

A Technical Comprehensive Survey For User Input Models For Mathematical Expression

¹Sachin Naik and ^{*2}Pravin Metkewar

¹*Assistant Professor, Symbiosis Institute of Computer Studies and Research (SICSR),
Affiliated to Symbiosis International University (SIU), Atur Centre,
Ghokhale Cross Road, Model Colony, PUNE-411016, Maharashtra State, India
Email: sachin.naik@sicsr.ac.in(1)*

^{*2}*Associate Professor, Symbiosis Institute of Computer Studies and Research (SICSR),
Affiliated to Symbiosis International University (SIU), Atur Centre,
Ghokhale Cross Road, Model Colony, PUNE-411016, Maharashtra State, India
Email: pravin.metkewar@sicsr.ac.in(2)*

Abstract

Mathematical Expression (ME) is an input to the computers is always a complex task. The variety of mathematical expressions which includes factoring formulas, Areas, Quadratic Equations, Arithmetic Progression, Geometric Progression, Elementary Algebra, Number Theory, Trigonometric Functions, Geometry, Differential Calculus, Integration etc. are the difficult one to provide input as well as to recognize the expression. In order to recognize any mathematical expression, first we need to perform pre-processing, segmentation, feature extraction and classification. Most of the user inputs models are developed for online handwritten mathematical expressions recognition but very few models were proposed for offline handwritten mathematical expression recognition. In this paper we have carried technical survey of existing user input models and benchmarking of these models has been performed by identifying its corresponding important factors. Finally, user input model problems and its challenges have been discussed thoroughly and its state of art is summarised.

Keywords: User input models, ME recognition, command based, string based, pen-based, structural analysis.

Introduction

Computer systems and handheld devices are widely used in our day to day life. Mathematical expression as an input to computers is a difficult and time consuming because of large set of mathematical symbols. There is a need to digitize mathematical

expressions into computer systems which are available in the scientific papers, reports, journals, library books, education systems etc. A variety of tools evolved to recognize and digitize the mathematical expressions. In this paper we have conducted a comparative study for variety of user input models for ME.

A mathematical expression can be given to computer in terms of offline and online way. Offline means the expression written on paper and scanned expression given to computer for the purpose of recognition. Online refers to drawing mathematical symbol through GUI and it is provided using pen or keyboard device.

Types of input Models

There are three types of input models have been observed in our technical comprehensive survey. They are command-string based, template based and pen based. Their corresponding literatures with respect to problems and challenges have been discussed thoroughly and independently.

Command-String Based

MathJournal is the first commercial application for doing mathematics on Tablet PC. It interprets diagrammatic and graphical representation. It has limited mathematical and problem solving capabilities. User depends on menus to perform manipulations [21].

XPRESS a novice interface for the real time communication of mathematical expressions proposed which is a open source AJAX/SVG web based users interface for expression entry by novices, called XPRESS (Transformation of Pictorial Representation of Expression Spatial Structure), using this user can quickly layout the mathematical expression which is converted using TEX spatial analysis algorithm. This is browser based front end and graphics server. The usability of the interface will be studied more extensively in a first-year online calculus course in the next academic year as both an input method for online office hours and for inputting expressions in online assignments and tests. [13, 3].

INFTY Editor is an integrated OCR system for mathematical documents based on four procedures these are layout analysis, character recognition and structure analysis of mathematical expressions, and manual errors. In those procedures, several novel techniques are utilized for better recognition performance. It is mathematical document digitization software which is used read mathematical documents and converts it into XML. INFTY requires about 10 s/page to perform all procedures (except for the manual error correction procedure) on a desk top PC (Pentium III, 1GHz) and about 20 s/page on a notebook PC (Pentium III, 700MHz). The manual error correction result showed that the operations could be reduced to about 1/3 by the automatic error correction using the result of the clustering. [4].

Diagram Recognition Application for Computer Understanding of Large Algebraic Expressions (DRACULAE) takes online input, contains user interface and a third party symbol recognizer provided by Freehand Formula Entry System (FEES). It deals with irregular symbol placements, alternative approaches and ambiguous layout, graph transformation [22].

EzMath is developed by Dave Raggett and Davy Batsalle, is a mathematical markup languages used to embed mathematical expressions in Web pages. It focus on semantics of mathematical notation .The structure of mathematical expression will be stored by syntax of the markup languages [23].

