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Denoising of MR images using curvelet transform

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Abstract- In this paper a new algorithm is developed to denoise the MR image using the edge detection and vanishing point detection with curvelet transform . Different Shrinkage functions such as Soft and Hard wavelet Shrinkage rules, curvelet and this new algorithm are used to remove noise and the performance of these results are compared. The algorithm is applied on brain MRIs with different noisy conditions by varying standard deviation of noise. Gaussian noise is taken as the additive noise. The results of proposed methods are compared with the results of wavelet and curvelet using peak signal to noise ratio, mean square error and edge keeping index .

Keywords- MRI;Wavelet transform;Curvelet transform; Denoising; vanishing point detection

1. Introduction

The recent advent of medical imaging has made a revolution in the field of medicine. Medical imaging has got special preferences in area of research extended up to the field of science, mathematics, engineering and medical Science. Digital imaging is widely used in medical science these days for the diagnosis of illness. With this wide spread use in medicine quality of image has a significant value. To get better diagnosis it is required that the images should be sharp, clear, free of artifacts and noise. De-noising is an important task in image processing. Biomedical signals are mostly random in nature where noise reduction becomes an important part**.** Edge preserving de-noising is of great interest in medical image processing. Wavelet has been extensively used for the last two decades for image processing. In recent years curvelets emerges and extended widely as an important image processing tool. Medical images have lots of curves, phase information and textured information. Edges are the most prominent feature for images. Edges defined the boundaries between different textures it shows discontinuities in image intensity from one pixel to other. In medical imaging detecting and enhancing the boundaries between cavities is an important task and curvelet transform is proved as good representation method for image with edges in the recent few years. Curvelet transform capture efficiently edges in an image; it is very useful tool for multiscale edge enhancement.

A new tight frame is analyzed and constructed for representing functions f(x1, x2), and establish an essential optimality of this system. Underlying these results is a mathematical insight concerning the central role, for the analysis and synthesis of objects with discontinuities along curves i.e. edges, played by parabolic scaling, in which analysis elements are supported in elongated regions obeying the relation width $\approx$ length^2 [1-15] . Once an image is constructed it is often desirable to process it to enhance the visibility of certain features such as the edges of a tumor. Noise in MR images consists of random signals that do not come from the tissues but from other sources in the machine and environment that do not contribute to the tissue differentiation. The noise of an image gives it a grainy appearance and mainly the noise is evenly spread and more uniform. There are many advanced methods of image processing involving techniques some of which were encountered in detail [16-19]. Those methods include digital image processing through Fourier transform, wavelet transform and curvelet transform. Image filters are designed for noise reduction in the MR images and better edge definition [20, 21].

N. Kingsbury in 1999 used complex wavelets for image processing [15] . In the same year Emmanuel J. Candès [10] applied monoscale ridgelets for the representation of images with edges. In 2002 J. Starck, E. Cand`es, D. Donoho [22] used curvelet first time to de-noise the image. After that curvelet transform was used for image enhancement, and noise attenuation, this technique can be used to efficiently encode edge features of the object being imaged. J. L. Starck, E. J. Candès and D. L. Donoho in 2001 did Image Restoration by Combining Wavelets and Curvelets in 2002 J. Starck, E. Cand`es, D. Donoho, denoised the images using curvelet transform in 2002 and in 2003[22,23] they did Gray and color image contrast enhancement by the curvelet transform. R. Willett, K. Nowak [24] Platelets applied multi-scaling approach for recovering edges and surfaces in photon-limited medical imaging in 2003. S. Mallat [25] did geometrical image approximation with bandlets in 2005 and M. Do, M. Vetterli [26] uses countourlet transform for directional multiresolution image representation G. Plonka, J. Ma [27] used Nonlinear regularized reaction-diffusion filters for denoising of images with textures in 2008.R. Neelamani etal [28] did Coherent and random noise attenuation using the curvelet transform, 2008. In 2012 Kunyu Tsai, Jianwei Ma, Datian Ye , Jian Wu [29]did curvelet processing of MRI for local image enhancement . They extract features through curvelet coefficients and the gradient of the original image, then they utilize fuzzy cluster method to classify the whole image scope into the ‘edge’ region and the ‘non edge’ region. In this paper we have given a denoising algorithm which combines edge detection in curvelet transform with vanishing point detection .The result of proposed methods are compared with wavelet traditional methods using PSNR.

