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Performance Analysis of Robust Controllers for Inertially Stabilized Platform

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Abstract
In this paper, direct line of sight (LOS) stabilization of a two axis gimbaled system has been analysed with different disturbances. It also includes modeling, simulation and control designing aspects of gimbal. Robust control algorithms using PI(proportional integral) & LQG(linear quadratic gaussian)/LTR(loop transfer recovery) have been designed & simulated for inertially stabilized platform. A comparison has been made between proportional Integral controller(PI) and LQG/LTR controllers in terms of frequency & time domain.

Keywords: Gimbal, LOS, LQG/LTR.

Introduction
Inertially stabilized platforms (ISPs) are widely used to stabilize the line of sight of the gimbaled payload mounted on a moving vehicle from satellites to submarines, and are even used on some handheld and ground-mounted, airplanes, helicopters and Unmanned Air Vehicles (UAVs). LOS stabilization systems has got the ability to maintain the LOS of a sensor or instrument when it is subjected to external disturbances i.e..mass imbalance, cable restraint, viscous friction, air friction etc. The several disturbance torques are included in the two axis gimbal model. The performance of the system is analysed with two controllers i.e PI & LQG/LTR. The PI controller is designed for the stabilization system. LQG/LTR controller is also presented in the present investigation to improve robustness. The control system meets its performance & stability objectives in the presence of disturbances & uncertainties, the system is called robust. In this paper a robust control techniques such as linear quadratic Gaussian with loop transfer recovery (LQG/LTR) have been used in the line of sight stabilization for land vehicle.

System Description
ISPs usually consist of an assembly of structure, bearings, and motors called a gimbal to which a gyroscope, or a set of gyroscopes, is mounted. The sensor or payload to be stabilized is mounted directly on the gimbal assembly in some configurations, while in others, mirrors or other optical elements are mounted to the gimbal, and the sensor is fixed to the vehicle. Typically, the gimbal must be designed to point and stabilize about two or more axes, and, therefore, most applications require at least two orthogonal gimbals. However, more than two gimbals are often required to provide additional degrees of freedom or to achieve better isolation from the host vehicle. Gimbal-sensor assemblies can range in weight from under a pound to several tons, and, although the weight of an ISP depends primarily on the size of the payload being controlled, the size and weight of the ISP also increase dramatically as additional gimbal axes are added.
Although requirements for ISPs vary widely depending on the application, they all have a common goal, which is to hold or control the line of sight (LOS) of one object relative to another object or inertial space. The LOS can be the aim point of a beam or weapon, the center of the field of view (FOV) of a telescope, or the direction a sensor is pointed. A typical scenario is shown in Figure 2 to illustrate the basic concepts. Much of the terminology surrounding ISP technology has evolved over the years.

Dynamics of gimbal kinematics
One can consider three co-ordinate frames i.e. base co-ordinate frame, outer co-ordinate frame & inner co-ordinate frame. LOS elevation control is proposed through the inner gimbal y-Axis & Azimuth control is via the outer gimbal Z-Axis. In this case stabilization about the inner gimbal Z-Axis (Cross elevation axis) is required. On the basis of kinematics, the gimbal dynamics is divided into two dynamics, namely inner gimbal & outer gimbal dynamics. The relation between the two gimbals frames i.e. base and outer gimbal frame is expressed by the rotational transformation matrix, $R\theta_{BO}$ (given by Euler's rotational matrix).

