International Journal of Applied Engineering Research
ISSN 0973-4562 Volume 10, Number 11 (2015) pp. 27651-27659

http://www.ripublication.com

A Profile Analysis of an Accelerated Thermal Life Test

Kyu-bark Shim[1] and Jae-geol Yim[2]

[1]*Department of Applied Statistics, Dongguk University at Gyeongju, Korea*
E-mail: shim@dongguk.ac.kr
[2]*Department of Computer Engineering, Dongguk University at Gyeongju, Korea*
E-mail: yim@dongguk.ac.kr

Abstract

In 1987, guidelines for statistically analyzing the thermal life test base on the accelerated life test(ALT) were proposed as ANSI/IEEE Std 101, and since then, these guidelines have been widely used to analyze diverse types of experimental data. Previous research has employed the Monte Carlo simulation method to compare the life of two different systems or materials, based on statistical values obtained from ANSI/IEEE Std101 data. This paper proposes, a profile analysis for comparing the life of two different systems or materials such as computers and smartphones and provides an analysis example, using pre-existing data.

Keywords: Arrhenius Model, Accelerated Life Data, Electrical Insulating Materials, Profile Analysis, Thermal Life Data

Introduction

Reliability is the possibility or ability of a product to perform a certain function under a given condition for an intended period. To accurately evaluate this reliability, functions and environments required for products should be regulated. The Accelerated Life Test(ALT) is, a method for detecting a product's life or fault rate and, accelerates degradation in terms of both time and physical aspects. The main purpose of the ALT is to obtain data on a product's life, and statistically estimate the life of the product during its use.

If a product's life and stress have a linear relationship, then parameters need to be estimated to determine the regression line. However, if there are some problems in data, then estimated parameters and confidence intervals can seriously affected. Shim (2004) proposes that for data following a log-normal life distribution or whose numbers are small, using the simulation method to estimate parameters and confidence intervals can be more accurate than using the method proposed in ANSI/IEEE Std 101.

Based on Shim (2004) and ANSI/IEEE Std 101, this paper employs a profile analysis to compare the life of two different systems or materials in the case of a small data set following a log-normal life distribution. For the life-stress relationship, the Arrhenius model is used, and life data are constructed based on statistics from ANSI/IEEE Std. 101 data.

The Arrhenius Lognormal Model

The most general environmental stress in the ALT on an electronic device is the temperature. The Arrhenius model is used when the product's life is a function of the temperature. The following equation shows the relationship between the temperature and the life of an insulator:

$$K = S' \exp\left(-E/\theta T\right) \quad . \qquad \textbf{(1)}$$

where K is the chemical reaction rate, E is the activation energy of the reaction with varying values with respect to a failure mechanism, its unit is eV(electron volt). θ is the Boltzmann constant $\theta = 8.617 \times 10^{-5} eV/°K$, T is the Kelvin temperature $°C + 273.16$, and S' is a constant showing a product's characteristic or test condition. The Arrhenius model applies the Arrhenius equation to the ALT, and assumes that if the reaction reaches a critical point, there is a fault.

Therefore, the critical value can be obtained as follows:

Critical value = reaction rate × time to failure

Here the time to failure t can be expressed as follows:

$$t = \text{Critical value}/\text{reaction rate} = S \exp\left(E/\theta T\right)$$

Through the log transformation of the model, the median life of insulator data becomes proportional to 1/t:

$$\log(L) = \log(S) + (E/\theta T) \quad . \qquad \textbf{(2)}$$

Here the log is a common logarithm. Equation (2) can be expressed in an algebraic form.

$$Y = A + BX \quad . \qquad \textbf{(3)}$$

In the equation, X=1/T, A=log(S), B=E/θ and Y is log(L) which can be expressed as a log-linear equation showing a nominal log life. In Equation (3),

coefficients A and B can be estimated from experiment data, and both a and b are sample estimators.
L_{ij} is the life of sample j under the temperature i, and if its log value is Y_{ij}, i=1,...,n, j=1,...,m, then $Y_{ij}= \log(L_{ij})$. An under some arbitrary temperature i, the mean of Y_{ij} is $\overline{Y}_i$, and the standard deviation is s_i.