Wavelet Transform

The wavelet transform performs multiresolution analysis which is widely used for signal or image processing. In the wavelet domain thresholding is used to extract information from noise.There are two main approaches hard thresholding and soft thresholding. Donoho and Johnstone [20,21] introduced various denoising methods by thresholding the wavelet coefficient arising from DWT. Let x(i,j) is noise free image and y(i,j) is noisy image corrupted by Gaussian noise z(i,j) then according to Donoho’s denoising method first transform noisy image into orthogonal domain by DWT then apply soft or hard threshold to the resulting wavelet coefficients by the threshold $t = s(2lognm)^{1/2}$ where $[n, m]$ is the image size finally perform inverse DWT to obtained denoised image.

Curvelet Transform

The curvelet transform is a higher dimensional generalization of the wavelet transform designed to represent images at different scales and different angles. Wavelets use shape changing but area fixed window but the curvelet transform uses angled polar wedges or angled trapezoid windows in frequency domain which help to resolve directional feature curvelet transform is a technique for multi-scale object representation. It is a multiscale directional transform with frame elements indexed by scale and location parameters. It provides a multidirectional analysis and allow a sparse representation of smooth objects containing smooth discontinuities [1,2]. Curvelet preserves the same time frequency localization property as for wavelets and at the same time, with their elongated support in the Fourier domain, curvelet become directional. In the spatial domain, the curvelet is Gabor along the width and Gaussian along the length [1].

Curvelet basis function:

$$\gamma(x_1, x_2) = \psi(x_1).\varphi(x_2) \qquad (1)$$
$$\psi(x_1) = Gabor\ (x_1)$$
$$\varphi(x_2) = Gaussian\ (x_2)$$

ψ is a smooth wavelet function and φ is a smooth scaling function.

Curvelet has been shown to be very efficient tool for many different applications in image processing, seismic data exploration, fluid mechanics, and solving partial differential equations .The discrete curvelet transform is very efficient in representing curve-like edges. It is an appropriate basis for representing images. Since edges are an integral part of images and they are usually not straight lines Curvelet allows an almost optimal non-adaptive sparse representation of objects with edges. Curvelet transform consist representation of an image using a series of energy measurement ranging across scale, orientation and position [1-6].

Curvelet can be defined as a function

$$\gamma_{j,l,k}(x_1, x_2) = 2^{\left(\frac{3j}{2}\right)}.\gamma(D_j R_{\theta_l}(x_1, x_2) - k_\delta \qquad (2)$$

Where $j = 0,1,2 \ldots\ldots$ is a scale parameter,
$k = (k_1, k_2) \in Z$ is a translation parameter,
$l = 0,1,2^j \ldots\ldots$ is an orientation parameter

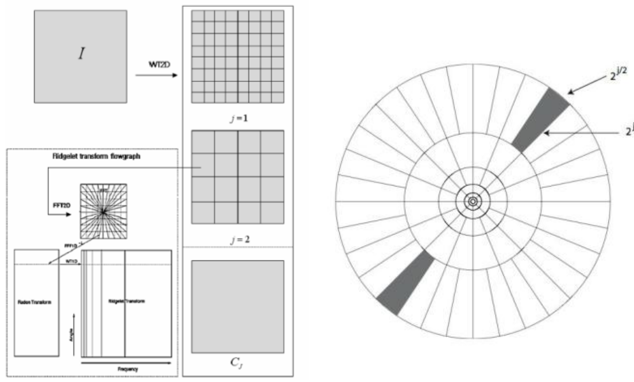

Fig1. (a) curvelet flow graph, (b) Frequency tiling

On images with boundaries, non optimal systems have the following rates

$\|f - f_m^W\|_{L_2}^2 \approx O(m^{-1})$ Wavelet approximation

$\|f - f_m^C\|_{L_2}^2 \approx O((logm)^3)m^{-2})$ Curvelet approximation

As seen from the m-term approximations, the curvelet Transform offers the closest m-term approximation to the lower bound. Therefore, in images with a large number of curve (i.e. an image with a great number of long edges); it would be found an advantageous to use the curvelet algorithm.

Edge detection using curvelet transform

In medical imaging, detecting and enhancing boundaries between different cavities is of prime importance. Neuroscientists have identified edge processing neurons in the earliest and most fundamental stages of the processing. Thus, in medical imaging, detecting and enhancing boundaries between different cavities is of prime importance. In an image, the presence of an edge feature is usually accompanied by a drastic change of the pixels grey value. As a result, edge features often appear at the higher frequency band in the frequency domain. Curvelet is an edge based multi-scale representation of the image, with the edge information corresponding to larger curvelet coefficients. Consequently, the edge information can be represented with the larger coefficients in the high frequency band. The highest band, however, usually corresponds to the frequency response of isolated points and noise; thus the frequency response of the edge features should be located at the sub-highest frequency band, as shown in following figure.

