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Visco-Elastic Free Convection Hydro Magnetic Flow Past: An Infinite Plate with Constant Suction

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Abstract

The purpose of the present paper is to study the Visco-elastic free convection hydro magnetic flow past an infinite plate. We have investigated the behavior of unsteady free convective flow of a Visco-elastic fluid past an infinite porous plate with constant suction under the action of the time-dependent plate temperature. A uniform magnetic field has been applied transversely to the porous plate and the magnetic lines of force are taken to be fixed relative to the fluid. We have considered that the plate temperature oscillates about a constant mean in magnitude but not in direction. We have calculated and tabulated the real part M_r and Imaginary part M_i of the fluctuating part of the velocity and some variations are represented graphically. It has been concluded that the velocity distribution inside the boundary layer lags behind the wall fluctuation by a constant angle ϕ.

Keywords: Visco-Elastic fluid, Infinite porous plate, velocity distribution, velocity fluctuation, unsteady convective flow

2000 AMS Subject Classification No.: 76R10, 76A10, 76W05

Introduction

The problem of free convection hydro magnetic flow past an infinite plate in presence of constant suction, attracted many scientists in view of its application in Aerodynamics, Astrophysics, Geophysics, Ground water hydrology and Engineering.

Many papers have been published on the theory of laminar boundary layers in unsteady flow. Nanda and Sharma [7] have discussed possibility similarity solution of unsteady free convection flow past a vertical plate with suction. Pop [8] has studied effect of periodic suction on the unsteady free convective flow past a vertical porous flat plate. Lal [6] obtained solutions for unsteady free convection flow of laminar power law fluids past a porous vertical wall. He found that if the wall temperature and suction velocity vary as $t^{-1/2}$, the similarity solution is possible. Soundalgekar [14] analysed unsteady free convection flow of an elastico-viscous fluid past an infinite plate with suction. Free convection and mass transfer flow through a porous medium bounded by an infinite vertical porous plate with constant heat flux have been discussed by Raptis et al. [9]. Sen [11] developed on visco-elastic free convection boundary layer flow past an infinite plate with constant suction. Kumari et al. [5] studied free convection boundary layer flow of a Non-Newtonian fluid along a vertical wavy wall.

Chowdhury and Islam [2] discussed MHD free convection flow of visco-elastic fluid past an infinite vertical porous plate. Ghosh and Ghosh [3] investigated MHD flow of a visco-elastic fluid through porous medium. Singh and Gupta [12] have also studied unsteady free convective flow of an electrically conducting fluid along a porous hot vertical plate in the presence of heat source with mass transfer. Bhargava et al. [1] obtained numerical solution of free convection MHD Micropolar fluid between two parallel porous vertical plates. Samaria et al. [10] analysed Hydro Magnetic free- convection laminar flow of an elasto-viscous fluid past an infinite plate. Hazem Attia [4] developed unsteady flow of a dusty conducting fluid between parallel porous plates with temperature dependent viscosity. Sreehari Reddy et al. [13] investigated Hydro Magnetic elastic free convection of a conducting elastico-viscous liquid between heated vertical plates. Recently, Sharma et al [12] studied effect of Radiation on MHD flow of viscoelastic (Rivlin-Ericksen) fluid through porous medium.

In this paper, we have derived the separate expression for the real part M_r and the imaginary part M_i of the fluctuating part of the velocity and their numerical study is presented. The phase angle $|\phi|$ and $M = \sqrt{M_r^2 + M_i^2}$ are calculated, tabulated and also represented graphically.

Nomenclature

B_0	= magnetic induction
c'_p	= specific heat at constant pressure
f_x	= acceleration due to gravity
G	= Grashof number
k_0	= elastic constant
M_r, M_i	= fluctuating parts of the velocity for $P \neq 1$
P	= Prandtl number
T'	= temperature in the boundary layer

T'_∞ = temperature of the fluid far away from the plate
T'_w = plate temperature
t' = time
(u', v') = velocity components
v_0 = suction velocity
x, y = co-ordinate axes along and normal to the plate

Greek symbols
ρ' = density
β = coefficient of volume expansion
η_0 = viscosity
λ' = thermal conductivity
υ = kinematic viscosity
ω = frequency
ε = constant
ϕ = phase angle