MathWebSearch 0.4 is a semantic search engine for mathematics, in this, Queries are constructed via keyboard and templates and Symbol types can be constrained. It has capability of indexing mathematical content by using formula structure.It is a scalable search engine for mathematical formulae over internet[9,10].

Template-based

InftyEditor is software which is used to recognize mathematical expressions. It is developed in Suzuki Laboratory. This software recognizes scanned images and outputs the results in IML, LaTeX, HR-TeX, XHTML and MS Word 2007 format. It has a powerful OCR engine which can recognize tables which consists of mathematical expressions [24].

Mathematica is software for doing mathematics. It support numeric and symbolic work and has word processing capabilities. In this searching,calculating,performing complex graphics are much simpler. It supports data, functions, graphics, and programs are represented in a uniform way as a symbolic expression. The mathematica notebook can be linked using mathlink to external program like Fortran 90 and results are captured using mathelink again to mathematica notebook [25].

Maplesofts maple is a powerful tool for mathematics. It consists of interface can analyse, explore,visualize and solve mathematical expressions.Maple 2015 has more than 5000 functions which can handle any type of mathematical expression. It automatically captures technical knowledge which consists of calculations, explanatory text and math, graphics, sound and diagrams. Math Equation Editor where we can express complicated mathematical equations and has over 1000 symbols[25].

LiveMath software allows to create interactive web pages on which the users , students can make changes to variables and functions and see results of these mathematical functions immediately. It consists of palate, interactive window where user can do dragging and dropping. It supports basic graphing, quadratic formula, square formula, translations and reflections, different quotient formulas, Product rule, and Linear transformation[26].

MathType is a intelligent mathematical equation editor designed for PC using Window of Apple MacOS. It supports for creating complex mathematical equations and use them into various documents. It consists over 175 templates which include fractions,radicals,sum,integrals, products,matrices etc. It supports over 214 mathematical symbols and intelligent in understanding mathematics and converting it into TeX,LaTeX, MathML or other textual languages as well as equations can be saved in WMF, EPS and GIF files [27].

Pen-Based

A Freehand Formula Entry system (FEES) was introduced by Steve Smithies (1999) as a pen based editor, implemented in C++ and Tcl/Tk. It is distributed under GNU

Public Licence. This supports entry and editing of mathematical expressions using pen and tablet. This support automatic interpretation of errors and recognized expressions are interpretation into LaTeX. It is based on existing character recognition and formula parsing technologies. The time taken to enter, edit, parse and preview the five expressions ranged from 291 to 503 seconds. An overall recognition rate of 77% was achieved for the 583 characters entered by all the test users. For a serious user of the system, taking the time to train the character recogniser is necessary. Once properly trained; they have found that the character recogniser offers a recognition rate of over 95%. The future work is refinement, optimization, faster and reliable parsing techniques and possibility of parsing incomplete expressions. [1, 2, 14]

MathBrush a system for doing math on pen based devices on a table computer, recognizes math expressions and support for mathematical transformation and problem solving using back-end computer Algebra Systems (CAS). Which includes modules for symbol recognition, semantic analysis and transformation to back end CAS and interface techniques for interacting with CAS output. The main system modules that make up MathBrush consist of a user interface, a Character Recognizer (CR), a Structural Analyzer (SA) that interacts with a Matrix Analyzer (MA), a CAS interface tool, and finally a mathematics rendering tool. MathBrush has been designed both to do math and to evaluate different recognition algorithms and semantic analysis approaches. MathBrush contains several other features of value during mathematical problem solving. These include plotting, logging features, and the ability to swap CAS. [5]. An Electronic chalkboard (E-Chalk) system is a online multimedia system for distance teaching. It includes algebraic formula manipulation and function plotting. SVM is used for classification of MEs. This model contains four symbols classes: Limit (sum, product or integral) square root, non-scripted (operators, open brackets and horizontal line) and plain (all other symbols). The formulae recognition process based on ordering, filtering, data reduction, smoothing, equation resampling, scaling and segmentation. A feature vector is constructed over the sequence of strokes and local features called coordinates, Turing angle, length position, enter of gravity, relative length, and accumulated angle. These features are integrated at the classification stage. This system uses Directed Acyclic Graphs SVM DAG-SVM as multi-class SVM classifier. The modification one of the Sequential Minimal Optimization (SMO) algorithm was used to train the classification nodes with the use of RBF kernel. [4, 6, 7].

