0}^{\infty}[W_n(t) - W_{n+1}(t)]\ [\bar{h}_A(\alpha + \beta)\bar{h}_B(\alpha + \beta)]^n \end{aligned}$$

On simplification and since $w_n(t) = \frac{\mu_2{}^n e^{-\mu_2 t} t^{n-1}}{(n-1)!}$ by hypothesis, it can be shown that

$$[1 - \bar{h}_A(\alpha + \beta)\bar{h}_B(\alpha + \beta)]\sum_{n=1}^{\infty}W_n(t)[\bar{h}_A(\alpha + \beta)\bar{h}_B(\alpha + \beta)]^{n-1} = 1 - e^{-\mu_2[1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta)]t}$$

and hence

$$\begin{aligned} D_{\alpha,\beta}(t) = {} & e^{-\mu_2[1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta)]t} - [1 - \bar{g}_A(\alpha + \beta)\bar{g}_B(\alpha + \beta)] \\ & X\sum_{m=1}^{\infty}F_m(t)e^{-\mu_2[1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta)]t}[\bar{g}_A(\alpha + \beta)\bar{g}_B(\alpha + \beta)]^{m-1} \end{aligned}$$

Since $l(t) = \frac{d}{dt}[L(t) = 1 - P(T > t)]$ and $E(T^r) = (-1)^r\left[\frac{d^r}{ds^r}(\bar{l}(s))\right]_{s=0}$, the first two moments of time to recruitment are given below.

$$\begin{aligned} E(T) = {} & C_1E_{\theta_{A1},\theta_{B1}} + C_2E_{\theta_{A2},\theta_{B2}} + C_3E_{\theta_{A3},\theta_{B3}} - C_4E_{\theta_{A1},\theta_{B2}} + C_5E_{\theta_{A1},\theta_{B3}} \\ & - C_6E_{\theta_{A2},\theta_{B1}} - C_7E_{\theta_{A2},\theta_{B3}} + C_8E_{\theta_{A3},\theta_{B1}} - C_9E_{\theta_{A3},\theta_{B2}} \end{aligned} \qquad (5)$$

and

$$E(T^2) = C_1E^2{}_{\theta_{A1}.\theta_{B1}} + C_2E^2{}_{\theta_{A2}.\theta_{B2}} + C_3E^2{}_{\theta_{A3}.\theta_{B3}} - C_4E^2{}_{\theta_{A1}.\theta_{B2}} + C_5E^2{}_{\theta_{A1}.\theta_{B3}} - C_6E^2{}_{\theta_{A2}.\theta_{B1}} - C_7E^2{}_{\theta_{A2}.\theta_{B3}} + C_8E^2{}_{\theta_{A3}.\theta_{B1}} - C_9E^2{}_{\theta_{A3}.\theta_{B2}} \quad (6)$$

where

$$E_{\alpha,\beta} = \frac{1}{\mu_2[1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta)]} - \frac{[1-\bar{g}_A(\alpha+\beta)\bar{g}_B(\alpha+\beta)]}{\mu_2[1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta)]} X\sum_{m=1}^{\infty}\bar{f}_m[\mu_2(1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta_{B1}))][\bar{g}_A(\alpha+\beta)\bar{g}_B(\alpha+\beta)]^{m-1} \quad (7)$$

and

$$E^2{}_{\alpha,\beta} = \frac{2}{\left[\mu_2[1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta)]\right]^2} - \frac{2[1-\bar{g}_A(\alpha+\beta)\bar{g}_B(\alpha+\beta)]}{\left[\mu_2[1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta)]\right]^2} X\sum_{m=1}^{\infty}\bar{f}_m\left[\mu_2\left(1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta)\right)\right][\bar{g}_A(\alpha+\beta)\bar{g}_B(\alpha+\beta)]^{m-1} + \frac{2[1-\bar{g}_A(\alpha+\beta)\bar{g}_B(\alpha+\beta)]}{\mu_2[1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta)]} X\sum_{m=1}^{\infty}\bar{f}'_m\left[\mu_2\left(1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta)\right)\right][\bar{g}_A(\alpha+\beta)\bar{g}_B(\alpha+\beta)]^{m-1} \quad (8)$$

When $K_i, i = 1,2,\ldots,m$ are exchangeable and constantly correlated exponential random variables with correlation R, Gurland [21] has obtained the expression for the cumulative distribution function of the partial sum $S_m = K_1 + K_2 + \cdots + K_m$ as

$$P(S_m \le x) = (1-R)\sum_{i=0}^{\infty}\frac{(mR)^i}{(1-R+mR)^{i+1}}\frac{\psi\left(m+i, {}^{x}/_{b}\right)}{(m+i-1)!}$$

where $\psi(n,x) = \int_0^x e^{-u}u^{n-1}du$ and $b = a(1-R), a$ being the parameter of the exponential distribution.

