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Effect Thermal Relaxation Time (L-S Theory) and Initial Stress on Reflection and Transmission Ql-Waves in Piezo-Thermoelastic Media

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Abstract

The present paper represents the reflection and transmission of quasi-longitudinal waves in a homogeneous, transversely isotropic, generalized piezo-thermoelastic media. Based on the Lord and Shulman (L-S) with one relaxation time. The generalized thermoelastic– piezoelectric governing equations are formulated. There are four types of plane waves quasi-longitudinal (qL), quasi-transverse (qSV) and thermal wave (Θ -mode), and potential electric waves ($\varphi - \text{mode}$). The appropriate boundary conditions are satisfied at the interface to obtain the reflection and transmission coefficients of various reflected and transmitted waves during incidence of qL wave. A particular numerical example is considered to show the effect of initial stresses and relaxation time on these coefficients for two dimension model. The obtained results are presented graphically.

Keywords:. Piezo-thermoelastic; Longitudinal waves; Reflection and transmission coefficients; Initial stresses; Lord and Shulman theory, Relaxation time.

Nomenclature

E_i, φ are the electric field and electric potential,

u_i, is the mechanical displacement vector,

Θ is absolute temperature,

D_i is the electric displacement vector,

$\sigma_{ij}, \sigma_{kj}^{o}$ are the stress and initial stress tensors,

S_{ij} is the strain tensor,

P_{ij} is the thermal elastic coupling tensors,

ρ is the density of the medium,

C_{ijkl} is the elastic parameters tensor,

C_{Lo} is the velocity of incident Longitudinal wave,

C_{L1}, C_{T2} are the velocity of reflected L and SV waves,

C_{L3} , C_{T4} are the velocity of transmitted L and SV waves

e_{ijk} is the piezoelectric tensor,

ϵ_{ij} is the dielectric moduli tensor,

d_i is the pyroelectric moduli vector,

τ_o is thermal relaxation time,

K_{ij} is the heat conduction tensor,

Θ_o is the reference temperature,

C_E is the specific heat at constant strain,

δ_{ik} is the Kronecker delta tensor.

Introduction

Reflection and transmission phenomena of a plane waves at the interface between two media is a essential topic in many fields such as, engineering, geophysics, earthquake, seismology, Industry, non-destructive evaluation, etc. It is impossible to embrace all the related references and textbooks on this matter. We just mention some of them, Achenbach [1] studied the reflection and transmission in the context of the isotropic elastic bodies, Dey and Addy [2] investigated the phenomena of reflection and refraction of plane elastic waves at a plane interface between two semi-infinite elastic solid media in contact, when both the media are initially stressed. Khurana and Tomar [3] presented the propagation of plane elastic waves from a stress free plane boundary of an electro-microelastic solid half space. Pang et al [4] discussed the reflection and refraction of plane waves at the interface between two transversely isotropic piezoelectric and piezomagnetic media. Kumar and Singh [5] studied a problem concerned with the reflection and transmission of plane waves at an imperfectly bonded interface of two orthotropic generalized thermoelastic half-spaces with different elastic and thermal properties. Abd-alla and Alsheikh [6] studied the reflection and refraction longitudinal waves at an interface of two piezoelectric media under initial stresses.

Generalized theories of thermoelasticity were introduced in order to eliminate the shortcomings of the classical dynamic thermoelasticity. The theory of generalized thermoelasticity with one relaxation time was first introduced by Lord and Shulman [7], who obtained a wave-type heat equation by postulating a new law of heat conduction instead of the classical Fourier's law. One can refer to Hetnarski and Ignaczak [8] for a review and presentation of the generalized theories of thermoelasticity. Many authors concentrate in studying the propagation waves in thermoelastic media with relaxation times, like Deresiewicz [9], Sinha and Elsibai [10,11], Abd-Alla and Al-Dawy [12], Wang and Dong [13].

The theory of piezo-thermoelasticity was first explored by Mindlin [14]. The physical laws for the piezo-thermoelastic materials have been proposed by Nowacki [15]. Chandrasekharaiah [16] has generalized Mindlin's theory of piezo-thermoelasticity to account for the finite speed of propagation of thermal disturbances on the basis of the first and the second thermodynamics laws. Sharma and Kumar [17] investigated plane harmonic waves in piezo-thermoelastic materials. Kuang and Yuan [18] discussed the reflection and transmission theories of waves in pyroelectric and piezoelectric medium. Alshaikh [19] studied the reflection and refraction longitudinal waves from the interface of the piezo-thermoelastic materials under initial stresses influence in Green and Lindsay theory context.

The aim of this paper is to study the reflection and transmission of longitudinal piezo-thermoelasticity waves in two anisotropic media under initial stresses. We employ the piezo-thermoelasticity equations with one relaxation time to solve this problem. The numerical calculations of amplitude ratios have been carried out by software MathCad for different materials as examples and the results are given in the form of graphs. Finally, some of particular cases are considered.

Governing equations of generalized piezo-thermoelastic of L-S model and general solution

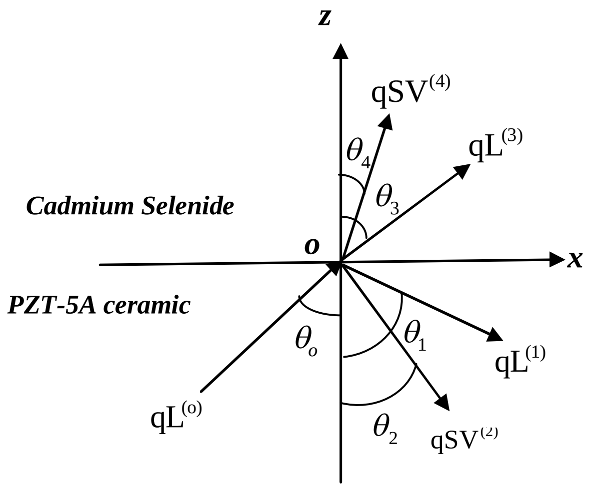


Figure1 Relation between the incident wave and the reflect/ transmit waves

Let the wave motion in these media be characterized by the displacement vector $\vec{u}(u, 0, w)$, the electric potential function φ, and thermal wave Θ all these quantities being dependent only on the variables *x, z, t.* (see figure 1). The governing equations for piezo-thermoelastic crystal class 6 mm for L-S theory are [17]:

$$\left.S_{kl} = (u_{k,l} + u_{l,k})/2, \qquad E_k = -\varphi_{,k}\right\} \qquad k, l = 1,2,3. \tag{1}$$

$$\left.\begin{aligned} \sigma_{11} &= C_{11}S_{11} + C_{13}S_{33} - e_{31}E_3 - P_1\Theta, \\ \sigma_{22} &= C_{21}S_{11} + C_{13}S_{33} - e_{31}E_3 - P_2\Theta, \\ \sigma_{33} &= C_{13}S_{11} + C_{33}S_{33} - e_{33}E_3 - P_3\Theta, \\ \sigma_{31} &= 2C_{44}S_{31} - P_{13}\Theta, \\ \sigma_{32} &= \sigma_{12} = 0 \end{aligned}\right\} \tag{2}$$

$$\left.\begin{aligned} D_1 &= 2e_{15}S_{31} - \epsilon_{11} E_1 + d_1\Theta, \\ D_2 &= 0, \\ D_3 &= 2(e_{31}S_{11} + e_{33}S_{33}) + \epsilon_{33} E_3 + d_3\Theta \end{aligned}\right\} \tag{3}$$

where 1, 2, 3 denote x, y, z variables.

The equations of motion, Gauss's divergence equation, and heat conduction equation of hexagonal 6mm material under initial stress and relaxation time effect can be written as

$$(C_{11} - \sigma_{11}^o)u_{,11}^{(n)} + (C_{44} + \sigma_{33}^o)u_{,33}^{(n)} + (C_{13} + C_{44})w_{,13}^{(n)}$$

$$+(e_{13} + e_{15})\varphi_{,13}^{(n)} - P_1\Theta_{,1}^{(n)} = \rho\ddot{u}^{(n)} \tag{4}$$

$$(C_{44} + C_{31})u_{,13}^{(n)} + (C_{44} + \sigma_{11}^o)w_{,11}^{(n)} + (C_{33} + \sigma_{33}^o)w_{,33}^{(n)}$$

$$+e_{15}\varphi_{,11}^{(n)} + e_{33}\varphi_{,33}^{(n)} - P_3\Theta_{,3}^{(n)} = \rho\ddot{w} \tag{5}$$

$$(e_{15} + e_{31})u_{,13}^{(n)} + e_{15}w_{,11}^{(n)} + e_{33}w_{,33}^{(n)} - \epsilon_{11}\varphi_{,11}^{(n)} - \epsilon_{33}\varphi_{,33}^{(n)} + d_3\Theta_{,3}^{(n)} = 0 \tag{6}$$

$$K_1\Theta_{,11}^{(n)} + K_3\Theta_{,33}^{(n)} - \rho C_E\,(\dot{\Theta}^{(n)} + \tau_o\ddot{\Theta}^{(n)}) = {\Theta_o}^{(n)}[P_1\left(\dot{u}_{,1}^{(n)} + \tau_o\delta\ddot{u}_{,1}^{(n)}\right)$$

$$+P_3\left(\dot{w}_{,3}^{(n)} + \tau_o\delta\ddot{w}_{,3}^{(n)}\right) - d_3\left(\dot{\varphi}_{,3}^{(n)} + \tau_o\delta\ddot{\varphi}_{,3}^{(n)}\right) \tag{7}$$