Mathematical Formulation of the Problem

The basic equations are:

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\rho'\frac{\partial u'}{\partial t'} + \rho' v' \frac{\partial u'}{\partial y'} = \rho' f_x \beta\left(T' - T'_\infty\right) + \eta_0 \frac{\partial^2 u'}{\partial y'^2} - k_0\left\{\frac{\partial^3 u'}{\partial y'^2 \partial t'} + v'\frac{\partial^3 u'}{\partial y'^3} - 3\frac{\partial u'}{\partial y'}.\frac{\partial^2 v'}{\partial y'^2}\right.$$
$$\left. -2\frac{\partial v'}{\partial y'}\frac{\partial^2 u'}{\partial y'^2}\right\} - \sigma B_0{}^2 u' \tag{2}$$

$$\rho'\frac{\partial v'}{\partial t'} + \rho' v' \frac{\partial v'}{\partial y'} = -\frac{\partial p'}{\partial y'} + 2\eta_0\frac{\partial^2 v'}{\partial y'^2} - 2k_0\left\{\frac{\partial^3 v'}{\partial y'^2 \partial t'} + v'\frac{\partial^3 v'}{\partial y'^3} - 3\frac{\partial v'}{\partial y'}\frac{\partial^2 v'}{\partial y'^2}\right\} \tag{3}$$

$$\rho' c'_p\left\{\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'}\right\} = \lambda'\frac{\partial^2 T'}{\partial y'^2} \tag{4}$$

The origin is taken at any point of a flat vertical porous infinite plate. The x'– axis is chosen along the plate vertically upwards and the y'– axis perpendicular to it. u' is the velocity in the direction of x'– axis and v' the velocity normal to the plate; t' the time variable; η_0 the limiting viscosity at small rate of shear and k_0 the elastic constant. Also, f_x is the acceleration due to gravity; β the coefficient of volume expansion; c'_p the specific heat at constant pressure; λ' thermal conductivity; T' the temperature in the boundary layer and T'_∞ the temperature of fluid far away from the plate. In the equation (4), terms representing viscous and elastic dissipation are assumed to be neglected. $-\frac{\sigma B_0^{\ 2} u'}{\rho'}$ is the value of $\vec{J} \times \vec{B}$, $\vec{J}$ and $\vec{B}$ being given by maxwell's equation in Ohm's Law namely:

Curl $\vec{H} = 4\pi \vec{J}$

Div $\vec{B} = 0$

Curl $\vec{E} = -\frac{\partial \vec{B}}{\partial t'}$

where $\vec{J} = \sigma \left[\vec{E} + \left(\vec{u'} \times \vec{B} \right) \right]$

From (1), for constant suction velocity v_0,

$$v' = -v_0 \tag{5}$$

applying (5) in equations (2) and (3), we get

$$\frac{\partial u'}{\partial t'} - v_0 \frac{\partial u'}{\partial y'} = f_x \beta \left(T' - T'_\infty \right) + \upsilon \frac{\partial^2 u'}{\partial y'^2} - k^* \left(\frac{\partial^3 u'}{\partial y'^2 \partial t'} - v_0 \frac{\partial^3 u'}{\partial y'^3} \right) - mu' \tag{6}$$

where

$$\upsilon = \frac{\eta_0}{\rho'},\ k^* = \frac{k_0}{\rho'} \text{ and } m = \frac{\sigma B_0^{\ 2}}{\rho'}$$

$$\frac{\partial v'}{\partial t'} = -\frac{1}{\rho'} \frac{\partial p'}{\partial y'} \tag{7}$$

The boundary conditions are:

$$\begin{cases} u' = 0,\ T' = T'_w(t) \text{ at } y' = 0 \\ u' \to 0,\ T' \to 0 \text{ as } y' \to \infty \end{cases} \tag{8}$$

Introducing the following non-dimensional quantities (in eq. 8.):

$$\begin{cases} \eta = \frac{v_0 y'}{\upsilon},\ t = \frac{v_0^{\ 2} t'}{4\upsilon},\ u = \frac{u'}{G v_0} \\ k = k^* \frac{v_0^{\ 2}}{\upsilon^2},\ T = \frac{T' - T'_\infty}{T'_w - T'_\infty} \end{cases} \tag{9}$$