Therefore in this paper

$$\bar{f}_m(s) = \frac{1}{(1+bs)^m\left(1+\frac{mRbs}{(1-R)(1+bs)}\right)} \quad (9)$$

Hence from (9), (7) and (8) become

$$E_{\alpha,\beta} = \frac{1}{\mu_2[1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta)]} - \frac{1-\bar{g}_A(\alpha+\beta)\bar{g}_B(\alpha+\beta)}{\mu_2[1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta)]}\sum_{m=1}^{\infty}\left\{\begin{array}{c}\dfrac{(1-R)\left[1+b\mu_2\left(1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta)\right)\right]^{1-m}}{1-R+b\mu_2\left(1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta)\right)(1-R+mR)} \\ X\,[\bar{g}_A(\alpha+\beta)\bar{g}_B(\alpha+\beta)]^{m-1}\end{array}\right\} \quad (10)$$

and

$$E^2{}_{\alpha,\beta} = \frac{2}{\left[\mu_2[1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta)]\right]^2} - \frac{2[1-\bar{g}_A(\alpha+\beta)\bar{g}_B(\alpha+\beta)]}{\left[\mu_2[1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta)]\right]^2} X\sum_{m=1}^{\infty}\frac{(1-R)[1+b\mu_2(1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta))]^{1-m}}{1-R+b\mu_2(1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta))(1-R+mR)}[\bar{g}_A(\alpha+\beta)\bar{g}_B(\alpha+\beta)]^{m-1} + \frac{2[1-\bar{g}_A(\alpha+\beta)\bar{g}_B(\alpha+\beta)]b^2(1-R)}{\mu_2[1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta)]}\sum_{m=1}^{\infty}\left\{\begin{array}{c}\left(1+b\mu_2\left(1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta)\right)\right)^{-m} \\ X\left\{\dfrac{b\mu_2\left(1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta)\right)\left(1-m(1-R+mR)\right)-m}{\left[1-R+b\mu_2\left(1-\bar{h}_A(\alpha+\beta)\bar{h}_B(\alpha+\beta)\right)(1-R+mR)\right]^2}\right\} \\ X\,[\bar{g}_A(\alpha+\beta)\bar{g}_B(\alpha+\beta)]^{m-1}\end{array}\right\} \quad (11)$$

(5) gives the mean time to recruitment and (5) together with (6) give the variance of time to recruitment for case(i).

Case (ii)

$$\boldsymbol{Z = \max(Z_{A1} + Z_{A2} + Z_{A3},\ Z_{B1} + Z_{B2} + Z_{B3})}.$$

Since $P(Z \le z) = P(Z_{A1} + Z_{A2} + Z_{A3} \le z)P(Z_{B1} + Z_{B2} + Z_{B3} \le z)$, from the hypothesis and on simplification it can be shown that

$$\begin{aligned} k(z) = &-C_1(\theta_{A1}+\theta_{B1})e^{-(\theta_{A1}+\theta_{B1})z} - C_2(\theta_{A2}+\theta_{B2})e^{-(\theta_{A2}+\theta_{B2})z} \\ &-C_3(\theta_{A3}+\theta_{B3})e^{-(\theta_{A3}+\theta_{B3})z} + C_4(\theta_{A1}+\theta_{B2})e^{-(\theta_{A1}+\theta_{B2})z} \\ &-C_5(\theta_{A1}+\theta_{B3})e^{-(\theta_{A1}+\theta_{B3})z} + C_6(\theta_{A2}+\theta_{B1})e^{-(\theta_{A2}+\theta_{B1})z} \\ &+C_7(\theta_{A2}+\theta_{B3})e^{-(\theta_{A2}+\theta_{B3})z} - C_8(\theta_{A3}+\theta_{B1})e^{-(\theta_{A3}+\theta_{B1})z} \\ &+C_9(\theta_{A3}+\theta_{B2})e^{-(\theta_{A3}+\theta_{B2})z} + C_{10}\theta_{A1}e^{-\theta_{A1}z} \\ &-C_{11}\theta_{A2}e^{-\theta_{A2}z} + C_{12}\theta_{A3}e^{-\theta_{A3}z} + C_{13}\theta_{B1}e^{-\theta_{B1}z} \\ &-C_{14}\theta_{B2}e^{-\theta_{B2}z} + C_{15}\theta_{B3}e^{-\theta_{B3}z}\end{aligned}$$