where $n = 0,1,2,3,4$

The general solution of the piezo-thermoelastic (hexagonal 6mm) equations can be written into the following form (see figure 1):

$$(u^{(o)}, w^{(o)}, \varphi^{(o)}, \Theta^{(o)}) = (A_o\sin\theta_o, A_o\cos\theta_o, B_o, C_o)\,exp[\Lambda_o]$$

$$(u^{(1)}, w^{(1)}, \varphi^{(1)}, \Theta^{(1)}) = (A_1\sin\theta_1, -A_1\cos\theta_1, B_1, C_1)\,exp[\Lambda_1]$$

$$(u^{(2)}, w^{(2)}, \varphi^{(2)}, \Theta^{(2)}) = (A_2\cos\theta_2, A_2\sin\theta_2, B_2, C_2)\,exp[\Lambda_2]$$

$$(u^{(3)}, w^{(3)}, \varphi^{(3)}, \Theta^{(3)}) = (A_3\sin\theta_3, A_3\cos\theta_3, B_3, C_3)\,exp[\Lambda_3]$$

$$(u^{(4)}, w^{(4)}, \varphi^{(4)}, \Theta^{(4)}) = (-A_4\cos\theta_4, A_4\sin\theta_4, B_4, C_4)\,exp[\Lambda_4] \tag{8}$$

where

$$\Lambda_0 = ik_o(x\sin\theta_o + z\cos\theta_o - C_{Lo}t),\ \Lambda_1 = ik_1(x\sin\theta_1 - z\cos\theta_1 - C_{L1}t)$$

$$\Lambda_2 = ik_2(x\sin\theta_2 - z\cos\theta_2 - C_{T2}t),\ \Lambda_3 = ik_3(x\sin\theta_3 + z\cos\theta_3 - C_{L3}t)$$

$\Lambda_4 = ik_4(x\sin\theta_4 + z\cos\theta_4 - C_{T4}t)$

$C_{L0} = \omega/k_0\,, C_{L1} = \omega/k_1\,, C_{T2} = \omega/k_2\,, C_{L3} = \omega/k_3\,, C_{T4} = \omega/k_4$

4. Continuous conditions in L-S model.

Consider the L-S model of two piezo-thermoelastic media with the interface $z = 0$ subjected to incident qL wave of frequency ω with an incident angle θ_o. The mechanical, electrical potential, and thermal boundary conditions are as follow:

$$\left.\begin{aligned} \sigma_{33}^{(O)} + \sigma_{33}^{(1)} + \sigma_{33}^{(2)} = \sigma_{33}^{(3)} + \sigma_{33}^{(4)}, \\ \sigma_{31}^{(O)} + \sigma_{31}^{(1)} + \sigma_{31}^{(2)} = \sigma_{31}^{(3)} + \sigma_{31}^{(4)} \end{aligned}\right\} \tag{9}$$

$$\varphi^{(O)} + \varphi^{(1)} + \varphi^{(2)} = \varphi^{(3)} + \varphi^{(4)} \tag{10}$$

$$\Theta^{(O)} + \Theta^{(1)} + \Theta^{(2)} = \Theta^{(3)} + \Theta^{(4)} \tag{11}$$

Substituting equations (1, 2, 8) into equations (9)-(11), we obtain the following set of equations:

$$\begin{aligned} &ik_o[(C_{13}\sin^2\theta_o + C_{33}\cos^2\theta_o)A_o + e_{33}\cos\theta_o B_o - (P_3/ik_o)C_o]\exp[\Lambda_0] \\ &\quad - ik_1[(C_{13}\sin^2\theta_1 + C_{33}\cos^2\theta_1)A_1 \\ &\quad - e_{33}\cos\theta_1 B_1 - (P_3/ik_1)C_1]\exp[\Lambda_1] \\ &\quad + ik_2[(C_{13} - C_{33})sin\theta_2\cos\theta_2 A_2 - e_{33}\cos\theta_2 B_2 \\ &\quad - (P_3/ik_2)C_2]\exp[\Lambda_2] \\ &\quad - ik_3[(C_{13}^m\sin^2\theta_3 + C_{33}^m\cos^2\theta_3)A_o + e_{33}^m\cos\theta_3 B_3 \\ &\quad - (P_3^m/ik_3)C_3]\exp[\Lambda_3] \\ &\quad - ik_4[(C_{33}^m - C_{13}^m)sin\theta_4\cos\theta_4 A_4 + e_{33}^m\cos\theta_4 B_4 \\ &\quad - (P_3^m/ik_4)C_4]\exp[\Lambda_4] = 0 \end{aligned} \tag{12}$$

$$\begin{gathered} ik_o[C_{44}\sin 2\theta_o\, A_o + e_{15}\sin\theta_o\, B_o]\exp[\Lambda_0] \\ -ik_1[C_{44}\sin 2\theta_1\, A_1 - e_{15}\sin\theta_1\, B_1]\exp[\Lambda_1] \\ -[C_{44}\cos 2\theta_2\, A_2 - e_{15}\sin\theta_2\, B_2]\exp[\Lambda_2] = \\ ik_3[C_{44}^m\sin 2\theta_3\, A_3 + e_{15}^m\sin\theta_3\, B_3]\exp[\Lambda_3] \\ -[C_{44}^m\cos 2\theta_4\, A_4 - e_{15}^m\sin\theta_4\, B_4]\exp[\Lambda_4] \end{gathered} \tag{13}$$

$$B_o\exp[\Lambda_0] + B_1\exp[\Lambda_1] + B_2\exp[\Lambda_2] = B_3\exp[\Lambda_3] + B_4\exp[\Lambda_4] \tag{14}$$

$$C_o\exp[\Lambda_0] + C_1\exp[\Lambda_1] + C_2\exp[\Lambda_2] = C_3\exp[\Lambda_3] + C_4\exp[\Lambda_4] \tag{15}$$

Equations (12)-(15) must be valid for all values of t and x, hence

$\Lambda_0 = \Lambda_1 = \Lambda_2 = \Lambda_3 = \Lambda_4$

$k_o\sin\theta_o = k_1\sin\theta_1 = k_2\sin\theta_2 = k_3\sin\theta_3 = k_4\sin\theta_4$

$$k_oC_{Lo} = k_1C_{L1} = k_2C_{T2} = k_3C_{L3} = k_4C_{T4} = \omega \tag{16}$$

From the above relations, we get

$$k_o = k_1, \qquad \theta_o = \theta_1, \qquad C_{Lo} = C_{L1}$$
$$\vartheta_1 = k_2/k_o, \vartheta_2 = k_3/k_o, \vartheta_3 = k_4/k_o$$
$$sin\theta_2 = sin\theta_o/\vartheta_1, \; sin\theta_3 = sin\theta_o/\vartheta_2, sin\theta_4 = sin\theta_o/\vartheta_3 \tag{17}$$

So, substituting from equations (8) (when $z = 0$) into equations (4) for the incident qL wave, the reflected and transmitted waves, we get

$$\left.\begin{aligned} \gamma_o A_o + H_o B_o + J_o C_o = 0, \\ \gamma_1 A_1 + H_1 B_1 + J_1 C_1 = 0, \\ \gamma_2 A_2 + H_2 B_2 + J_2 C_2 = 0, \\ \gamma_3 A_3 + H_3 B_3 + J_3 C_3 = 0, \\ \gamma_4 A_4 + H_4 B_4 + J_4 C_4 = 0 \end{aligned}\right\} \tag{18}$$

where

$$\gamma_o = -\sin\theta_o[\rho C_{Lo}^2 - (C_{11} + \sigma_{11}^o)\sin^2\theta_o - (C_{13} + 2C_{44} + \sigma_{33}^o)\cos^2\theta_o]$$
$$H_o = (e_{13} + e_{15})\sin\theta_o\cos\theta_o, \; J_o = iP_1\sin\theta_o/k_o, \; \gamma_1 = -\gamma_o, \; H_1 = H_o, J_1 = -J_o$$
$$\gamma_2 = \cos\theta_2[\rho C_{T2}^2 - (C_{11} + \sigma_{11}^o)\sin^2\theta_2 - (C_{44} + \sigma_{33}^o)\cos^2\theta_2 + (C_{13} + C_{44})\sin^2\theta_2]$$
$$H_2 = (e_{13} + e_{15})\sin\theta_2\cos\theta_2, \; \mu_2 = -iP_1\sin\theta_2/k_2$$
$$\gamma_3 = \sin\theta_3[\rho^m C_{L3}^2 - (C_{11}^m + \sigma_{11}^o)\sin^2\theta_3 - (C_{44}^m + \sigma_{33}^o)\cos^2\theta_3 - (C_{13}^m + C_{44}^m)\cos^2\theta_3]$$
$$H_3 = -(e_{13}^m + e_{15}^m)\sin\theta_3\cos\theta_3, \; J_3 = -\sin\theta_3\, iP_1^m/k_3$$
$$\gamma_4 = \cos\theta_4[(C_{11}^m - C_{13}^m - C_{44}^m + \sigma_{11}^o)\sin^2\theta_4 + (C_{44}^m + \sigma_{33}^o)\cos^2\theta_4 - \rho^m C_{T4}^2]$$
$$H_4 = -(e_{13}^m + e_{15}^m)\sin\theta_4\cos\theta_4, \; J_4 = -iP_1^m\sin\theta_4/k_4$$