Where

$$G = \frac{\upsilon f_x \beta \left(T'_w - T'_\infty \right)}{v_0^{\ 3}} \text{ (Grashof number)},\ P = \frac{\eta_0 c'_p}{\lambda'} \text{ (Prandtl number)}$$

Equations (6) & (4) then reduce to,

$$\frac{\partial^2 u}{\partial \eta^2} + \frac{\partial u}{\partial \eta} - \frac{1}{4} \frac{\partial u}{\partial t} - k \left[\frac{1}{4} \frac{\partial^3 u}{\partial t \partial \eta^2} - \frac{\partial^3 u}{\partial \eta^3} \right] - Mu = -T \tag{10}$$

where

$$M = \frac{m\upsilon}{v_0^{\ 2}}$$

$$\frac{\partial^2 T}{\partial \eta^2} + P\frac{\partial T}{\partial \eta} - \frac{P}{4}\frac{\partial T}{\partial t} = 0 \tag{11}$$

The boundary conditions (8) now reduce to,

$$\begin{cases} u = 0, \ \ T = T_W(t) \text{ at } \eta = 0 \\ u \to 0, \ \ T \to 0 \text{ as } \eta \to \infty \end{cases} \tag{12}$$

Let the temperature and the velocities in the neighborhood of the plate are assumed to be Pop [8]:

$$T(\eta, t) = \{1 - f_1(\eta)\} + \varepsilon e^{i\omega t}\{1 - f_2(\eta)\} \tag{13}$$

and

$$u(\eta, t) = g_1(\eta) + \varepsilon e^{i\omega t} g_2(\eta) \tag{14}$$

respectively.

Substituting (13) in (11) and solving after comparing the harmonic terms, we have

$$f_1(\eta) = 1 - e^{-P\eta} \tag{15}$$

$$f_2(\eta) = 1 - e^{-PH\eta} \tag{16}$$

Where

$$H = \frac{1}{2}\left\{1 + \left(1 + \frac{i\omega}{P}\right)^{1/2}\right\}$$

and f_1, f_2 satisfy the reduced boundary conditions:

$$\begin{cases} f_1 = f_2 = 0 \text{ at } \eta = 0 \\ f_1 \to 1, \ \ f_2 \to 1 \text{ as } \eta \to \infty \end{cases} \tag{17}$$

Substituting (13) and (14) in (10) and comparison of harmonic terms yield to,

$$kg_1''' + g_1'' + g_1' - Ng_1 = f_1 - 1 = -e^{-P\eta} \tag{18}$$

$$kg_2''' + \left(1 - \frac{k}{4}i\omega\right)g_2'' + g_2' - \frac{i\omega}{4}g_2 - Mg_2 = f_2 - 1 = -e^{-PH\eta} \tag{19}$$

Where corresponding boundary conditions now become:

$$\begin{cases} g_1 = g_2 = 0 \text{ at } \eta = 0 \\ g_1 \to 0, \ \ g_2 \to 0 \text{ as } \eta \to \infty \end{cases} \tag{20}$$

Equation (18) & (19) reduce to those governing the flow of a Newtonian fluid, if

$k=0$.

Let us assume that the solution in the forms Soundalgekar [14]:

$$g_1 = g_{01} + kg_{11} + O\left(k^2\right) \tag{21}$$

$$g_2 = g_{02} + kg_{12} + O\left(k^2\right) \tag{22}$$

Inserting (21) & (22) in (18) & (19) and equating the coefficient of k, we obtain after substituting for f_1 & f_2 respectively from (15) & (16):

$$g''_{01} + g'_{01} - Mg_{01} = -e^{-P\eta} \tag{23}$$

$$g''_{11} + g'_{11} - Mg_{11} = -g'''_{01} \tag{24}$$

$$g''_{02} + g'_{02} - \frac{i\omega}{4}g_{02} - Mg_{02} = -e^{-PH\eta} \tag{25}$$

$$g''_{12} + g'_{12} - \frac{i\omega}{4}g_{12} - Mg_{12} = -g'''_{02} + \frac{i\omega}{4}g''_{02} \tag{26}$$