where

$C_i, i = 1,2,3,\ldots,9$ are given by (4) and

$$C_{10} = \frac{\theta_{A2}\theta_{A3}}{(\theta_{A2}-\theta_{A1})(\theta_{A3}-\theta_{A1})}, C_{11} = \frac{\theta_{A1}\theta_{A3}}{(\theta_{A2}-\theta_{A1})(\theta_{A3}-\theta_{A2})},$$

$$C_{12} = \frac{\theta_{A1}\theta_{A2}}{(\theta_{A3}-\theta_{A2})(\theta_{A3}-\theta_{A1})}, C_{13} = \frac{\theta_{B2}\theta_{B3}}{(\theta_{B2}-\theta_{B1})(\theta_{B3}-\theta_{B1})},$$

$$C_{14} = \frac{\theta_{B1}\theta_{B3}}{(\theta_{B2}-\theta_{B1})(\theta_{B3}-\theta_{B2})}, C_{15} = \frac{\theta_{B1}\theta_{B2}}{(\theta_{B3}-\theta_{B2})(\theta_{B3}-\theta_{B1})}$$

Proceeding as in case (i), we get

$$E(T) = C_{10}E_{\theta_{A1}} - C_{11}E_{\theta_{A2}} + C_{12}E_{\theta_{A3}} + C_{13}E_{\theta_{B1}} - C_{14}E_{\theta_{B2}} + C_{15}E_{\theta_{B3}} - C_1E_{\theta_{A1}.\theta_{B1}} - C_2E_{\theta_{A2}.\theta_{B2}} - C_3E_{\theta_{A3}.\theta_{B3}} + C_4E_{\theta_{A1}.\theta_{B2}} - C_5E_{\theta_{A1}.\theta_{B3}} + C_6E_{\theta_{A2}.\theta_{B1}} + C_7E_{\theta_{A2}.\theta_{B3}} - C_8E_{\theta_{A3}.\theta_{B1}} + C_9E_{\theta_{A3}.\theta_{B2}} \quad (12)$$

and

$$E(T^2) = C_{10}E^2{}_{\theta_{A1}} - C_{11}E^2{}_{\theta_{A2}} + C_{12}E^2{}_{\theta_{A3}} + C_{13}E^2{}_{\theta_{B1}} - C_{14}E^2{}_{\theta_{B2}} + C_{15}E^2{}_{\theta_{B3}} - C_1E^2{}_{\theta_{A1}.\theta_{B1}} - C_2E^2{}_{\theta_{A2}.\theta_{B2}} - C_3E^2{}_{\theta_{A3}.\theta_{B3}} + C_4E^2{}_{\theta_{A1}.\theta_{B2}} - C_5E^2{}_{\theta_{A1}.\theta_{B3}} + C_6E^2{}_{\theta_{A2}.\theta_{B1}} + C_7E^2{}_{\theta_{A2}.\theta_{B3}} - C_8E^2{}_{\theta_{A3}.\theta_{B1}} + C_9E^2{}_{\theta_{A3}.\theta_{B2}} \quad (13)$$

where $E_{\alpha,\beta}$ and $E^2{}_{\alpha,\beta}$ are given by (10) and (11) respectively.