Vanishing Point Detection

Vanishing points are elements of great interest in the image and computer vision, since they are the main source of information about the geometry of the image and the projection. First an edge detection algorithm is performed on the original image through an isotropic operator which is composed of the following two masks[30,31].

$$D_x = \begin{bmatrix} -1 & 0 & 1 \\ -\sqrt{2} & 0 & \sqrt{2} \\ -1 & 0 & 1 \end{bmatrix} \quad (3)$$

$$D_y = \begin{bmatrix} 1 & \sqrt{2} & 1 \\ 0 & 0 & 0 \\ -1 & -\sqrt{2} & 1 \end{bmatrix} \quad (4)$$

The result image is then normalized and threshold against a very high value, to eliminate redundant information. This way, a binary image is obtained. For each point P within it, the tangent is calculated, by means of the following expression

$$m = tg\theta = \frac{|D_x|}{|D_y|} \quad (5)$$

where D_x and D_y are the x and y components resulting from the application of the isotropic operator in the point. Now that θ is known, it can be used to draw a straight line, with slope m, passing through P. Such line is then accumulated in the (x, y) parameter space. At the end of the process, those points having the greatest numbers of votes will be marked as candidates. Images of both indoor and outdoor scenes are often full of (nearly) vertical and horizontal lines. Since, for the most part, they would identify vanishing points at the infinite (which are not useful for our investigation), we introduce the following constraint:

$$\frac{1}{p} < |tg\theta| < q$$

where p and q have values greater than one and are chosen according to the particular needs (for example, p= q= 64) [30,31]

Algorithm

In this paper a new algorithm is proposed where after acquisition of MR image of brain detection of edges were performed using Sobel or Canny then apply curvelet transform and do the vanishing point detection finally apply inverse curvelet transform to get resultant image which is de-noised MR image of the brain . It is as follows

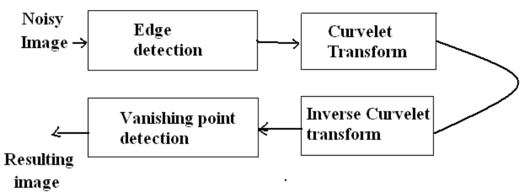

Fig 2. Algorithm for denoising of MRI images.

Edge detection techniques (sobel, canny etc.) returns a low threshold of the edges in the image. Edge thinning algorithm eliminates areas of high density in the edge image. Curvelet algorithm works well at picking out edges surrounded by noise. Using the curvelet algorithm, it is easy to remove any strictly horizontal or vertical edge (as these would not help in determining vanishing points), returning edges are only at the finest scale (representing long, narrow edges), so the N largest coefficients which returns the N most dominant edges in the image is a series of edges that represent a collection of edges that can be vanish.

2. Methodology

First take image which is to be de-noised find dominant line segment where we do edge detection then thinning of the edge after that apply curvelet transform on edges in the image (find lines) then corners of line segment then find line segments (find line segments) then mean vanishing point of line segment where average intersection point will be selected then check the distance from two lines find the mean of the intersection point and then cluster the intersection point finally apply de-noised algorithm to the image with determined vanishing point. All programmes are written in MAT LAB.

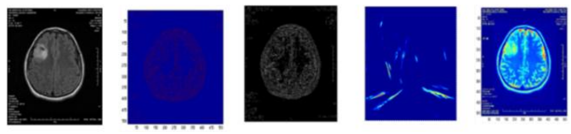

Fig. 3. Methodology Steps.

3. Result

In this study we used brain CT images acquired from the images from website. Each image was of 512 × 512 in size. All the images are corrupted by additive Gaussian noise with standard deviation as 10, and 20. We examine all the denoising methods with different quality performance parameters not only for noise suppression but also for edge preservation these parameters are as follows:

$$MSE = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}\left(x_{i.j} - y_{i.j}\right)^2 \quad (6)$$

$$PSNR = 20log\left(255/\sqrt{MSE}\right) \quad (7)$$

$$EKI = \frac{\sum_{i=1}^{M}(\Delta x_i - \Delta\mu_x)(\Delta y_i - \Delta\mu_y)}{\sqrt{\sum_{i=1}^{N}(\Delta x_i - \Delta\mu_x)^2 \sum_{i=1}^{N}(\Delta y_i - \Delta\mu_y)^2}} \quad (8)$$

where x_i and y_i are original and reconstructed images respectively. Δx and Δy are found by filtering x and y through high pass Laplacian filter with mean value as μ_x and μ_y respectively.