By using the solutions from equations (8) into equations (6), we get:

$$\left.\begin{aligned} \xi_o A_o + \eta_o B_o + \lambda_o C_o = 0, \\ \xi_1 A_1 + \eta_1 B_1 + \lambda_1 C_1 = 0, \\ \xi_2 A_2 + \eta_2 B_2 + \lambda_2 C_2 = 0, \\ \xi_3 A_3 + \eta_3 B_3 + \lambda_3 C_3 = 0, \\ \xi_4 A_4 + \eta_4 B_4 + \lambda_4 C_4 = 0 \end{aligned}\right\} \tag{19}$$

Where

$$\xi_o = -[(e_{13} + 2e_{15})\sin^2\theta_o\cos\theta_o + e_{33}\cos^3\theta_o], \eta_o = \epsilon_{11}\sin^2\theta_o + \epsilon_{33}\cos^2\theta_o$$
$$\lambda_o = id_3\cos\theta_o/k_o, \; \xi_1 = -\xi_o, \; \eta_1 = \eta_o, \; \lambda_1 = -\lambda_o$$
$$\xi_2 = [(e_{13} + e_{15} - e_{33})\sin\theta_2\cos^2\theta_2 - e_{15}\sin^3\theta_2], \eta_2 = \epsilon_{11}\sin^2\theta_2 + \epsilon_{33}\cos^2\theta_2$$
$$\lambda_2 = -id_3\cos\theta_2/k_2, \; \xi_3 = -[(e_{13}^m + 2e_{15}^m)\sin^2\theta_3\cos\theta_3 + e_{33}^m\cos^3\theta_3]$$
$$\eta_3 = \epsilon_{11}^m\sin^2\theta_3 + \epsilon_{33}^m\cos^2\theta_3, \lambda_3 = -id_3^m\cos\theta_3/k_3$$
$$\xi_4 = [(e_{13}^m + e_{15}^m - e_{33}^m)\sin\theta_4\cos^2\theta_4 - e_{15}^m\sin^3\theta_4],$$

$\eta_4 = \in_{11}^{m} \sin^2\theta_4 + \in_{33}^{m} \cos^2\theta_4$, $\lambda_4 = -i d_3^m \cos\theta_4 / k_4$

By using the solutions from Eqs.(8) into Eqs.(7), we get

$$\left.\begin{aligned}
£_o A_o + \Omega_o B_o + F_o C_o &= 0,\\
£_1 A_1 + \Omega_1 B_1 + F_1 C_1 &= 0,\\
£_2 A_2 + \Omega_2 B_2 + F_2 C_2 &= 0,\\
£_3 A_3 + \Omega_3 B_3 + F_3 C_3 &= 0,\\
£_4 A_4 + \Omega_4 B_4 + F_4 C_4 &= 0
\end{aligned}\right\} \tag{20}$$

where

$£_o = \Theta_o (1 - i k_o \tau_o \delta\, C_{Lo})(P_1 \sin^2\theta_o + P_3 \cos^2\theta_o)$

$\Omega_o = -\Theta_o d_3 (1 - i k_o \tau_o \delta\, C_{Lo}) \cos\theta_o$

$F_o = [(K_1 \sin^2\theta_o + K_3 \cos^2\theta_o)/C_{Lo}] - i\rho C_E (1 - i k_o \tau_o C_{Lo})$

$£_1 = -£_o,\quad \Omega_1 = \Omega_o,\qquad F_1 = -F_o$

$£_2 = -\Theta_o (1 - i k_2 \tau_o\, \delta C_{T2})(P_1 - P_3) \sin\theta_2 \cos\theta_2$

$\Omega_2 = -\Theta_o d_3 (1 - i k_2 \tau_o\, \delta C_{T2}) \cos\theta_2$

$F_2 = i\rho C_E (1 - i k_2 \tau_o C_{T2}) - [(K_1 \sin^2\theta_2 + K_3 \cos^2\theta_2)/C_{T2}]$

$£_3 = -\Theta_o^m (1 - i k_3 \tau_o \delta C_{L3})(P_1^{\mathrm{m}} \sin^2\theta_3 + P_3^{\mathrm{m}} \cos^2\theta_3)$

$\Omega_3 = \Theta_o^m d_3^m (1 - i k_3 \tau_o \delta C_{L3}) \cos\theta_3$

$F_3 = i\rho^m C_E^m (1 - i k_3 \tau_o C_{L3}) - [(K_1^m \sin^2\theta_3 + K_3^m \cos^2\theta_3)/C_{L3}]$

$£_4 = -\Theta_o^m (1 - i k_4 \tau_o \delta C_{T4})(P_3^{\mathrm{m}} - P_1^{\mathrm{m}}) \sin\theta_4 \cos\theta_4$

$\Omega_4 = \Theta_o^m d_3^m (1 - i k_4 \tau_o\, \delta C_{T4}) \cos\theta_4$

$F_4 = i\rho^m C_E^m (1 - i k_4 \tau_o C_{T4}) - [(K_1^m \sin^2\theta_4 + K_3^m \cos^2\theta_4)/C_{T4}]$

From equations (18-20), It is easy to see that

$$\left.\begin{aligned}
(\Psi_{11} A_1 + \Psi_{12} A_2 + \Psi_{13} A_3 + \Psi_{14} A_4)/A_o &= l_1,\\
(\Psi_{21} A_1 + \Psi_{22} A_2 + \Psi_{23} A_3 + \Psi_{24} A_4)/A_o &= l_2,\\
(\Psi_{31} A_1 + \Psi_{32} A_2 + \Psi_{33} A_3 + \Psi_{34} A_4)/A_o &= l_3,\\
(\Psi_{41} A_1 + \Psi_{42} A_2 + \Psi_{43} A_3 + \Psi_{44} A_4)/A_o &= l_4
\end{aligned}\right\} \tag{21}$$

where

$\Psi_{12} = R_1/R,\ \Psi_{13} = R_2/R,\ \Psi_{14} = R_3/R$

$\Psi_{22} = \vartheta_1 (C_{44} \cos 2\theta_2 - \chi_2 e_{15} \sin\theta_2)/(C_{44} \sin 2\theta_o + \chi_o e_{15} \sin\theta_o)$

$\Psi_{23} = \vartheta_2 (C_{44}^m \sin 2\theta_3 + \chi_3 e_{15}^m \sin\theta_3)/(C_{44} \sin 2\theta_o + \chi_o e_{15} \sin\theta_o)$

$\Psi_{24} = -\vartheta_3 (C_{44}^m \cos 2\theta_4 - \chi_4 e_{15}^m \sin\theta_4)/(C_{44} \sin 2\theta_o + \chi_o e_{15} \sin\theta_o)$

$\Psi_{11} = \Psi_{21} = \Psi_{31} = \Psi_{41} = 1,\ \Psi_{32} = -\chi_2/\chi_o,\quad \Psi_{33} = \chi_3/\chi_o,\quad \Psi_{34} = \chi_4/\chi_o$

$\Psi_{42} = \upsilon_2/\upsilon_o,\ \Psi_{43} = -\upsilon_3/\upsilon_o,\ \Psi_{44} = -\upsilon_4/\upsilon_o,\ l_1 = l_4 = -1,\ l_2 = l_3 = 1$

$R_1 = \vartheta_1[(C_{13} - C_{33})\sin\theta_2\cos\theta_2 - \chi_2 e_{33}\cos\theta_2 - (P_3 \upsilon_2/ik_2)]$
$R_2 = -\vartheta_2[C_{13}^m\sin^2\theta_3 + C_{33}^m\cos^2\theta_3) + \chi_3 e_{33}^m\cos\theta_3 - (P_3^{\mathrm{m}}\upsilon_3/ik_3)]$
$R_3 = -\vartheta_3[(C_{33}^m - C_{13}^m)\sin\theta_4\cos\theta_4 + \chi_4 e_{33}^m\cos\theta_4 - (P_3^{\mathrm{m}}\upsilon_4/ik_4)$
$R = C_{13}\sin^2\theta_o + C_{33}\cos^2\theta_o + \chi_o e_{33}\cos\theta_o - (P_3\upsilon_o/ik_o)$
$R_1 = \vartheta_1[(C_{13} - C_{33})\sin\theta_2\cos\theta_2 - \chi_2 e_{33}\cos\theta_2 - (P_3 \upsilon_2/ik_2)]$
$R_2 = -\vartheta_2[C_{13}^m\sin^2\theta_3 + C_{33}^m\cos^2\theta_3) + \chi_3 e_{33}^m\cos\theta_3 - (P_3^{\mathrm{m}}\upsilon_3/ik_3)]$
$R_3 = -\vartheta_3[(C_{33}^m - C_{13}^m)\sin\theta_4\cos\theta_4 + \chi_4 e_{33}^m\cos\theta_4 - (P_3^{\mathrm{m}}\upsilon_4/ik_4)$
$R = C_{13}\sin^2\theta_o + C_{33}\cos^2\theta_o + \chi_o e_{33}\cos\theta_o - (P_3\upsilon_o/ik_o)$
$\chi_o = (E_o J_o - F_o\gamma_o)/(F_o H_o - \Omega_o J_o)$, $\chi_1 = (E_1 J_1 - F_1\gamma_1)/(F_1 H_1 - \Omega_1 J_1)$
$\chi_2 = (E_2 J_2 - F_2\gamma_2)/(F_2 H_2 - \Omega_2 J_2)$, $\chi_3 = (E_3 J_3 - F_3\gamma_3)/(F_3 H_3 - \Omega_3 J_3)$
$\chi_4 = (E_4 J_4 - F_4\gamma_4)/(F_4 H_4 - \Omega_4 J_4)$, $\upsilon_o = (M_o E_o - \Omega_o\gamma_o)/(\Omega_o J_o - F_o H_o)$
$\upsilon_1 = (H_1 E_1 - \Omega_1\gamma_1)/(\Omega_1 J_1 - F_1 H_1)$, $\upsilon_2 = (H_2 E_2 - \Omega_2\gamma_2)/(\Omega_2 J_2 - F_2 H_2)$
$\upsilon_3 = (H_3 E_3 - \Omega_3\gamma_3)/(\Omega_3 J_3 - F_3 H_3)$, $\upsilon_4 = (H_4 E_4 - \Omega_4\gamma_4)/(\Omega_4 J_4 - F_4 H_4)$