Life-Temperature Estimation

Employ the Arrhenius model to analyze thermal life data and make several assumptions to estimate a change in the life with respect to the temperature. Here the assumptions are as follow: 1) The relationship between the log life and the inverse Kelvin temperature is linear within the temperature range of interest. 2) Sample data are statistically independent. 3) The sample is selected arbitrarily from a population of interest. 4) The random variation of the log life follows a normal distribution with equal standard deviations under every temperature of interest.

Based on the aforementioned assumptions, a change in the Celsius temperature T to the inverse Kelvin temperature is as follows:

$$X_i = 1/(T_i + 273), \quad i = 1, \cdots, n. \tag{4}$$

Because each failure life in sample L is converted into Y (Y=log(L)), the sample estimator from populations A and B in Equation (3) can be expressed as follows:

$$a = \overline{Y} - b\overline{X} \ , \qquad b = \frac{\sum_{i=1}^{n} X_i Y_i - n\overline{X}\overline{Y}}{\sum_{i=1}^{n} X_i^2 - n\overline{X}^2} . \tag{5}$$

If $m(T_i)$ is the sample estimator of the mean log life of a population from the selected temperature T_i, then $m(T_i)$ can be calculated as follows:

$$m(T_i) = a + b[1/(T_i + 273)] . \tag{6}$$

The anti-log of $m(T_i)$ is the estimated value of the median life for the time unit under the temperature T_i. A sample standard deviation of the log life is as follows:

$$s = \sqrt{\frac{1}{n-2} \sum_{i=1}^{n} (Y_i - (a + bX_i))^2} . \tag{7}$$

For the selected temperature T_i, following equation can be obtained.

$$V(T_i) = \frac{(X_i - \overline{X})^2}{\sum_{i=1}^{n} (X_i - \overline{X})^2} . \tag{8}$$

where X_i is the inverse Kelvin temperature such that

$$X_i = 1/(T_i + 273), \quad i = 1, \cdots, n.$$

The upper confidence limit $m_U(T_i)$ and lower confidence limit $m_L(T_i)$ of the mean log life $m(T_i)$ can be calculated using Equations (7) and (8) as follows:

$$m_U(T_i) = (a + bX_i) + t_{n-2}(\alpha)s\sqrt{(1/n) + V(T_i)}. \tag{9}$$

$$m_L(T_i) = (a + bX_i) - t_{n-2}(\alpha)s\sqrt{(1/n) + V(T_i)}. \tag{20}$$

A Profile Analysis of Data from Two Group

For electrical insulation materials, thermal resistance is one of the most important properties. These materials can be classified into seven groups based on the upper temperature limit, and all seven groups are composed of different materials.

Table 1: Seven types of heat-resistant electrical insulation materials

Types	Y	A	E	B	F	H	C
Limit of the temperature increase (°C)	90 and less	91 ~105	106~ 120	121~ 130	131 ~155	156~ 180	180 And more

Therefore, if there is some difference in the mean log life between two equal types of insulators or if there is little difference between two different insulators, then they are possibly bad.

To compare the life of two different types of systems or materials, the thermal life test can be applied. After data on both insulating materials are obtained, the question of whether one material is clearly better than the other can be observed using the Arrhenius line. Observed differences can originate from an unexpected change. To assess any reasonable differences, two sets of data should be compared, and this is similar to the method of comparing two means.

A comparison between lines can be done under more than one temperature, and a comparison under a certain range of temperatures can sometimes be more useful.

The profile analysis method examines whether an effect is equal or not if p processes (e.g., tests and questions) are done for more than two groups. This method is similar to the analysis of variance based on an agreement analysis of average vectors, but the difference is that the profile analysis includes several steps.

If $\mu_1^T = [\mu_{11}, \mu_{12}, \cdots, \mu_{1p}]$ and $\mu_2^T = [\mu_{21}, \mu_{22}, \cdots, \mu_{2p}]$ represent the mean log life of response values for two populations, then the null hypothesis $H_0 : \mu_1 = \mu_2$ can be reconstituted through the following three steps:

First, are profiles of two groups parallel to each other? Second, assuming that these profiles are parallel, are the profiles coincident? Third, assuming that the profiles are coincident, are all means equal to the same constant?