Mathpad2 is a system for the creation and exploration of mathematical sketches is a novel, pen-based modeless gestural interaction paradigm for mathematical problem solving [7].

MathBox uses interactively showed boxes in which user can write symbol. It is a pen based interface for inputting mathematical expressions. It reduces the troublesome task of inputting mathematical expressions with mouse and keyboard. It skips the recognition of structures of mathematical expressions [15].

AlgoSketch is a pen based application for mathematics for entering and editing mathematical expressions. It is a paper like environment Mathematical expressions can be entered anywhere on the page for which recognition and computation feedback is given real-time (16).

LipiToolKit (LipiTk) is a general toolkit used for online handwriting recognition engines. It provides robust implementation of tools, algorithms, scripts and sample code necessary to support the activities of handwriting data collection, training and evaluation of recognizers. It focuses of isolated online handwritten shape and character recognition [18].

JMathNotes is a recognition system uses multi-class support vector machine (DAG-SVM) classifier using radial basis function (RBF) kernel. This system is written using java under Gnu public Licence. It has dependencies and it can run on windows or UNIX environment. The accuracy rate is not available for JMathNotes[6].

The Microsoft TablePC Recognizer is widely used commercial handwriting recognition engine. It uses very large database collected from variety of writers and a heuristic to resolve ambiguities between characters. It is designed for text input rather than mathematical symbols and notation. It uses neural network to identify input at word level. The accuracy rate is not reported [19].

AlgoSketch is a pen based mathematical expression recognition and it supports interactive computation. It allow user to enter and edit 2D handwritten mathematical expressions in the form of pseudo code-like descriptions. This tool can be used for image analysis and number theoretic calculation problems. It has a preliminary user feedback system. It supports users which are unknown about advanced programming languages[20].

Benchmarking of User Input Models(Tools):

Sr. No.	Tools/ User Input Methods	Structure Analysis	Character Recognition	Transform To Other Language/ Formats	Connected Components segmentation	Automatic Error Correction	Computation of ME
Pen Based							
1	FFES	Partially	Yes	Latex	No	gesture-based	No
2	PenCalc Project	No	No	TeX	No	No	yes
3	MathBrush	yes	Yes	MathML	No	No	Computer Algebra System (CAS)
4	E-Chalk	Yes	Yes	pdf	No	No	No
5	MathPad2	Yes	Yes	No	No	No	No
6	MathBox	NO	YES	TeX	NO	Partially	No
7	AlgoSketch	Yes	Yes	No	No	No	YES
8	LipiToolKit	yes	Yes	No	No	No	No
9	JMathNotes	Yes	Yes	No	No	No	No
10	Microsoft Tablet PC Recognizer	yes	yes	No	No	No	No
11	Algosketch	yes	Yes	Yes	No	No	Yes

Command-String Based							
1	MathJournalTM	No	Yes	No	No	No	Yes
2	DRACULA	Yes	Yes	LaTeX	No	No	No
3	XPRESS	yes	No	Latex	No	No	No
4	INFTY (offline)	Yes	Yes	No	No	Manual	No
5	EzMath	Yes	Yes	ML	No	No	NA
6	MathWebSearch 0.4	Yes	Yes	No	No	No	No
Template Based							
1	InftyEditor	Yes	Yes	LaTeX, ML	No	Yes	No
2	Mathematica	Yes	No	TeX	No	Yes	Yes
3	Maple	Yes	Yes	TeX	No	Manual	Yes
4	MathType	Yes	Yes	Yes	No	Partial	No
5	LiveMath	Yes	Yes	Yes	No	Yes	Computer Algebra System (CAS)

Essence of User Input Models:

On the basis of analysis so far, here we may conclude that out of twenty five identified user input models, twelve models deals with pen based recognition of mathematical expressions, there are seven models which talks about command string based models and template based models are six. The important factors which includes category of the tool (either it is offline or online),structure analysis(possible, not possible, partially possible),character recognition (yes or No),transformation to other languages (LateX, pdf,etc.),connected components for segmentation (yes or No),automatic error correction (yes or No,partially,using bounding box ,manually, gesture based), recognition time (seconds),future work (given or not given) and finally computation (Computer Algebra System (CAS), or Not CAS). These factors are common in pen based, command-string based and in template based user input models.Essence of these input models have been summarised below.