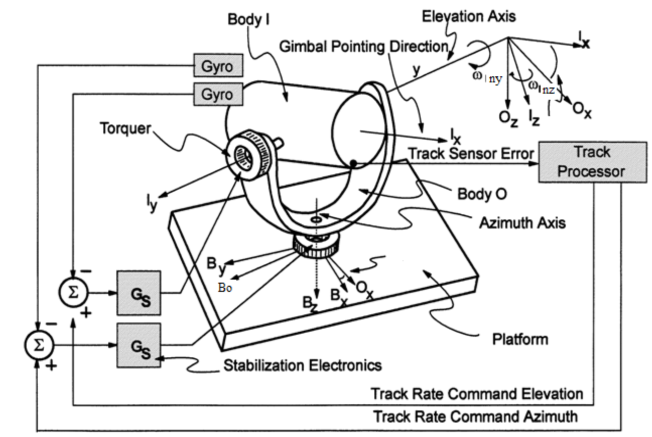

Fig.1 Inertially Stablized Platform (ISP)

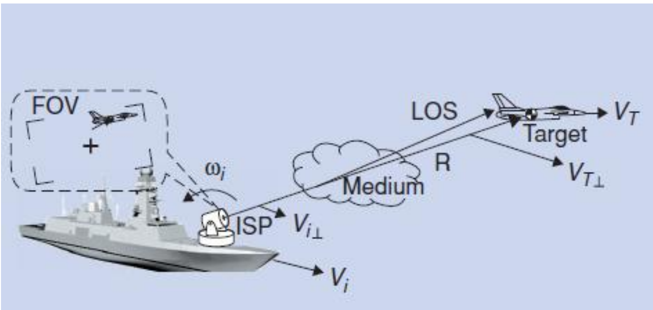

Fig. 2 ISP with LOS

$$R\theta_{BO} = \begin{bmatrix} Cos\theta_{BO} & Sin\theta_{BO} & 0 \\ -Sin\theta_{BO} & Cos\theta_{BO} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Relationship between outer gimbal angular velocity vector & base vector can be expressed as

$$\omega_o(t)=R\theta_{BO}(t)\omega_B(t)+\dot{n}\,(t) \quad (1)$$

For the outer and inner gimbal frame the rotational transformation matrix is written as

$$R\theta_{OI} = \begin{bmatrix} Cos\theta_{OI} & 0 & -Sin\theta_{OI} \\ 0 & 1 & 0 \\ Sin\theta_{OI} & 0 & Cos\theta_{OI} \end{bmatrix}$$

Relationship between inner Gimbal angular velocity vector & outer vector is written as

$$\omega_I(t)=R\theta_{OI}(t)\omega_o(t)+\dot{s}\,(t) \quad (2)$$

The relationship between angular acceleration of the inner & outer gimbal is written as

$$\dot{w_{In}} = R\theta_{OI} w_{out} + \dot{w_{out}}\, R_{Com}\, \dot{\theta_{OI}} + \ddot{\square}_{OI} \quad (3)$$

$$R_{Com} = \begin{bmatrix} -Sin\theta_{OI} & 0 & -Cos\theta_{OI} \\ 0 & 0 & 0 \\ cos\theta_{OI} & 0 & -Sin\theta_{OI} \end{bmatrix}$$

a) Inner gimbal dynamics

The goal is to control the elevation axis and cross elevation axis of inner gimbal. Since the direct control of cross elevation axis is not possible so it is controlled by outer gimbal z-axis. Inner gimbal dynamics will be calculated by the Euler's moment equations. The sum of kinematics torques about the inner gimbal is given by (As mentioned in fig.: gimbal structure).

$$M_{Ix} = T_{IX}(t) - T_{uIx}(t)$$

$$M_{Iy} = T_{elv}(t) - T_{uIy}(t) - T_{Ifr}$$

$$M_{Iz} = T_{Iz}(t) - T_{uIz}(t)$$

Friction and cable restraint torque generate the linear and nonlinear disturbances. Gimbal dynamic equations can be expressed as

$$T_{IX}(t) = I_{Ix}\dot{\omega_{Inx}} + \omega_{Iny}\omega_{Inz}(I_{Iz} - I_{Iy}) + T_{uIx} \quad (4)$$

$$I_{Iy}\dot{\omega}_{Iny} = -\omega_{Inz}\omega_{Inx}(I_{Ix} - I_{Iz}) + T_{elv} - T_{Ifric} - T_{uIy} - K_{Iw}\theta_{OI} - K_{If}\,\dot{\theta}OI \quad (5)$$

$$T_{Iz}(t) = I_{Iz}\dot{\omega_{Inz}} + \omega_{Inx}\omega_{Iny}(I_{Iy} - I_{Ix}) + T_{uIz} \quad (6)$$