Let $\bar{x}_1, \bar{x}_2$ be the sample mean vector of two samples of sizes n_1 and n_2, respectively, and independent of each other. Here C is a contrast matrix:

$$\mathbf{C} = \begin{bmatrix} -1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & 0 & & 0 & 0 \\ & \vdots & & & \ddots & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -1 & 1 \end{bmatrix}$$

A Parallel Test of Two Population Profile

To test whether two populations' profiles are parallel to each other, the null hypothesis can be written as follows:

$$H_{01} : \mathbf{C}\boldsymbol{\mu}_1 = \mathbf{C}\boldsymbol{\mu}_2 \ . \tag{11}$$

This null hypothesis can be examined based on modified observations $Cx_{1i}, i = 1, \cdots, n_1$ and $Cx_{2j}, j = 1, \cdots, n_2$. The sample mean vector of modified observations is $C\bar{x}_1 = C\bar{x}_2$, and the pooled sample covariance matrix is $CS_{\mathbf{pooled}}C^T$. Therefore, the null hypothesis (1) is rejected if the following conditions are satisfied:

$$T^2 = (\bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2)^T \mathbf{C}^T \left[\left(\frac{1}{n_1} + \frac{1}{n_2}\right) \mathbf{C}\mathbf{S}_{\text{pooled}} \mathbf{C}^T\right]^{-1} \mathbf{C}(\bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2) > c^2 \ . \tag{12}$$

and,

$$c^2 = \frac{(n_1+n_2-2)(p-1)}{n_1+n_2-p} F_{p-1, n_1+n_2-p\,(\alpha)}.$$

A Coincident Test of Values for Two Population Profile

If the null hypothesis H_{01} is not rejected and satisfies the parallel condition, then a coincident test should be conducted. Because profiles are coincident only if $\mu_{11} + \mu_{12} + \cdots + \mu_{1p} = 1^T\mu_1$ and $\mu_{21} + \mu_{22} + \cdots \mu_{2p} = 1^T\mu_2$ are equal, the null hypothesis

$$H_{02} : \mathbf{1}^T\boldsymbol{\mu}_1 = \mathbf{1}^T\boldsymbol{\mu}_2 \ . \tag{13}$$

should be examined. Here the null hypothesis (13) is rejected at a significant level if the following condition is satisfied:

$$T^2 = \mathbf{1}^T(\bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2)^T\left[\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\mathbf{C}\mathbf{S}_{pooled}\mathbf{C}^T\right]^{-1}\mathbf{1}(\bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2)\mathbf{1}^T(\bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2) > F_{1,n_1+n_2-2\,(\alpha)}. \quad \textbf{(14)}$$

An Equality Test of All Means Equal to the Same Constant

If the null hypotheses H_{01} and H_{02} are not rejected, then an equality test of all means should be conducted. The common mean vector μ for two groups can be estimated based on $n_1 + n_2$ as follows:

$$\bar{\mathbf{x}} = \frac{1}{n_1+n_2}\left(\sum_{j=1}^{n_1} x_{1j} + \sum_{j=1}^{n_2} x_{2j}\right) = \frac{n_1}{n_1+n_2}\bar{x}_1 + \frac{n_2}{n_1+n_2}\bar{x}_2. \quad \textbf{(15)}$$

If average values are equal, the null hypothesis can be written as follows because $\mu_1 = \mu_2 = \cdots = \mu_p$:

$$H_{03} : \mathbf{C}\boldsymbol{\mu} = \mathbf{0}. \quad \textbf{(16)}$$

The null hypothesis (6) can be rejected at a significant level if the following condition is satisfied:

$$(n_1 + n_2)\bar{x}^T C^T\left[CS_{pooled}C^T\right]^{-1} C\bar{x} > F_{p-1,n_1+n_2-p\,(\alpha)}. \quad \textbf{(17)}$$

Examples

Table 2 is based on Nelson's (2004) data from two different electrical insulators. The data are of type C in the table and show the life under test temperatures 200℃, 225℃, 250℃. The life time is the mean value obtained during the analysis period. Because the application to the atmosphere and space is sometimes related to 250℃, the purpose of a life test is to compare the mean log life within the whole range of test temperatures and the design temperature 200℃.