Essence of Pen Based User Input Models

Every user input model have been benchmarked with set of available and identified important factors which helps to determine the problems, challenges and its state of the art. As far as our technical survey is concerned maximum number of models has been falls in this category and majorly they are with online category.

Structure analysis can be possible in maximum number of models and partially observed in two models and not possible is observed that is very negligible.In this models character recognition is possible through almost all models except one.

Transformation to other languages like pdf, Latexetc. is possible almost 50% out of identified and analysed user input models. From our study it reveals that connected component segmentation is not possible in almost all the user input models. We have focused on auto error correction technique , few models performed it , few are not , few models have performed it partially , few models may adapt bounding box method , few models may depends upon manual technique ,and few may have gesture technique. In these models recognition time is given in seconds. User input models may give very less importance to its future work and finally computation is possible in very less user input models.

Essenceof Command-String Based User Input Models

Command-String baseduser input models have also deals with important factors which are accomplished in the pen based user input models. As far as our technical survey is concerned seven models has been falls in this category and majorly they are with online category.

Structure analysis can be possible in maximum number of models and partially observed in two models. In not possible criteria is observed that is very negligible. In this models character recognition is possible through almost all models except two models.

Transformation to other languages like LateX etc. is possible in almost 70% out of identified and analysed user input models. From our study it reveals that connected component segmentation is not possible in almost all the user input models. We have focused on auto error correction technique; in almost all the user input models is not having auto error correction technique. In these models recognition time is given in seconds. User input models may give very less importance to its future work and finally computation is possible in very less user input models.

Essence of Template Based User Input Models

Template baseduser input models have also deals with important factors which are accomplished in the pen based user input models as well as in command –string based user input models. As far as our technical survey is concerned six models has been falls in this category and majorly they are with online category.

Structure analysis and character recognition and transformation to other languages is possible in almost all the models.

From our study it reveals that connected component segmentation is not possible in almost all the user input models. We have focused on auto error correction technique , few models performed it , few are not , few models have performed it partially , few models may adapt bounding box method , few models may depends upon manual technique ,and few may have gesture technique. In these models recognition time is given in seconds. User input models may give very less importance to its future work and finally computation is possible in very less user input models.

Conclusion

Our technical comprehensive survey reveals that most of the user input models falls in pen based category which is compared with command string based and template based. The category of maximum number of available user input models which are observed from online model that is compared with offline. Structure analysis has been achieved 100% in case of template based models and 50% in pen based models as well as in command string based models respectively. Character recognition and transformation to other languages is almost achieved in all the models.

Connected component segmentation is achieved negligible in almost all the models. Automatic error correction is varied model to model that is some time it is achieved fully, sometimes it is not achieved, sometimes it is achieved partially, few tools come up with bounding box method, few models depends upon manual correction and few models with gesture adjustment.

From our above study, we have identified challenges which includes writer independence in terms of editing and viewing, constraints for writing an expression on writer, ambiguity in symbol, character recognition due to invariant strokes by user, connected component segmentation and transformation to other languages which are not easily understood for a naive user is still exists. This is current state of the art is summarized for identified user input models.

References

- [1] D. Blostein, E. Lank, A. Rose, and R. Zanibbi User interfaces for on-line diagram recognition. In *Selected Papers from the Fourth Int'l Work. Graphics Recognition Algorithms and Applications*, volume 2390 of LNCS, pages 92–103. Springer, 2002
- [2] S. Smithies. Equation entry and editing via handwriting and gesture recognition. *Behavior & Information Technology*, 20(1):53–67, 2001.
- [3] R. Plamondon and S.N. Srihari. On-line and off-line handwriting recognition: A comprehensive survey. *IEEE Trans. Pattern Analysis and Machine Intelligence*, 22(1):63–84, 2000.
- [4] M. Suzuki, F. Tamari, R. Fukuda, S. Uchida, and T. Kanahori. INFTY: An integrated OCR system for mathematical documents. In *Proc. Document Engineering*, pages 95–104, Grenoble, France, 2003.
- [5] G. Labahn, E. Lank, S. MacLean, M. Marzouk, and D. Tausky. Mathbrush: A system for doing math on pen based devices. In *Proc. Work. Document Analysis Systems*, pages 599–606, Nara, Japan, 2008.
- [6] E. Tapia and R. Rojas. Recognition of on-line handwritten mathematical formulas in the E-chalk system. In *Proc. Int'l Conf. Document Analysis and Recognition*, pages 980–984, Edinburgh, 2003.