Elevation control is done by inner gimbal y-Axis. The dynamics of y-Axis is given by equation (5). The inputs for this equation are the base rates and base acceleration. wIny & wInz are the controlled variables. Elevation axis dynamics in terms of base inputs is expressed as

$$I_{Iy}\dot{W}_{Iny} + K_{If}\,\omega_{Iny} + K_{Iw}\int_0^t \omega Iny\,(\tau)d\tau + \omega_{Inz} \left(\frac{\omega onx - sin\theta_{OI}\,\omega Iny}{cos\theta_{OI}}\right)(I_{Ix} - I_{Iz})$$

$$= T_{Irate} + T_{Ibase} - T_{UIy} - T_{Ifric} - T_{Icr} + K_{If}(Cos\theta_{BO}\,\omega_{Bny} - Sin\theta_{BO}\,\omega_{Bnx}) + K_{Iw}\int_0^t (Cos\theta_{BO}\,\omega_{Bny}(\tau) - Sin\theta_{BO}\,\omega_{Bnx}(\tau))\,d\tau \quad (7)$$

b) Outer Gimbal dynamics

Azimuth control is done by outer gimbal z-Axis. The outer gimbal dynamics is expressed as (as mentioned in gimbal structure fig.)

$$M_{ox} = T_{oX}(t) - T_{uox}(t)$$

$$M_{oy} = T_{oy}(t) - T_{uoy}(t)$$

$$M_{oz} = T_{az}(t) - T_{uoz}(t) - T_{ofr}$$

Rigid body torque dynamics equations are expressed as

$$Io\dot{\omega}o + (\omega o(t) \times Io\omega o(t)) + (M_I)o = Mo(t) \quad (8)$$

$$(M_I)o = R_{\theta OI}[(I_I\dot{\omega}_I + (\omega_I(t) \times I_I\omega_I(t))] \quad (9)$$

The resultant expression in terms of cross elevation (inner body z-Axis) is as

$Cos\theta_{OI}(I_{oz}+ \ I_{Iz})\,\dot{\omega}_{Inz}= sin\theta_{OI}(I_{oz}+ \ I_{Iox})\,\dot{\omega}_{Inx} - [\omega_o \times I_o \ \omega_o - R^T\theta_{OI}(t)]_3+[Io\ R^T\theta_{OI}(t)R_{com}\ \omega_o\ \dot{\theta}_{OI}]_3+[M_o]_3$

Overall kinematics in terms of angular acceleration

$I_T\dot{\omega}_{Inz} = sin\theta_{OI}(I_{oz}+ I_{Ix})\,\dot{\omega}_{onx} + (I_{oz}\ \omega_{inx} - sin\theta_{OI}\ I_{Ix}\ \omega_{onz})\,\dot{\theta}_{OI} - cos\theta_{OI}[\omega_{onx}\ \omega_{ony}(I_{oy} - I_{ox}) - sin\theta_{OI}\omega_{Iny}\ \omega_{Inz}\ I_{Iz} + \omega_{Iny}\ \omega_{onx}\ I_{Iy} - cos\theta_{OI}\ \omega_{Inx}\ \omega_{Iny}\ I_{Ix}] + cos\theta_{OI}[T_{az} - T_{uoz} - T_{ofric} - T_{ocr}] - cos\theta_{OI}\ K_{of}\ \dot{\theta}_{Bo} - cos\theta_{OI}K_{ow}\,\theta_{Bo}$

c) Plant dynamics

On the basis of IRDE-DRDO(Instrumental research & development establishment) lab data of different system parameters, the dynamics of the system i.e motor, gimbal & gyro can de expressed as