Solving equations (21), we can determine the reflection and transmission coefficients $A_1/A_o, A_2/A_o, A_3/A_o, A_4/A_o$ as:

$$A_1/A_o = \Delta_1/\Delta_o,\ A_2/A_o = \Delta_2/\Delta_o,\ A_3/A_o = \Delta_3/\Delta_o,\ A_4/A_o = \Delta_4/\Delta_o \tag{22}$$

Where

$$\Delta = \begin{vmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} \\ \Psi_{21} & \Psi_{22} & \Psi_{23} & \Psi_{24} \\ \Psi_{31} & \Psi_{32} & \Psi_{33} & \Psi_{34} \\ \Psi_{41} & \Psi_{42} & \Psi_{43} & \Psi_{44} \end{vmatrix},\ \Delta_1 = \begin{vmatrix} l_1 & \Psi_{12} & \Psi_{13} & \Psi_{14} \\ l_2 & \Psi_{22} & \Psi_{23} & \Psi_{24} \\ l_3 & \Psi_{32} & \Psi_{33} & \Psi_{34} \\ l_4 & \Psi_{42} & \Psi_{43} & \Psi_{44} \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} \Psi_{11} & l_1 & \Psi_{13} & \Psi_{14} \\ \Psi_{21} & l_2 & \Psi_{23} & \Psi_{24} \\ \Psi_{31} & l_3 & \Psi_{33} & \Psi_{34} \\ \Psi_{41} & l_4 & \Psi_{43} & \Psi_{44} \end{vmatrix},\ \Delta_3 = \begin{vmatrix} \Psi_{11} & \Psi_{12} & l_1 & \Psi_{14} \\ \Psi_{21} & \Psi_{22} & l_2 & \Psi_{24} \\ \Psi_{31} & \Psi_{32} & l_3 & \Psi_{34} \\ \Psi_{41} & \Psi_{42} & l_4 & \Psi_{44} \end{vmatrix},$$

$$\Delta_4 = \begin{vmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & l_1 \\ \Psi_{21} & \Psi_{22} & \Psi_{23} & l_2 \\ \Psi_{31} & \Psi_{32} & \Psi_{33} & l_3 \\ \Psi_{41} & \Psi_{42} & \Psi_{43} & l_4 \end{vmatrix}$$

By using equations (18-20) we get:

$$B_1/B_o = -\ A_1/A_o,\ B_2/B_o = \chi_2 A_2/\chi_o A_o,\quad B_3/B_o = \chi_3 A_3/\chi_o A_o,\ B_4/B_o = \chi_4 A_4/\chi_o A_o,\qquad C_1/C_o = \ A_1/A_o,\ C_2/C_o = \upsilon_2 A_2/\upsilon_o A_o,\ C_3/C_o = \upsilon_3 A_3/\upsilon_o A_o,\ C_4/C_o = \upsilon_4 A_4/\upsilon_o A_o \tag{23}$$

5. Numerical results and discussion

For the purpose of numerical computations, the following physical constants are considered for upper and lower generalized piezo-thermoelastic media under initial stress and relaxation time in the context of L-S theory:

The physical data for upper medium (Cadmium Selenide) is given as [20]:

$C_{11}^{m} = 7.41\times 10^{10}\ \mathrm{Nm^{-2}}, C_{12}^{m} = 4.52\times 10^{10}\ \mathrm{Nm^{-2}},$

$C_{13}^{m} = 3.93\times 10^{10}\ \mathrm{Nm^{-2}}, C_{33}^{m} = 8.36\times 10^{10}\ \mathrm{Nm^{-2}},$

$C_{44}^{m} = 1.32\times 10^{10}\ \mathrm{Nm^{-2}}, \rho^{m} = 5504\ \mathrm{Kgm^{-3}}, e_{13}^{m} = -0.160\ \mathrm{Cm^{-2}},$

$e_{33}^{m} = 0.347\ \mathrm{Cm^{-2}}, e_{15}^{m} = -0.138\ \mathrm{Cm^{-2}}, \Theta_{o}^{m} = 298\ \mathrm{K},$

$P_{1}^{m} = 0.621\times 10^{6}\ \mathrm{NK^{-1}m^{-2}}, P_{3}^{m} = 0.551\times 10^{6}\ \mathrm{NK^{-1}m^{-2}},$

$d_{3}^{m} = -2.94\times 10^{-6}\mathrm{CK^{-1}m^{-2}}, K_{1}^{m} = K_{3}^{m} = 9\ \mathrm{Wm^{-1}K^{-1}}$

$\epsilon_{11}^{m} = 8.26\times 10^{-11}\mathrm{C^{2}\ N^{-1}m^{-2}}, \epsilon_{33}^{m} = 9.03\times 10^{-11}\ \mathrm{C^{2}\ N^{-1}m^{-2}}$

$C_{E}^{m} = 260\ \mathrm{J\,Kg^{-1}K^{-1}}, \omega^{m} = 2.14\times 10^{13}\mathrm{s^{-1}}.$

The physical data for lower medium (Lead Zirconate Titanate ceramics) is given as [21]:

$C_{11} = 13.9\times 10^{10}\ \mathrm{Nm^{-2}}, C_{12} = 7.78\times 10^{10}\ \mathrm{Nm^{-2}},$

$C_{13} = 7.54\times 10^{10}\ \mathrm{Nm^{-2}}, C_{33} = 11.3\times 10^{10}\ \mathrm{Nm^{-2}},$

$C_{44} = 2.56\times 10^{10}\mathrm{Nm^{-2}}, \rho = 7750\ \mathrm{Kgm^{-3}}, e_{13} = -6.98\ \mathrm{Cm^{-2}},$

$e_{33} = 13.8\ \mathrm{Cm^{-2}}, e_{15} = 13.4\ \mathrm{Cm^{-2}}, \Theta_{o} = 298\ \mathrm{K},$

$P_{1} = 1.52\times 10^{6}\ \mathrm{NK^{-1}m^{-2}}, P_{3} = 1.53\times 10^{6}\ \mathrm{NK^{-1}m^{-2}},$

$d_{3} = -452\times 10^{-6}\mathrm{CK^{-1}m^{-2}}, K_{1} = K_{3} = 1.5\ \mathrm{Wm^{-1}K^{-1}},$

$\epsilon_{11} = 60.0\times 10^{-10}\mathrm{C^{2}\ N^{-1}m^{-2}}, \epsilon_{33} = 54.7\times 10^{-10}\ \mathrm{C^{2}\ N^{-1}m^{-2}}$

$C_{E} = 420\ \mathrm{J\,Kg^{-1}K^{-1}}, \omega = 2.14\times 10^{13}\mathrm{s^{-1}}.$

The variations of phase velocities computed from

$$c_{L\circ} = c_{L1} = \frac{\sqrt{C_{44}+C_{11}sin^{2}\alpha + C_{33}cos^{2}\alpha + v_{1}}}{\sqrt{2\rho}},\quad c_{T2} = \frac{\sqrt{C_{44}+C_{11}sin^{2}\alpha + C_{33}cos^{2}\alpha - v_{1}}}{\sqrt{2\rho}}$$

$$c_{L3} = \frac{\sqrt{C_{44}^{m}+C_{11}^{m}sin^{2}\beta + C_{33}^{m}cos^{2}\beta + v_{2}}}{\sqrt{2\rho^{m}}},\quad c_{L3} = \frac{\sqrt{C_{44}^{m}+C_{11}^{m}sin^{2}\beta + C_{33}^{m}cos^{2}\beta - v_{2}}}{\sqrt{2\rho^{m}}}$$

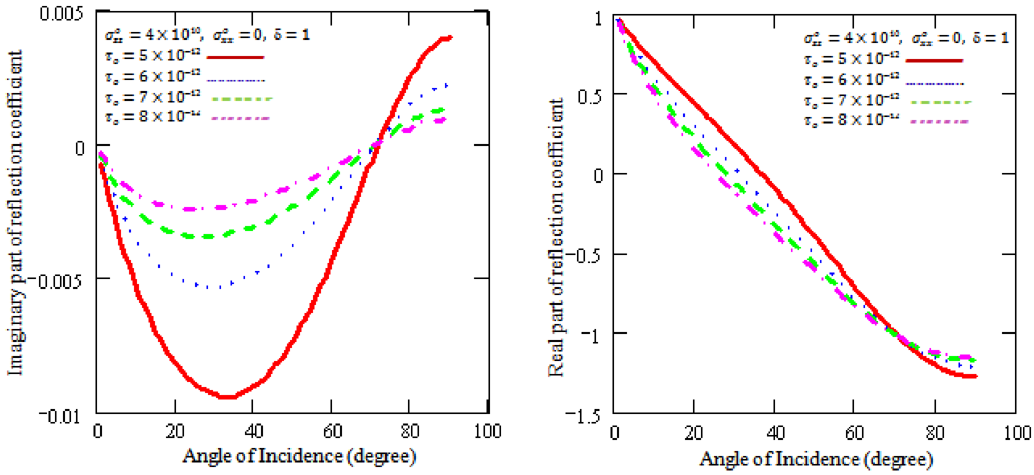
where

$$v_{1} = \sqrt{\left[\left(C_{11}-C_{44}\right)sin^{2}\alpha + \left(C_{44}-C_{33}\right)cos^{2}\alpha\right]^{2} + \left(C_{13}+C_{44}\right)^{2} sin^{2} 2\alpha}$$