Table 2: Nelson(2004)`s data from two types electrical insulators

Reference period	Insulator 1			Insulator 2		
	200℃	225℃	250℃	200℃	225℃	250℃
1st	1176	624	204	2520	816	300
2nd	1512	624	228	2856	912	324
3rd	1512	624	252	3192	1296	372
4th	1512	816	300	3192	1392	372
5th	3528	1296	324	3258	1488	444

A Profile analysis

This section provides the results of the three-step profile analysis for Hypotheses (12), (14), and (17) based on Equations (13), (15), and (18). This represents a profile analysis of the log life of insulators 1 and 2.

Table 3: profile analysis result of two groups' data

Hypothesis	Test Statistics	Critical Value ($\alpha = 0.05$)
$H_{01} : C\mu_1 = C\mu_2$	11.6343	6.9282
$H_{02} : 1^T\mu_1 = 1^T\mu_2$	19.5541	4.1961
$H_{03} : C\mu = 0$	18.9099	3.3404

The results provide no support for all hypotheses at the 5% level. The parallel of the profile under a given temperature is not valid, and both profile and mean values are not coincident.

Simulation

Given a given temperature for both insulators 1 and 2, 1,000 Monte Carlo simulations are performed to estimate the relationship between the inverse Kelvin temperature and the log life. The estimation results for Equation (3) as follows:

1) The estimated formula for insulator 1:

$$Y = -4.4504 + 3615.104X.$$

2) The estimated formula for insulator 2:

$$Y = -4.9549 + 3785.861X.$$

For the selected temperature, the value of an appropriate point can be obtained from Equation (7) by calculating the corresponding estimated value of the mean log life. From the anti-log of this calculated value, the estimated value of the mean log life in the unit of time can be obtained.

For example, for 200°C,

$$m(200) = -4.4504 + 3615.104[1/(273+200] = 3.1925$$

The anti-log of this is the estimated value of the mean log life: 1,558 hours.

Table 4: An estimates of mean log life

T_i	insulator 1	insulator 2
200℃	3.1926	3.4720
225℃	2.8092	3.0490
250℃	2.4618	2.5662

Conclusions

Based on ANSI/IEEE Std 101 data, Shim (2005) estimates a regression model of the inverse Kelvin temperature and the log life based on 10 measured values under 150℃, 6 values under 175℃, and 10 values under 200℃ but finds only some differences between estimated values, through a statistical analysis because of limited data.

This paper provides a profile analysis of the log life of thermal life test data based on the ALT to compare data of one type from Table 1. Because collecting large amounts of data from a thermal life test is hard, any estimation is considered to be more accurate based on a simulation using empirical data. It is crucial to conduct life tests in the early stages of computer system design, and therefore the proposed method is expected to reduce system design costs.

However, setting reasonable conditions for a simulation remain difficult because of limitations in using experimental data proposed in ANSI/IEEE Std 101. In this regard, this issue of an accurate simulation is left to future research.

Acknowledgments

This research was supported by the Basic Science Research Program of the National Research Foundation (NRF) of Korea funded by the Ministry of Education (NRF-2011-0006942) and by the Development of Global Culture and Tourism IPTV Broadcasting Station Project of the Industrial Infrastructure Program for Fundamental Technologies funded by the Ministry of Knowledge Economy (10037393).

References

[1] SHim, K., 2004, "Simulation about thermal life test data of two different insulating materials", Journal of the Korean Data Analysis Society, 6(2) pp.603-611.
[2] KINS/HR 551-2003, 2003, "A Study of Instrument Verification Suitability About Digital Measuring Control System".
[3] ANSI/IEEE Std 101-1987, 1987, "IEEE Guide for the Statistical Analysis of Thermal Life Test Data".

[4] Evans, J., and Olson, D., 2002, "Introduction to Simulation and Risk Analysis", Prentice Hall, New Jersey.
[5] Johnson, R., and Wichern, D., 2007, "Applied Multivariate Statistical Analysis 6th Ed"., Pearson Prentice Hall, New Jersey.
[6] Thas, O., 2010, "Comparing Distributions", Springer, New York.
[7] Nelson, W., 2004, "Accelerated testing: statistical models, test plans and data analyses", Wiley-Interscience, New Jersey.