The corresponding boundary conditions on g_{01}, g_{11}, g_{02} & g_{12} are:

$$\left\{\begin{array}{l} g_{01} = g_{11} = g_{02} = g_{12} = 0 \text{ at } \eta = 0 \\ g_{01},\ g_{11},\ g_{02},\ g_{12} \to 0 \text{ as } \eta \to \infty \end{array}\right. \tag{27}$$

On solving equations (23) to (26) which satisfy (27), the velocity field in the boundary layer is obtained for $P \neq 1$, as:

$$u = g_1 + \varepsilon e^{i\omega t} g_2 \tag{28}$$

Where

$$g_1 = \frac{e^{-\alpha_2\eta} - e^{-P\eta}}{P^2 - P - M} + kP^3\left\{\frac{e^{-\alpha_2\eta} - e^{-P\eta}}{\left(P^2 - P - M\right)^2}\right\}$$

$$g_2 = \frac{e^{-\beta_2\eta} - e^{-PH\eta}}{P^2H^2 - PH - M - \frac{i\omega}{4}} + \frac{kP^2H^2\left(e^{-\beta_2\eta} - e^{-PH\eta}\right)\left(PH + \frac{i\omega}{4}\right)}{\left(P^2H^2 - PH - M - \frac{i\omega}{4}\right)^2}$$

and

$$\beta_2 = \left(A_1 + B_1 i\right),\ \alpha_2 = \frac{1 + (1+4M)^{1/2}}{2},\ A_1 = \frac{1 + (1+4M)^{1/2} + \frac{\omega^2}{8(1+4M)^{3/2}}}{2}$$

$$B_1 = \frac{\omega}{4(1+4M)^{1/2}},\ H = C_1 + iD_1,\ C_1 = 1 + \frac{1}{16}.\frac{\omega^2}{P^2},\ D_1 = \frac{\omega}{4P}$$

We may write,

$$u = g_1 + \varepsilon\left(M_r \cos\omega t - M_i \sin\omega t\right) \tag{29}$$

(The imaginary part being neglected for obvious reasons) where M_r, M_i are the real and imaginary parts of the fluctuating part of the velocity when $P \neq 1$ and are given by,

$$M_r + iM_i = g_2 \tag{30}$$

Equating real and imaginary parts, we get

$$M_r = \frac{E_1G_1 + F_1H_1}{G_1^2 + H_1^2} + \frac{kP^2\left[\left(E_1I_1 - F_1J_1\right)\left(G_1^2 - H_1^2\right) + 2G_1H_1\left(E_1J_1 + F_1I_1\right)\right]}{\left(G_1^2 + H_1^2\right)^2} \tag{31}$$

$$M_i = \frac{F_1G_1 - E_1H_1}{G_1^2 + H_1^2} + \frac{kP^2\left[\left(E_1J_1 + F_1I_1\right)\left(G_1^2 - H_1^2\right) - 2G_1H_1\left(E_1I_1 - F_1J_1\right)\right]}{\left(G_1^2 + H_1^2\right)^2} \tag{32}$$

where

$$\begin{aligned}
I_1 &= \left(C_1^2 - D_1^2\right)PC_1 - 2C_1D_1\left(PD_1 + \frac{\omega}{4}\right)\\
J_1 &= \left(C_1^2 - D_1^2\right)\left(PD_1 + \frac{\omega}{4}\right) + 2PC_1^2D_1\\
G_1 &= P^2G_1^2 - P^2D_1^2 - PC_1 - M\\
H_1 &= 2P^2C_1D_1 - PD_1 - \frac{\omega}{4}\\
E_1 &= e^{-A_1\eta}\cos\left(B_1\eta\right) - e^{-PC_1\eta}\cos\left(PD_1\eta\right)\\
F_1 &= e^{-PC_1\eta}\sin\left(PD_1\eta\right) - e^{-A_1\eta}\sin\left(B_1\eta\right)\\
C_1 &= 1 + \frac{1}{16}.\frac{\omega^2}{P^2}\\
D_1 &= \frac{1}{4}.\frac{\omega}{P}
\end{aligned} \tag{33}$$

Equations (31) & (32) together yield,

$$\begin{aligned}
M &= \left|M_r + iM_i\right|\\
&= \sqrt{M_r^{\,2} + M^2_{\,i}}\\
&= \frac{\left[\left(E_1^2 + F_1^2\right)\left\{\left(G_1 + kP^2I_1\right)^2 + \left(H_1 + kP^2J_1\right)^2\right\}\right]^{1/2}}{\left(G_1^2 + H_1^2\right)}
\end{aligned} \tag{34}$$

Results and Discussion

The unsteady free convective MHD flow of an elastico-viscous fluid past an infinite porous plate with constant suction under the action of time dependent plate temperature has been studied, when the plate temperature oscillates about a constant mean in magnitude but not in direction.