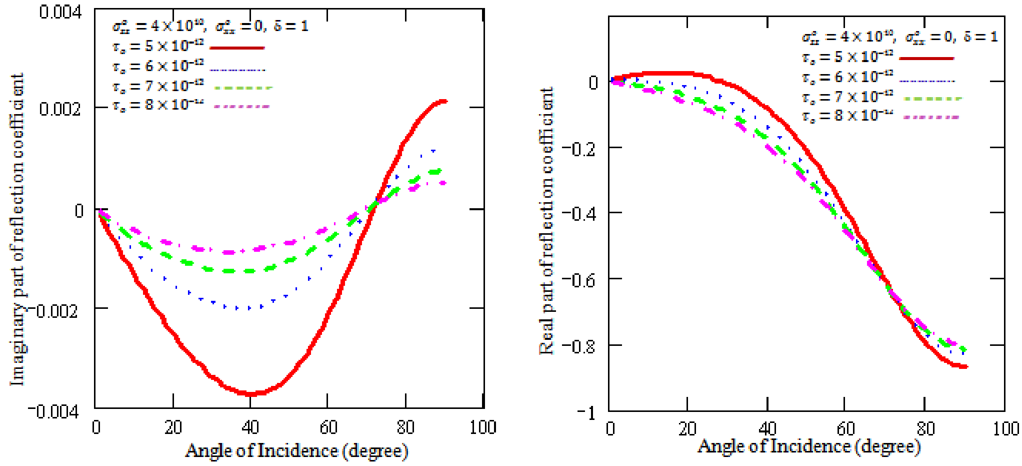
$$v_2 = \sqrt{\left[\left(C_{11}^m - C_{44}^m\right)\sin^2\beta + \left(C_{44}^m - C_{33}^m\right)\cos^2\beta\right]^2 + \left(C_{13}^m + C_{44}^m\right)^2 \sin^2 2\beta}.$$

The variations of the reflection and transmission coefficients (A_i/A_o, C_i/C_o, B_i/B_o (*i=1,2,3,4*) corresponds on qL, qSV ,Θ , and φ waves with the angle of incidence wave under thermal relaxation time τ_o = (5,6,7,8) .10^{-12} pico – sec. when $\sigma_{33}^o = 4 \cdot 10^{10}$ are shown graphically in figures (2-5). We also show the thermal relaxation time effect on electric potential and thermal waves in figures (6-11).

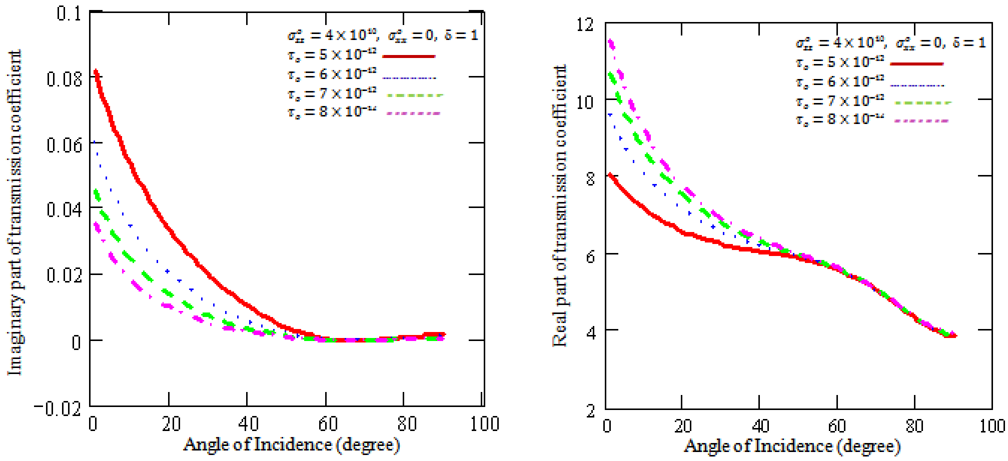
Figures (12-21) depict the variations for reflection and transmission coefficients of reflected and transmitted qL, qSV, Θ ,and φ waves under various values of initial stresses ($\sigma_{11}^o = 0$, $\sigma_{33}^o = (2,3,4,5) \times 10^{10}$), when $\tau_o = 4\,.10^{-12}$. The reflection coefficients of reflected qL wave are shown in figures 12. The reflection coefficients for this wave decrease with the increase in the angle of incidence until it reaches the minimum value near $\theta_o = 43^o$ for $Im(A_1/A_o)$ and near $\theta_o = 90^o$ for $Re(A_1/A_o)$. Also, the imaginary coefficient decreases as long as the initial stresses increases, whereas the real coefficient increases. We show the same behavior for reflection coefficients of reflected qSV wave in figures 13. The transmission coefficients of transmitted qL wave (real and imaginary parts) are shown graphically in figures 14. The transmission coefficient for this wave has its minimum value near $\theta_o = 90^o$. The transmission coefficients of transmitted qSV wave (real and imaginary parts) are shown graphically in figures 15. The transmission coefficient for this wave has its maximum value near $\theta_o = 20^o$ for imaginary part and near $\theta_o = 35^o$ for real part. Also, the A_3/A_o and A_4/A_o coefficients decrease as long as the initial stresses increases in figures 16 and 17. It can get some of particular case through neglecting the thermal effect and the relaxation time as [19].



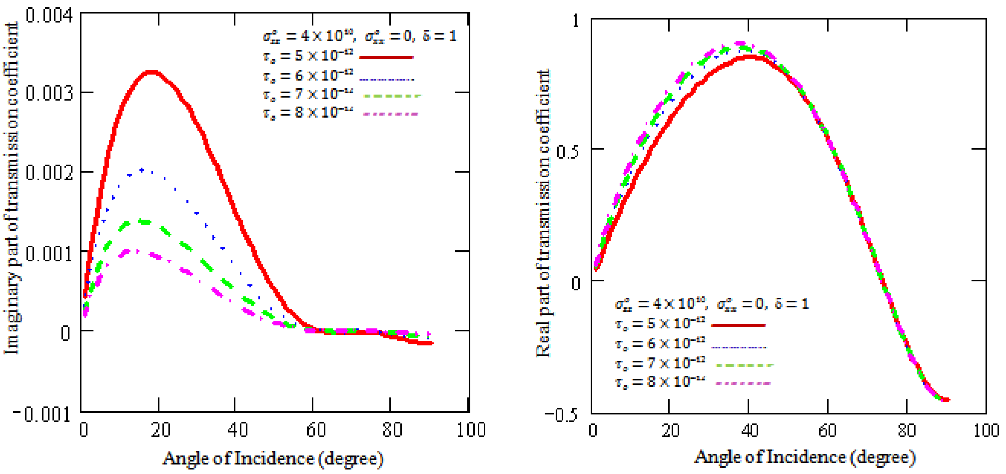
Figures 2. Imaginary and real parts of reflection coefficient (A_1/A_o) as a function of incidence angle θ_o for various values of the relaxation times for (L-S) model.



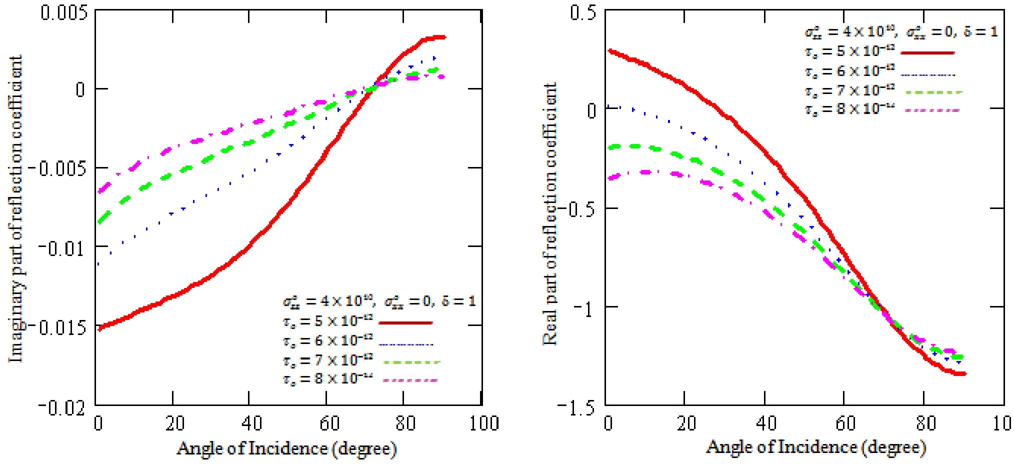
Figures 3. Imaginary and real parts of reflection coefficient (A_2/A_o) as a function of incidence angle θ_o for various values of the relaxation times for (L-S) model.