$\ddot{V} = -879.65\dot{V} - 394784.17V + 2258165.5Ra$ or $\ddot{V} = -879.65\dot{V} - 394784.17V + 2258165.5\,R(t) + 2258165.5d(t)$ (10)

$\dot{R} = -172.30R(t)+3.1T(t)$ (11)

$\ddot{T} = 2842446.06\,I(t) - 188.49\,\dot{T}(t) - 3553057.58\,T(t)$ (12)

$\dot{I} = -2400I(t)-160R(t)+200u(t)$ (13)

The state space representation of the plant with matrices A,B,C,D and E is

Design & Development Of Robust Controller

Designing of Linear Quadratic Regulator:

Linear quadratic regulator (LQR) system is dealing with state regulation, output regulation, tracking and also design of optimal linear systems with quadratic performance index. The optimal state feedback gain matrix for the augmented plant is determined in such a way that the performance and stability objectives of Kalman Bucy Filter loop transfer function $G_{bf}(s)$ are retained at the output of the plant. This is referred as loop transfer recovery(LTR). The quadratic performance index to be minimized as

$$J = \int_0^{\infty} (X^T a(t)\ Qc\ Xa(t) + U^T a(t)\ Rc\ Ua(t))dt$$

Control law for LQR, $K_{ca} = R_c^{-1} B_a^T P_c$

Symmetric and positive definite solution of the Control Ricaati Equation (CRE) is given as

$A_a^T\ P_c + P_c\ A_a - P_c B_a R_c^{-1} B_a^T Pc + Qc = 0$; $Q_c = q\ C_a^T C_a$, q>0 and $R_c = \alpha\ I$, $\alpha > 0$

The scalar design parameters are q and α. The overall open loop transfer function G_{OL} is given by

$G_{OL}(s) = G_a(s).\ K_{LQG/LTR}(s)$

$K_{LQG/LTR}(s) = K_{ca}[\ sI - A_a + B_a.K_{ca} + K_{bf}.C_a\ ]^{-1}.K_{bf}$

Various values of the design parameters q and α are to be tried so that open loop transfer function $G_{OL}(s)$ approaches the Kalman-Bucy filter loop transfer function $G_{bf}(s)$ point wise in s.

The closed loop configuration of the system can be expressed by the following state space equations.

$G_{CL}(s) = C_{CL}\ (sI - A_{CL})-1\ B_{CL}$

Where $A_{CL} = \begin{bmatrix} A_a & -B_aK_{ca} \\ K_{bf}C_a & A_a - B_aK_{ca} - K_{bf}C_a \end{bmatrix}$

$B_{CL} = \begin{bmatrix} 0 \\ -K_{bf} \end{bmatrix}$ and $C_{CL} = [C_a \quad 0]$

The closed loop transfer function from the disturbance input d(t) to the output y(t) is given by

$G_D(s) = C_{CL}\ (sI - A_{CL})^{-1}\ B_D$; $B_D = \begin{bmatrix} Ea \\ 0 \end{bmatrix}$

The closed loop responses for the command inputs and the disturbance inputs has been developed and simulated using the above equations, shown in fig 4.