- [7] J. LaViola and R.C. Zeleznik. Mathpad2: A system for the creation and exploration of mathematical sketches. *ACM Transactions on Graphics*, 23(3):432–440, 2004.
- [8] C. Li, R.C. Zeleznik, T. Miller, and J.J. LaViola. Online recognition of handwritten mathematical expressions with support for matrices. In *Proc. Int’l Conf. Pattern Recognition*, pages 1–4, Tampa, Florida, 2008.
- [9] M. Kohlhase, S. Anca, C. Jucovschi, A.G. Palomo, and I. Sucan. MathWebSearch 0.4: A semantic search engine for mathematics. (Unpublished manuscript, <http://kwarc.info/kohlhase/publications.html>), 2008.
- [10] M. Kohlhase and I. Sucan. A search engine for mathematical formulae. In *Proc. Artificial Intelligence and Symbolic Computation*, volume 4120 of *LNAI*, pages 241–253.
- [11] M. Altamimi and A.S. Youssef. An extensive math query language. In *ISCA Int’l Conf. Software Engineering and Data Engineering*, pages 57–63, Las Vegas, USA, 2007.
- [12] B.R. Miller and A.S. Youssef. Technical aspects of the digital library of mathematical functions. *Annals of Mathematics and Artificial Intelligence*, 38:121–136, 2003.
- [13] Macro Pollanen, Thomas and Xiao “XPRESS: A Novice Interface for the Real-Time communication of Mathematical Expressions”.
- [14] Steve Smithies, Freehand Formula Entry System. Master’s thesis, university of Otago, Dunedin, New Zealand, May 1999.
- [15] Yuji Kasuya Waseda, Hayato Yamana "MathBox: Interactive Pen-Based Interface for Inputting Mathematical Expressions" "IUI’07, January 28–31, 2007, Honolulu, Hawaii, USA. Copyright 2007 ACM 1-59593-4812/07/0001...\$5.00."
- [16] Theresa O’Connell^{1†}, Chuanjun Li², Timothy S. Miller², Robert C. Zeleznik², and Joseph J. LaViola Jr "A Usability Evaluation of AlgoSketch: A Pen-Based Application for Mathematics" *EUROGRAPHICS Symposium on Sketch-Based Interfaces and Modelling* (2009) C. Grimm and J. J. LaViola Jr. (Editors). (www.cs.rit.edu).
- [17] Free Formula Entry System. <http://www.cs.queensu.ca/drl/ffes/>
- [18] Sriganesh et.al. LipiTk: A Generic Toolkit for online Handwriting Recognition. Source: Tenth International workshop on Frontiers in Handwriting Recognition, Oct 2006 La Baule (France) Suvisoft.
- [19] L. Anthony, J. Yang, and K.R. Koedinger, Adapting handwriting recognition for applications in algebra learning. In *proceedings of ACM Workshop on Educational Multimedia and Multimedia Education*, pages 47–56, ACM press, New York, NY, USA, 2007.

- [20] ChuanJun Li, Timothy S, Miller , Robert C, Zeleznik and Josph Laviola ,
Algosketch: Algorithm sketching and Interactive Computation,
EuroGraphics Workshop on Sketching-Based Interfaces and
Modelling, 2008
- [21] <http://www.xthink.com/MathJournal.html>.
- [22] Rechard Zanibbi, Dorothein, Recognizing Mathematical Expressions using
Tree Transformation, IEEE Transaction on Pattern analysis and machine
intelligence, Vol. 24 No. 11 November 2002.
- [23] Lucy Zhang and Richard Fateman, Survey of User Input Models for
Mathematical Recognition: Keyboard, Mice, Tablets, Voice. Computer
science division, University of California m Berkeley CA.
- [24] www.Inftyproject.org/en/software.html
- [25] Maple: www.maplesoft.com
- [26] Barbara, Carl, Introduction to LiveMath, International Conference On
Technology in Collegiate Mathematics, November 2001.
- [27] MathType: www.desswci.com/en/products/mathtype/