The value of real part M_r for different values of k, considering $P = 0.1$ and $\omega = 0.2$ as constants are entered in Table-1 and also shown graphically in Fig.-1. We observe that M_r is positive throughout. It is zero at $\eta = 0$ for every value of k. Its value is maximum at different values of k from 0.05 to 1 for $\eta = 2$. It does not show any remarkable change for any value of η in the region $0 \le k \le 0.05$ but comparatively very minuet change in the region after it.

In Table-2, imaginary part M_i is entered & we drawn graph as shown in Fig.-2. An examination of this table & graph shows that M_i is negative throughout. It is zero at $\eta = 0$ for every value of k. Its magnitude is maximum at $k = 1$ for $\eta = 0.5$.Its magnitude does not show an appreciable change for any value of η in the region $0 \le k \le 0.05$ but shows comparatively an appreciable change in the region after it.

In the Table-3 and as shown in Fig.-3, the value of $M = \sqrt{M_r^2 + M_i^2}$ for various sets of values of η and k have been incorporated. The study of this table and graph indicates that M is zero at $\eta = 0$ for every value of k. Its magnitude is maximum at $k = 0$ for $\eta = 0.5$.

In the velocity distribution equation (29), the coefficient of ε is $(M_r \cos \omega t - M_i \sin \omega t) = \sqrt{M_r^2 + M_i^2} \cos(\omega t + \phi)$. The term lags or leads over fluctuations by an angle ϕ. In our case ϕ is negative. The values of $|\tan \phi|$ for various k are entered in Table-4 and also shown in Fig.-4.