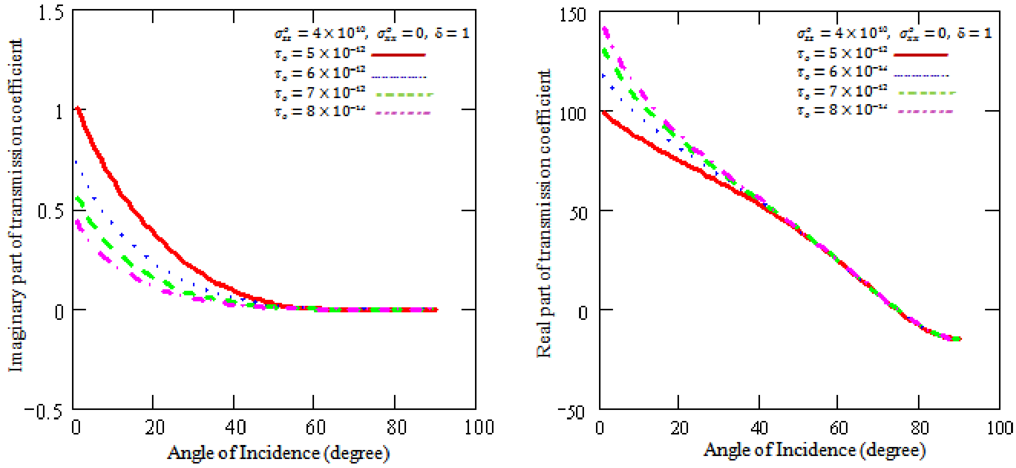
Figures 4. Imaginary and real parts of transmission coefficient (A_3/A_o) as a function of incidence angle θ_o for various values of the relaxation times for (L-S) model.



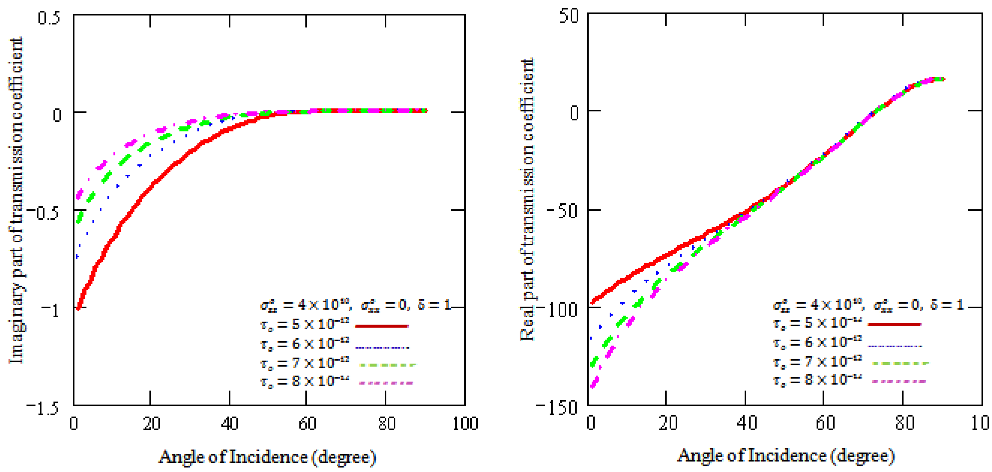
Figures 5. Imaginary and real parts of transmission coefficient (A_4/A_o) as a function of incidence angle θ_o for various values of the relaxation times for (L-S) model.



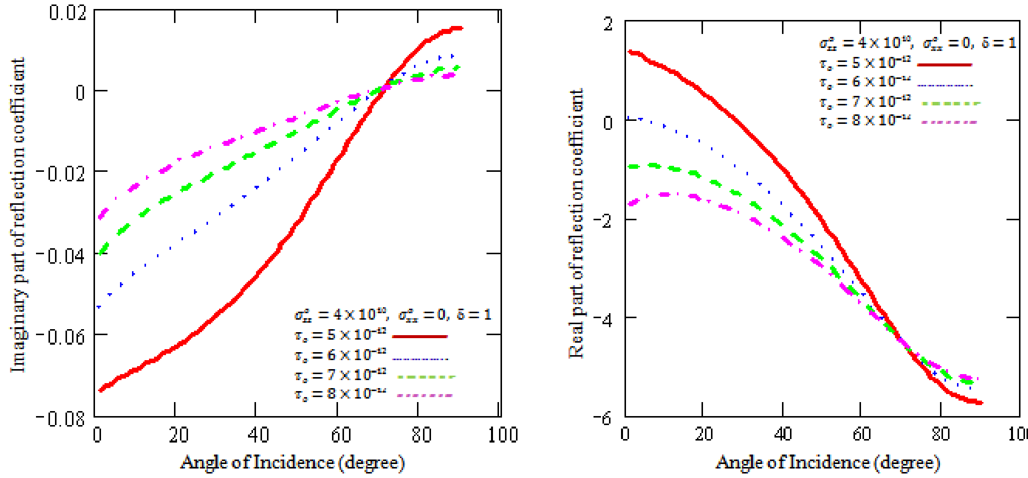
Figures 6. Imaginary and real parts of reflection coefficient (B_2/B_o) as a function of incidence angle θ_o for various values of the relaxation times for (L-S) model.



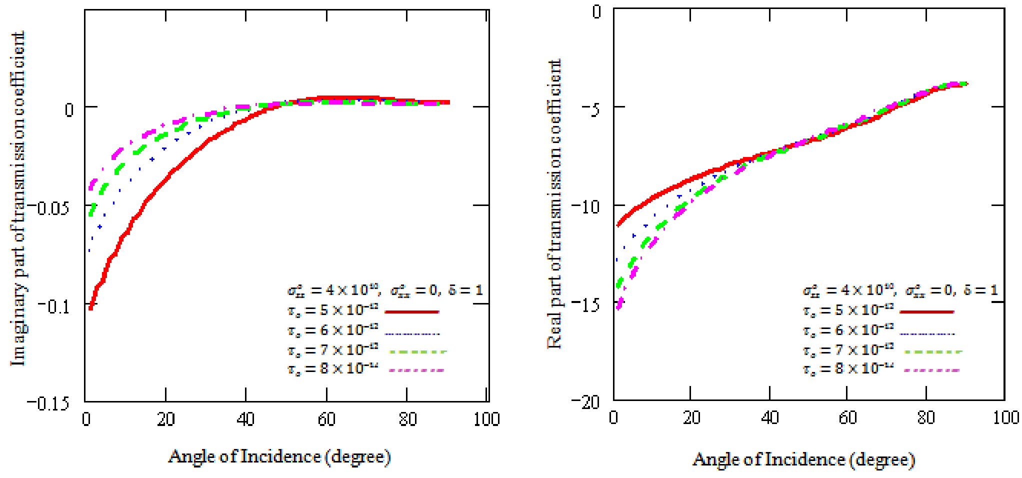
Figures 7. Imaginary and real parts of transmission coefficient (B_3/B_o) as a function of incidence angle θ_o for various values of the relaxation times for (L-S) model.



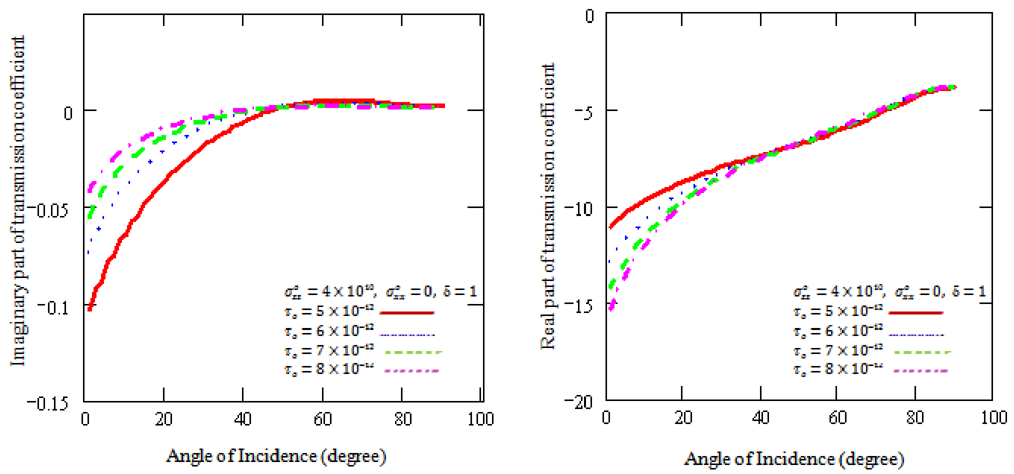
Figures 8. Imaginary and real parts of transmission coefficient (B_4/B_o) as a function of incidence angle θ_o for various values of the relaxation times for (L-S) model.



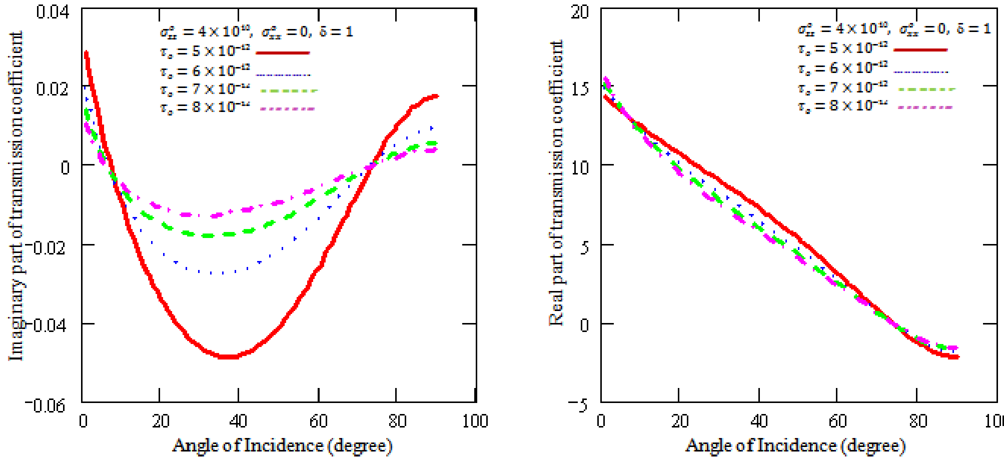
Figures 9. Imaginary and real parts of reflection coefficient (C_2/C_0) as a function of incidence angle θ_0 for various values of the relaxation times for (L-S) model.