and

$$E_\alpha = \frac{1}{\mu_2[1-\bar{h}_A(\alpha)\bar{h}_B(\alpha)]} - \frac{1-\bar{g}_A(\alpha)\bar{g}_B(\alpha)}{\mu_2[1-\bar{h}_A(\alpha)\bar{h}_B(\alpha)]}\sum_{m=1}^{\infty}\frac{(1-R)\left[1+b\mu_2\left(1-\bar{h}_A(\alpha)\bar{h}_B(\alpha)\right)\right]^{1-m}}{1-R+b\mu_2\left(1-\bar{h}_A(\alpha)\bar{h}_B(\alpha)\right)(1-R+mR)}[\bar{g}_A(\alpha)\bar{g}_B(\alpha)]^{m-1} \quad (14)$$

$$E^2{}_\alpha = \frac{2}{[\mu_2[1-\bar{h}_A(\alpha)\bar{h}_B(\alpha)]]^2} - \frac{2[1-\bar{g}_A(\alpha)\bar{g}_B(\alpha)]}{[\mu_2[1-\bar{h}_A(\alpha)\bar{h}_B(\alpha)]]^2} X\sum_{m=1}^{\infty}\frac{(1-R)\left[1+b\mu_2\left(1-\bar{h}_A(\alpha)\bar{h}_B(\alpha)\right)\right]^{1-m}}{1-R+b\mu_2\left(1-\bar{h}_A(\alpha)\bar{h}_B(\alpha)\right)(1-R+mR)}[\bar{g}_A(\alpha)\bar{g}_B(\alpha)]^{m-1} + \frac{2[1-\bar{g}_A(\alpha)\bar{g}_B(\alpha)]b^2(1-R)}{\mu_2[1-\bar{h}_A(\alpha)\bar{h}_B(\alpha)]}\sum_{m=1}^{\infty}\left\{\begin{array}{c}\left(1+b\mu_2\left(1-\bar{h}_A(\alpha)\bar{h}_B(\alpha)\right)\right)^{-m} \\ X\left\{\dfrac{b\mu_2\left(1-\bar{h}_A(\alpha)\bar{h}_B(\alpha)\right)\left(1-m(1-R+mR)\right)-m}{\left[1-R+b\mu_2\left(1-\bar{h}_A(\alpha)\bar{h}_B(\alpha)\right)(1-R+mR)\right]^2}\right\} \\ X\,[\bar{g}_A(\alpha)\bar{g}_B(\alpha)]^{m-1}\end{array}\right\} \quad (15)$$

(12) gives the mean time to recruitment and (12) together with (13) give the variance of time to recruitment for case(ii).

Case (iii)

$$\boldsymbol{Z = Z_{A1} + Z_{A2} + Z_{A3} + Z_{B1} + Z_{B2} + Z_{B3}}$$

In this case it can be shown that

$$k(z) = C_{16}\theta_{A1}e^{-\theta_{A1}z} - C_{17}\theta_{A2}e^{-\theta_{A2}z} + C_{18}\theta_{A3}e^{-\theta_{A3}z} - C_{19}\theta_{B1}e^{-\theta_{B1}z} + C_{20}\theta_{B2}e^{-\theta_{B2}z} - C_{21}\theta_{B3}e^{-\theta_{B3}z}$$

where

$$C_{16} = \frac{\theta_{A2}\theta_{A3}\theta_{B1}\theta_{B2}\theta_{B3}}{(\theta_{A2}-\theta_{A1})(\theta_{A3}-\theta_{A1})(\theta_{B1}-\theta_{A1})(\theta_{B2}-\theta_{A1})(\theta_{B3}-\theta_{A1})},$$

$$C_{17} = \frac{\theta_{A1}\theta_{A3}\theta_{B1}\theta_{B2}\theta_{B3}}{(\theta_{A2}-\theta_{A1})(\theta_{A3}-\theta_{A2})(\theta_{B1}-\theta_{A2})(\theta_{B2}-\theta_{A2})(\theta_{B3}-\theta_{A2})},$$

$$C_{18} = \frac{\theta_{A1}\theta_{A2}\theta_{B1}\theta_{B2}\theta_{B3}}{(\theta_{A3}-\theta_{A2})(\theta_{A3}-\theta_{A1})(\theta_{B1}-\theta_{A3})(\theta_{B2}-\theta_{A3})(\theta_{B3}-\theta_{A3})},$$

$$C_{19} = \frac{\theta_{A1}\theta_{A2}\theta_{A3}\theta_{B2}\theta_{B3}}{(\theta_{B1}-\theta_{A1})(\theta_{B1}-\theta_{A2})(\theta_{B1}-\theta_{A3})(\theta_{B2}-\theta_{B1})(\theta_{B3}-\theta_{B1})},$$