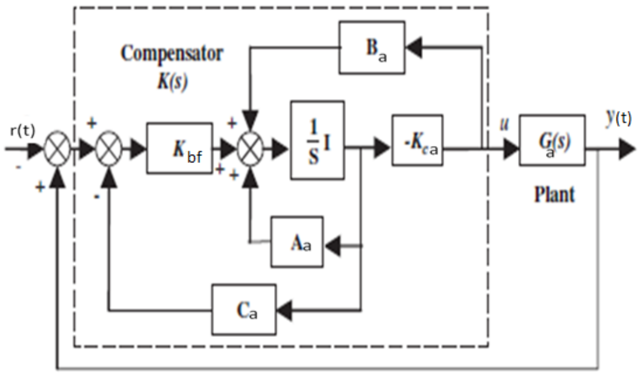

Fig. 3 LQG/LTR control loop

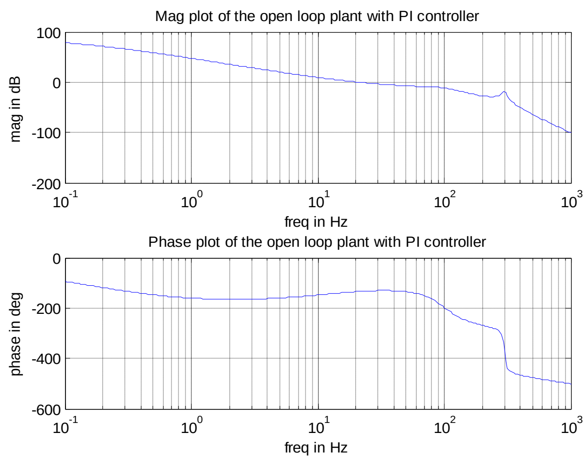

Fig.4 Open loop bode response with PI controller

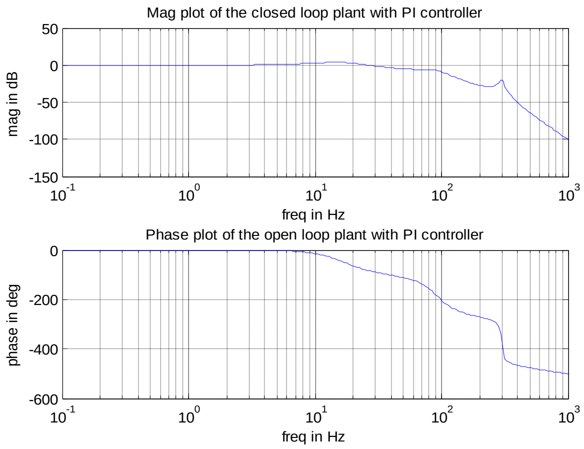

Fig.5 closed loop bode response with PI controller

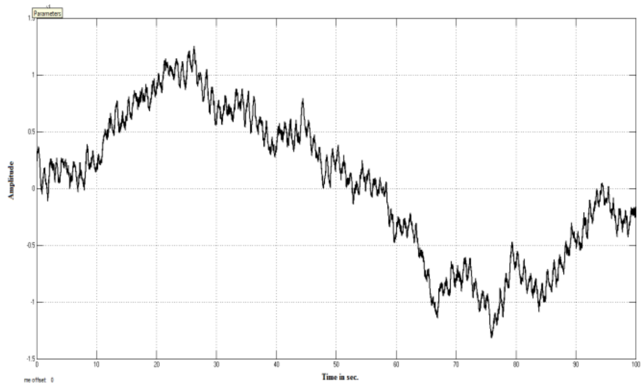

Fig. 6 LOS jitter with PI controller

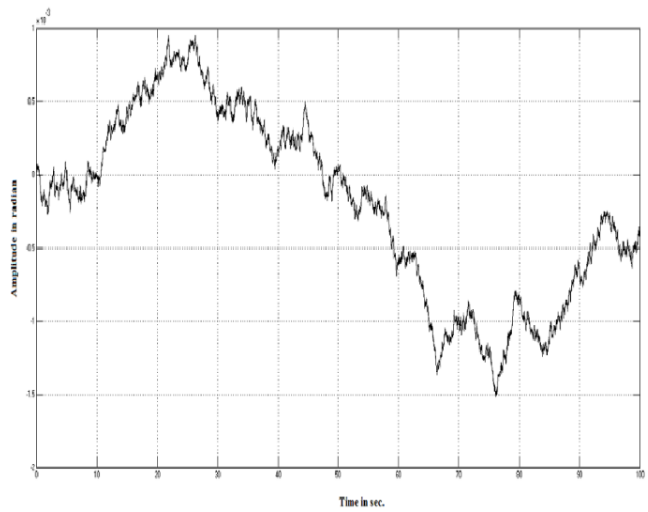

Fig.7 LOS jitter with LQG/LTR controller

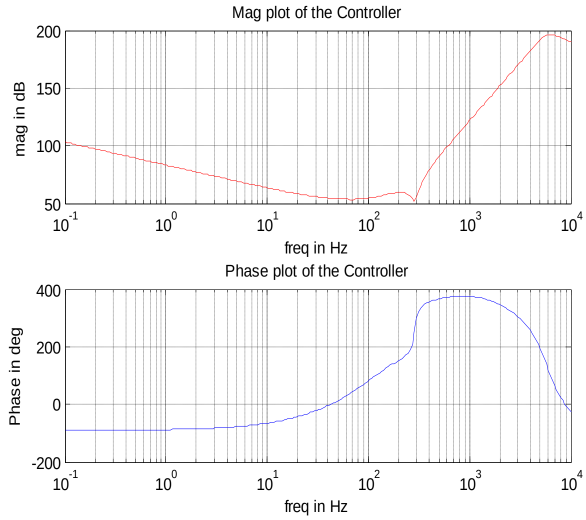

Fig.8 Bode plot with LQG/LTR controller

Result & Analysis

LOS jitter and LOS rate responses with LQG/LTR controller are shown in fig.4.5 &4.6 respectively.The standard deviation of LOS jitter is of the order of 13µrad in case of LQG/LTR controller and improves the performance of the system.

Conclusion

In the proposed work, one can design & develop robust models for two axis gimbal with PI controller & LQG/LTR controller for inert ally stabilized platform. The comparison has been made in terms of performance & stability bjectives of the system. The comparative analysis, shown in above table proves that the LQG/LTR controller gives the better performance & stability objectives compare to PI controller.
Figs 3.6 & 4.7 show the decoupling performances by the stabilization system with respect to body motion at 1Hz.The gain at 1Hz is 47.36 dB and 67.46 dB for the PI and LQG/LTR respectively.
The comparative analysis (frequency & time response analysis) is shown in the following table.

TABLE-1 performance analysis

Parameters	**PI controller**	**LQG/LTR controller**
GM(dB)	10.28	21.8
PM(deg)	46	58
Bandwidth	37	118.1
Los jitter(Std. Deviation)	22.15µrad	13 µrad