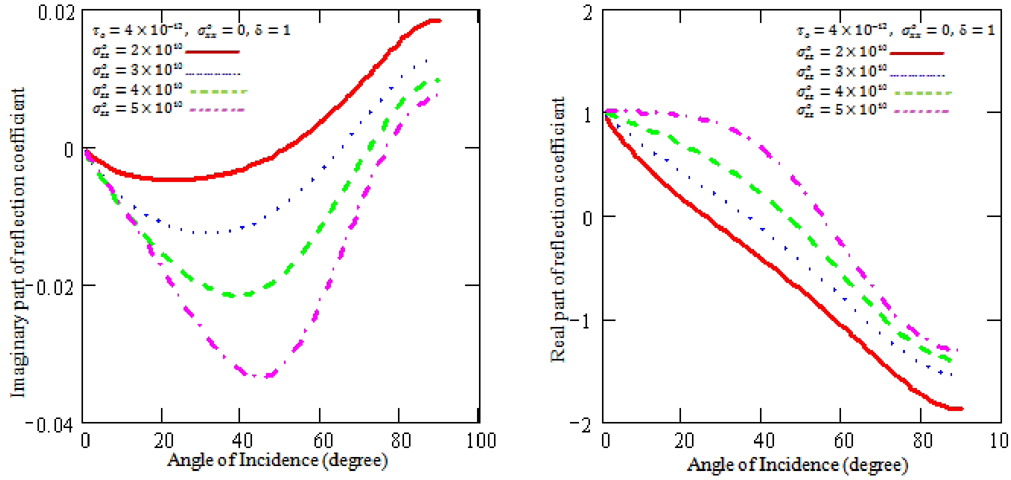
Figures 10. Imaginary and real parts of transmission coefficient (C_3/C_0) as a function of incidence angle θ_0 for various values of the relaxation times for (L-S) model.



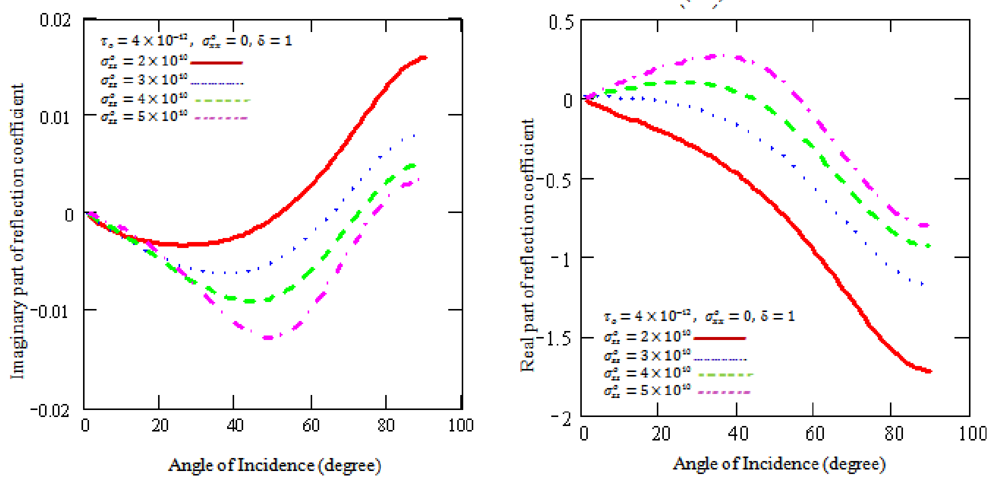
Figures 10. Imaginary and real parts of transmission coefficient (C_3/C_0) as a function of incidence angle θ_0 for various values of the relaxation times for (L-S) model.



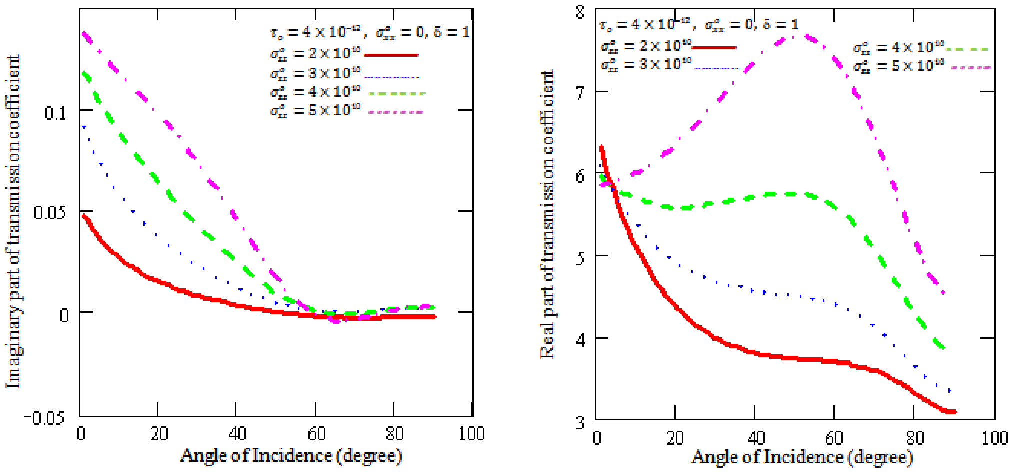
Figures 11. Imaginary and real parts of transmission coefficient (C_4/C_o) as a function of incidence angle θ_o for various values of the relaxation times for (L-S) model.



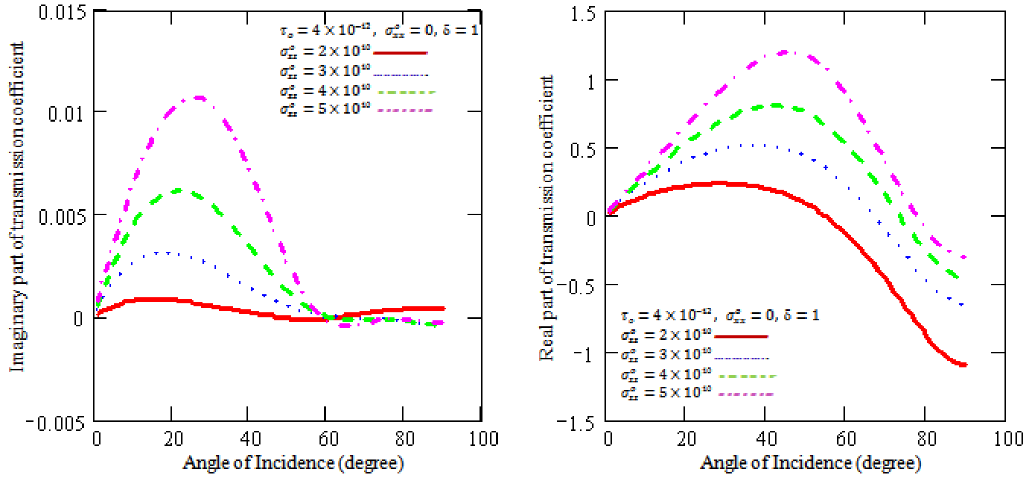
Figures 12. Imaginary and real parts of reflection coefficient (A_1/A_o) as a function of incidence angle θ_o under effect of different values of the initial stresses for (L-S) model.



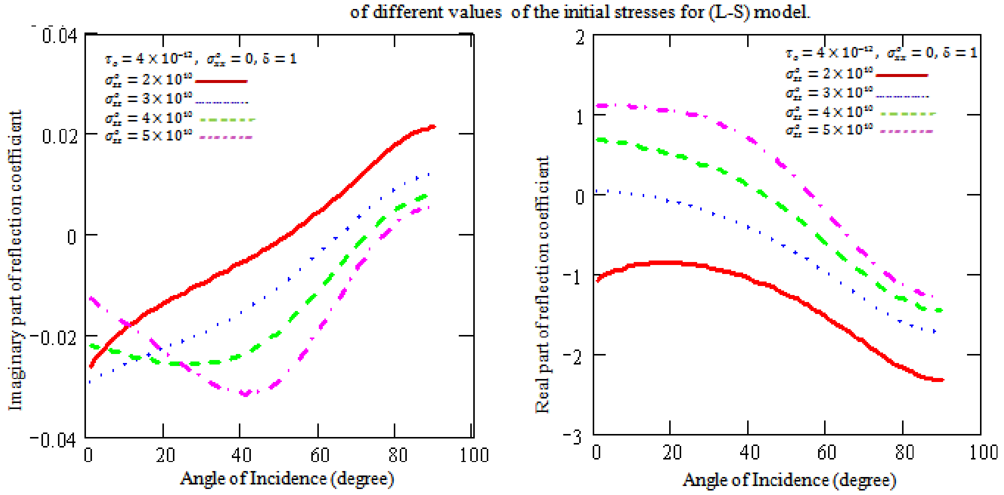
Figures 13. Imaginary and real parts of reflection coefficient (A_2/A_o) as a function of incidence angle θ_o under effect of different values of the initial stresses for (L-S) model.