$$C_{20} = \frac{\theta_{A1}\theta_{A2}\theta_{A3}\theta_{B1}\theta_{B3}}{(\theta_{B2}-\theta_{A1})(\theta_{B2}-\theta_{A2})(\theta_{B2}-\theta_{A3})(\theta_{B2}-\theta_{B1})(\theta_{B3}-\theta_{B2})},$$

$$C_{21} = \frac{\theta_{A1}\theta_{A2}\theta_{A3}\theta_{B1}\theta_{B2}}{(\theta_{B3}-\theta_{A1})(\theta_{B3}-\theta_{A2})(\theta_{B3}-\theta_{A3})(\theta_{B3}-\theta_{B1})(\theta_{B3}-\theta_{B2})}$$

Proceeding as in case (i) we get

$$E(T) = C_{16}E_{\theta_{A_1}} - C_{17}E_{\theta_{A_2}} + C_{18}E_{\theta_{A_3}} + C_{19}E_{\theta_{B_1}} - C_{20}E_{\theta_{B_2}} + C_{21}E_{\theta_{B_3}} \quad (16)$$

and

$$E(T^2) = C_{16}E^2{}_{\theta_{A_1}} - C_{17}E^2{}_{\theta_{A_2}} + C_{18}E^2{}_{\theta_{A_3}} + C_{19}E^2{}_{\theta_{B_1}} - C_{20}E^2{}_{\theta_{B_2}} + C_{21}E^2{}_{\theta_{B_3}} \quad (17)$$

where E_α and $E^2{}_\alpha$ are given by (14) and (15) respectively.

(16) gives the mean time to recruitment and (16) together with (17) give the variance of time to recruitment for case(iii).

4. Special Case

Suppose $X_{Ai}, X_{Bi}, Y_{Aj} and\ Y_{Bj}$ follow exponential distribution with parameters $\alpha_{1A}, \alpha_{1B}, \alpha_{2A}$ and α_{2B} respectively.

In this case $\bar{g}_A(\theta) = \frac{\alpha_{1A}}{\alpha_{1A}+\theta}$, $\bar{g}_B(\theta) = \frac{\alpha_{1B}}{\alpha_{1B}+\theta}$, $\bar{h}_A(\theta) = \frac{\alpha_{2A}}{\alpha_{2A}+\theta}$, $\bar{h}_B(\theta) = \frac{\alpha_{2B}}{\alpha_{2B}+\theta}$ (18)

Using (18) in (6), (13), (14), (16) and (17), we get explicit form of mean and variance of time to recruitment for all the three cases.

Note:

Some performance measures related to time to recruitment are presented below.

1. The average number of policy decisions taken until the time to recruitment T is

$$E\big(N_P(T)\big) = \int_0^\infty E\big(N_P(t)\big)\, l(t)dt$$

2. The average number of transfer decisions taken until the time to recruitment T is

$$E\big(N_{Trans.}(T)\big) = \int_0^\infty E\big(N_{Trans.}(t)\big)\, l(t)dt = \mu_2 E(T)$$

3. The cumulative loss of manpower due to $N_P(T)$ policy decisions is

$$\bar{X}_{N_P(T)} = E(X_i)E\big(N_P(T)\big) = E\big(N_P(t)\big)[E(X_{Ai}) + E(X_{Bi})]$$

4. The cumulative loss of manpower due to $N_{Trans.}(T)$ transfer decisions is

$$\bar{X}_{N_{Trans.}(T)} = E(X_i)E\big(N_{Trans.}(T)\big) = \mu_2 E(T)[E(Y_{Aj}) + E(Y_{Bj})]$$

5. Conclusion

The manpower planning model developed in this paper is new in the context of incorporating a more realistic assumption of dependence in the form of correlation between inter-policy decision times for a manpower system with two grades having three components for the breakdown threshold. This model can be used to plan for the adequate provision of manpower for the organization at graduate, professional and management levels in the context of attrition. There is a scope for studying the applicability of the designed model using simulation. Further, by collecting relevant data, one can test the goodness of fit for the distributions assumed in this paper. The findings given in this paper enable one to estimate manpower gap in future, thereby facilitating the assessment of manpower profile in predicting future manpower development not only on industry but also in a wider domain.