Peak overshoot	11.5%	7%

References

[1] Hilkert J. M., "Inertially stabilised platform technology", IEEE control system magazine,

Feb.2008, pp 26-46.

[2] Ho-Pyeong Lee and Inn-Eark Yoo, "Robust Control Design for a Two-axis Gimbaled Stabilization System", IEEE AC paper # 1010, Version 3, Updated December 31, 2007.

[3] Leghmizi, Liu Sheng, Maeim farouk, "Modeling design and control of a ship carried 3-DOF stabilized platform", Research Journal of applied sciences, Engineering and Technology 4(19): 3843- 3851, 2012 ISSN: 2040-7467.

[4] Martin Rezac and Zdenek Hurak, "Vibration rejection for inertially stabilized double gimbal platform using acceleration feed forward", IEEE International conference on control Applications (CCA), 978-1-4577-1063-6/11/$26.00 2011.

[5] Masten M.K. et al, "Inertially stabilised platforms for optical imaging systems", IEEE control system magazine, Feb.2008, pp 47-64.

[6] Mokbel et al, "Design optimization of the inner gimbal for dual axis inertially stabilised platform using finite element model analysis", International journal of modern engg. Research, vol-2, issue-2, Mar-April 2012, pp239-44.

[7] P. Apkarian, D. Noll (2006). "Non-smooth H infinity synthesis". IEEE TAC, AC-51:71-86.

[8] Roshdy A. A et al, "Design a robust PI controller for LOS stabilization system", International journal of Modern Engg. Research, vol-2, issue-2, Mar-April 2012, PP144-48.

[9] Steven T. Jenkins and Hilkert, "Line of sight stabilization using image motion compensation, Texas instruments Incorporated", Control system technology center, SPIE vol.1111 Acquisition, Tracking and pointing III (1989*).*