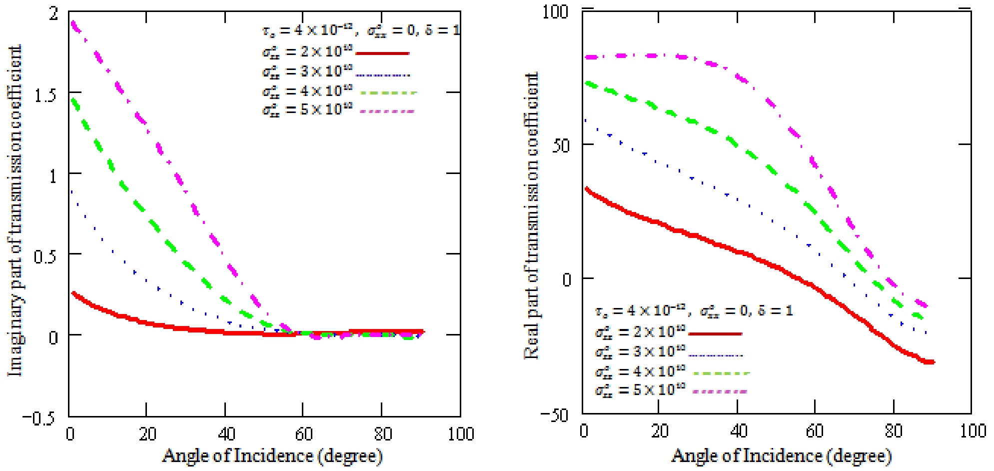
Figures 14. Imaginary and real parts of transmission coefficient (A_3/A_o) as a function of incidence angle θ_o under effect of different values of the initial stresses for (L-S) model.



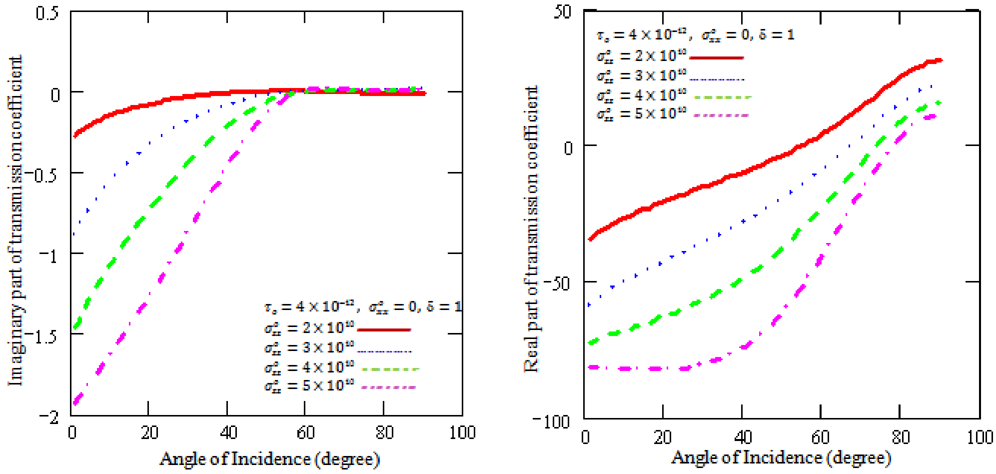
Figures 15. Imaginary and real parts of transmission coefficient (A_4/A_o) as a function of incidence angle θ_o under effect of different values of the initial stresses for (L-S) model.



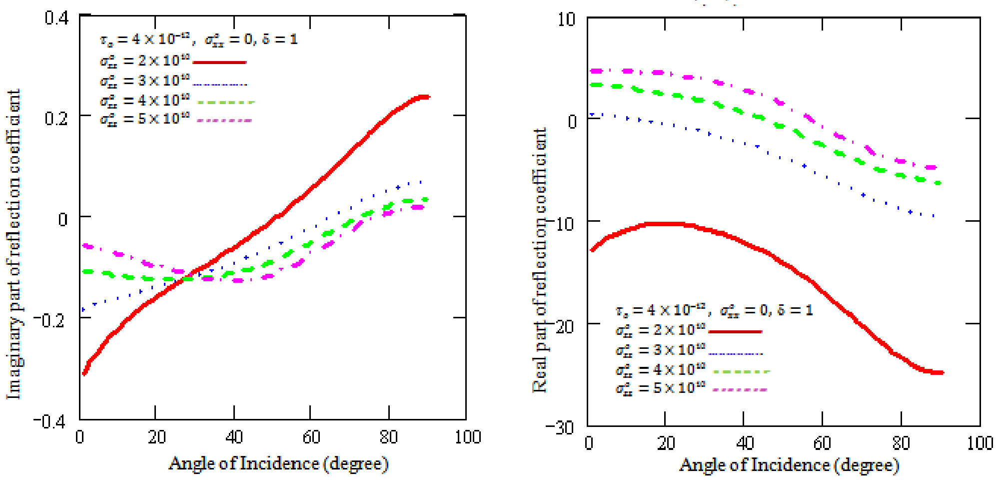
Figures 16. Imaginary and real parts of reflection coefficient (B_2/B_o) as a function of incidence angle θ_o under effect of different values of the initial stresses for (L-S) model.



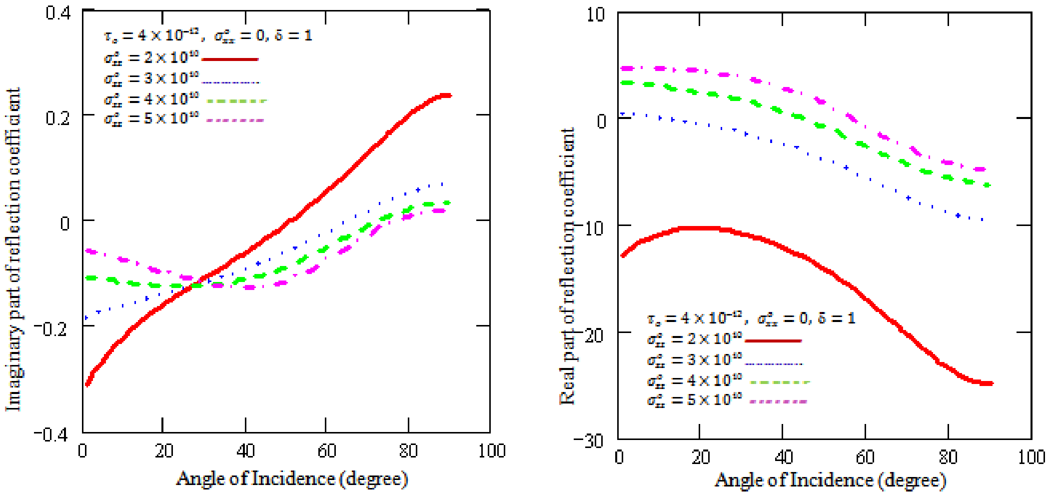
Figures 17. Imaginary and real parts of transmission coefficient (B_3/B_o) as a function of incidence angle θ_o under effect of different values of the initial stresses for (L-S) model.



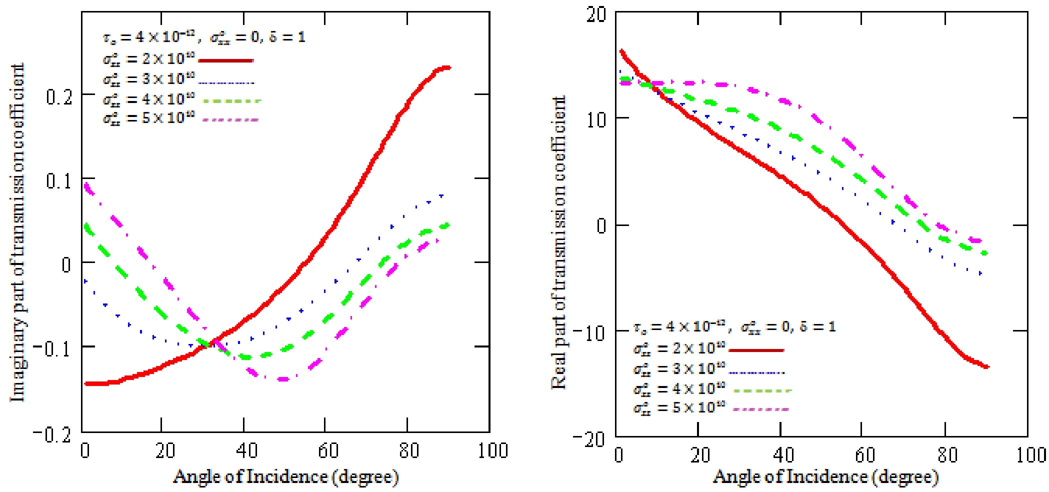
Figures 18. Imaginary and real parts of transmission coefficient (B_4/B_o) as a function of incidence angle θ_o under effect of different values of the initial stresses for (L-S) model.



Figures 19. Imaginary and real parts of reflection coefficient (C_2/C_o) as a function of incidence angle θ_o under effect of different values of the initial stresses for (L-S) model.



Figures 19. Imaginary and real parts of reflection coefficient (C_2/C_o) as a function of incidence angle θ_o under effect of different values of the initial stresses for (L-S) model.



Figures 21. Imaginary and real parts of transmission coefficient (C_4/C_o) as a function of incidence angle θ_o under effect of different values of the initial stresses for (L-S) model.

CONCLUSIONS

The aim of present paper is to study the problem of wave propagation (reflection/ transmission) of plane waves incident under initial stress effect and relaxation time of a homogeneous, hexagonal generalized piezo-thermoelastic media. For two dimensional model in L-S theory, there exist four waves namely, qL, qSV, φ and Θ. These waves are dispersive in nature and their velocity of propagation also depends on angle of incidence. The amplitude ratios of reflected and transmitted waves to that of incident waves have been obtained and presented graphically. It can get some previous studies as a special case through neglecting the thermal effects and the relaxation times as [19].

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