References

[1] Bartholomew, D.J., and Forbes, A.F., 1979, "*Statistical Techniques for Manpower Planning*", John Wiley, NewYork .

[2] Young, A., and Almond, G., 1961, "*Predictions of distributions of staff*", Computer Journal, *3,246-250.*

[3] Bartholomew, D.J., and Morries, B.R., 1971, "*Aspects of Manpower Planning*", Elsevier Publishing Company, New York.

[4] Young, A., and Vassiliou, P.C.G., 1974, "*A non-linear model on the promotion of staff*", Journal of Royal Statistical Society, *A137, 584-595.*

[5] Grinold, R.C., 1976, "*Manpower planning with uncertain recruitments*", Operations Research, *24, 387-400.*

[6] Sathiyamoorthi, R., and Elangovan, R., 1998 "*Shock model approach to determine the expected time for recruitment*", Journal of Decision and Mathematika Sciences, 3(1-3), 67-78.

[7] Saavithri, V., and Srinivasan, A., 2002, "*Cost analysis on univariate policies of recruitment*", International Journal of Management Systems,18(3), 249-264.

[8] J.B.Esther Clara, 2012, "*Contributions to the study on some stochastic models in manpower planning*", Ph.D. thesis, Bharathidasan University, Tiruchirappalli.

[9] Muthaiyan, A., and Sathiyamoorthi, R., 2010, "*A stochastic model using geometric process for inter-arrival time between wastage*", Acta Ciencia Indica, *36 M(4),479-486.*

[10] Parthasarathy, S., and Vinoth, R., 2009, "*Determination of expected time to recruitment in manpower planning*", Recent Research in Science and Technology, 1(3), 147-150.

[11] Sendhamizh Selvi, S., 2009, "*A study on expected time to recruitment in manpower models for a multi grated system associated with an univariate policy of recruitment*", Ph.D thesis, Bharathidasan University, Tiruchirappalli.

[12] Vidhya, S., and Srinivasan, A., 2010, "*A stochastic model for the expected time to recruitment in a two grade manpower system having correlated inter-decision times and constant combined thresholds*", Applied Mathematical Sciences, 4(54), 2653 – 2661.

[13] Elangovan, R., Sathiyamoorthi, R., and Susiganeshkumar, E., 2011, "*Estimation of expected time to recruitment under two sources of depletion of manpower",* Proceedings of the International Conference on Stochastic Modelling and Simulation, Allied PublishresPvt.Ltd.,Chennai, *99-104*.

[14] Usha, K., Leelathilagam, P., and Ramanarayanan, R., 2013, "*A stochastic model for estimation of expected time to recruitment with two sources of depletion of manpower under correlated inter arrival times,* International Journal of Pure and Applied Mathematics, *82(5), 683-693*.

[15] Dhivya, S., and Srinivasan, A., 2013, " *Stochastic model for time to recruitment under two sources of depletion of manpower using univariate policy of recruitment", International J. of Multidispl. Research & Advcs. in Engg, 5(4), 17-26*.

[16] Dhivya, S., and Srinivasan, A., 2013, "*Stochastic model for time to recruitment under two Sources of depletion of manpower associated with different renewal process", International Journal of Revolution in Science and Humanity, 2(1), 45-51*.

[17] Dhivya, S., and Srinivasan, A.,2015,"*Estimation of variance of time to recruitment for a two grade manpower system with two sources of depletion and two types of policy decisions when the breakdown threshold has three components",*International Journal of Pure and Applied Mathematics,*100(2),211-224*.

[18] Dhivya, S., and Srinivasan, A., 2015, "*Time to recruitment in a two grade manpower system under two sources of depletion associated with different renewal processes"*, Annals of Pure and Applied Mathematics, *9(2), 183-189*.

[19] Dhivya, S., and Srinivasan, A., 2014, "*Estimation of variance of time to recruitment for a two grade manpower system with two sources of depletion and two types of policy decisions",* Proceedings of the International Conference on Mathematics and its Applications, Anna University, *1230-1241*.

[20] Medhi, J., 2009, "*Stochastic processes",* Third edition, New Age International (P) Limited, New Delhi.

[21] Gurland, J., 1955, "*Distribution of the maximum of the arithmetic mean of correlated random variables"*, Auu. Math. Statist., 26, 294-300.