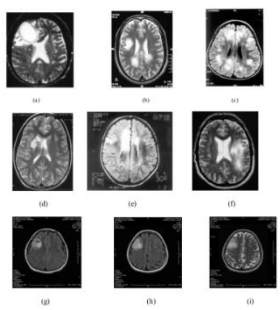

Fig. 4. Test Images

(images taken from www.surgicalneurologyint.com, www.streamlyner.com and www.cpmc.org)

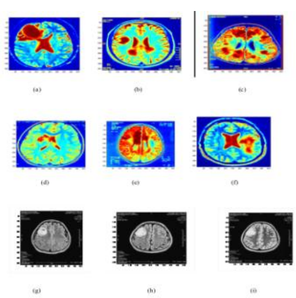

Fig.5. Result Images.

Performance Evaluation	Wavelet Soft - threshold	Wavelet hard- threshold	Curvelet transform	Curvelet transform (New algorithm of curvelet with edge detection and Vanishing point detection)
Fig:5 (a)				
Ω=10				
PSNR	31.32	32.84	34.09	37.78
MSE	21.78	17.45	15.23	12.62
EKI	.773	.810	.880	.948
Ω=20				
PSNR	27.98	29.67	30.75	31.85
MSE	60.96	54.12	49.64	33.07
EKI	.523	.542	.798	.881
Fig:5(b)				
Ω=10				
PSNR	29.36	30.72	32.34	34.47
MSE	19.94	15.55	13.87	10.71
EKI	.586	.621	.734	.809
Ω=20				
PSNR	26.87	27.32	29.56	30.91
MSE	59.85	57.83	43.65	32.10
EKI	.567	.591	.675	.760
Fig:5(c)				
Ω=10				
PSNR	32.41	33.54	36.07	38.60
MSE	20.45	19.20	15.89	11.91
EKI	.809	.739	.879	.998
Ω=20				
PSNR	29.34	30.11	31.08	33.85
MSE	58.75	57.02	53.67	30.78
EKI	.691	.729	.798	.952
Fig:5 (d)				
Ω=10				
PSNR	34.32	34.79	35.76	38.65
MSE	20.90	19.23	16.23	11.02
EKI	.892	.914	.914	1.23
Ω=20				
PSNR	28.34	28.86	30.79	33.15
MSE	58.78	58.18	51.65	30.07
EKI	.721	.792	.860	.981
Fig:5(e)				
Ω=10				
PSNR	30.21	31.73	33.73	36.67
MSE	20.68	16.36	12.36	10.89
EKI	.721	.825	.943	1.69
Ω=20				
PSNR	26.80	27.97	29.57	31.76
MSE	55.81	54.23	48.23	34.54
EKI	.677	.710	.792	.892
Fig:5(f)				
Ω=10				
PSNR	27.92	29.06	32.78	35.61
MSE	19.53	17.45	15.45	10.59
EKI	.642	.696	.812	.991
Ω=20				
PSNR	23.74	25.56	28.56	30.93
MSE	59.94	57.72	56.72	43.07

EKI	.541	.637	.765	.874

Fig:5(g)				
Ω=10				
PSNR	28.31	29.63	30.77	35.92
MSE	20.65	19.24	16.38	12.12
EKI	.703	.791	.804	.990
Ω=20				
PSNR	24.98	25.28	27.55	31.09
MSE	59.96	58.34	53.22	32.86
EKI	.681	.703	.782	.802
Fig:5(h)				
Ω=10				
PSNR	30.21	30.95	31.95	36.90
MSE	21.62	20.23	18.23	13.83
EKI	.669	.686	.776	.849
Ω=20				
PSNR	25.08	26.55	28.21	30.79
MSE	62.96	60.27	56.43	31.87
EKI	.598	.602	.647	.799
Fig:5(i)				
Ω=10				
PSNR	33.41	33.84	35.73	37.78
MSE	23.67	18.45	16.59	14.56
EKI	.679	.660	.741	.805
Ω=20				
PSNR	28.77	28.04	29.06	30.13
MSE	60.08	58.12	55.12	31.15
EKI	.598	.697	.732	.873

In this section we analyzed the performance of different denoising methods on brain CT images with Gaussian noise of standard deviation 10 and 20. Table 1 shows the vales of different quality assessment parameters like MSE and PSNR for noise suppression and EKI for edge preservation It can be observed from the table1 that CS_CT method outperforms all other methods in terms of both noise suppression and edge preservation. Further for low noise (Std. dev. 10) TI_WT method gives better results as compared to both Hard_CT as well as Hard_WT. As the noise in the image increase (Std. dev. 20), noise suppression capability of Hard_CT becomes better but still TI_WT is better in case of edge preservation.

4. Conclusion

It can be observed from the above table that new method outperforms all other methods in terms of both noise suppression and edge preservation. Further for low noise (Std. dev. 10) new method gives better results as compared to Soft_WT, Hard_WT as well as curvelet transform. As the noise in the image increase (Std. dev. 20), noise suppression capability of Hard_WT becomes better but still the proposed algorithm is better in case of edge preservation and other performances.

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