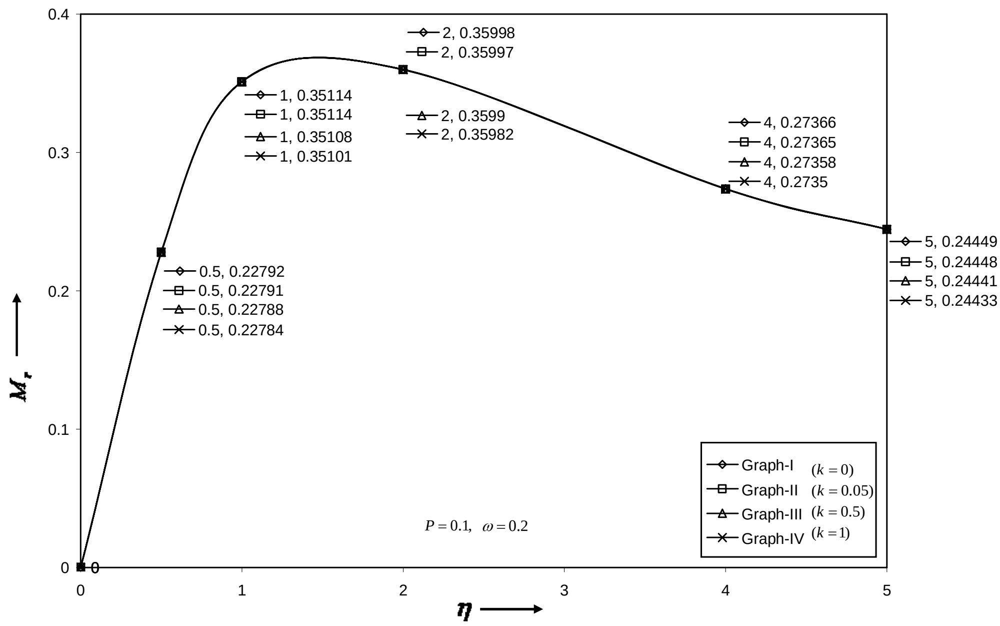

Figure 1

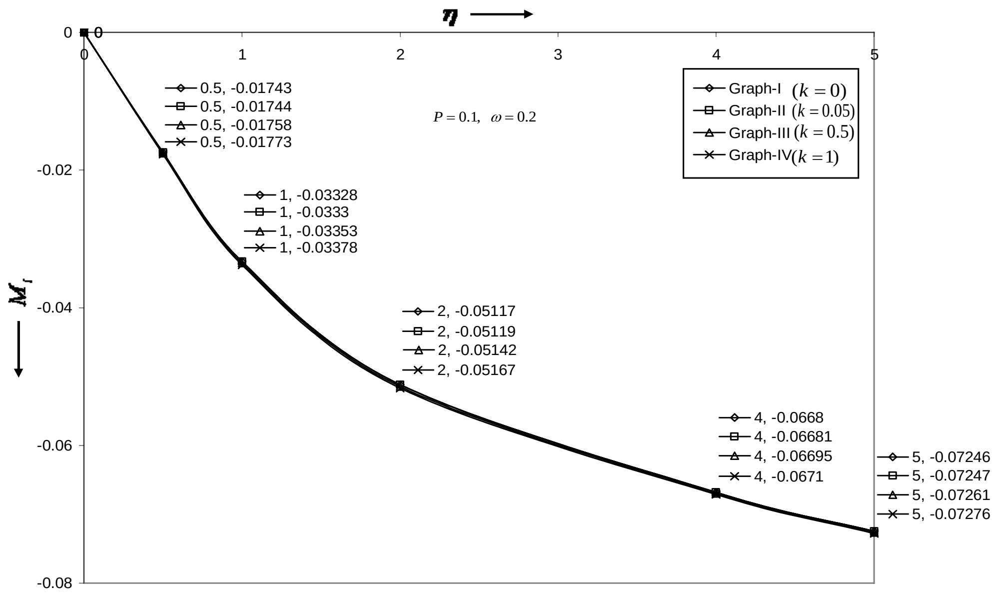

Figure 2

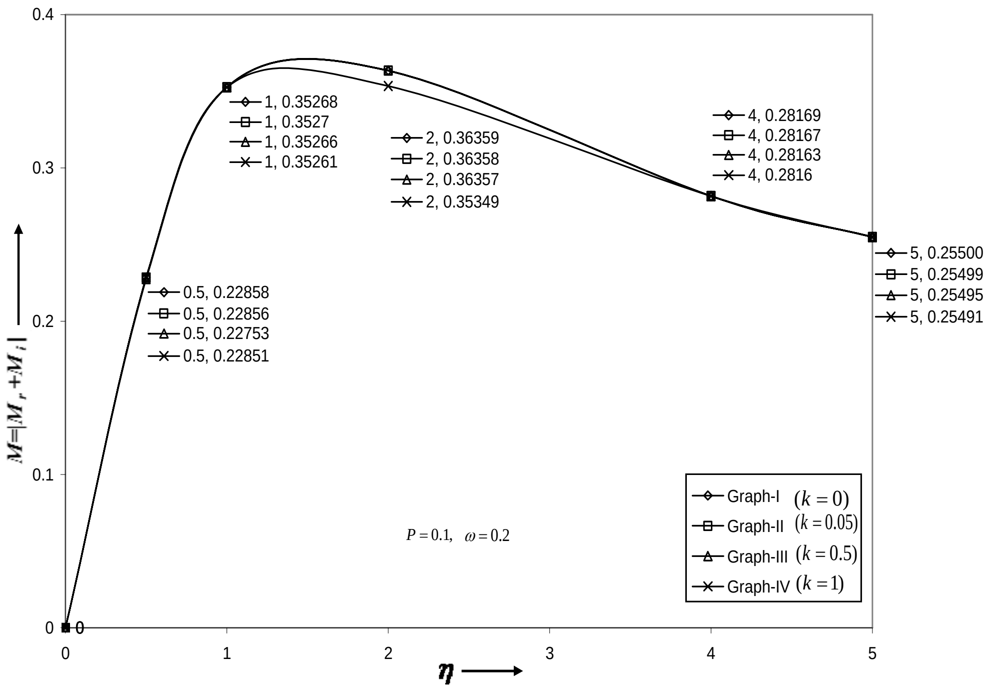

Figure 3

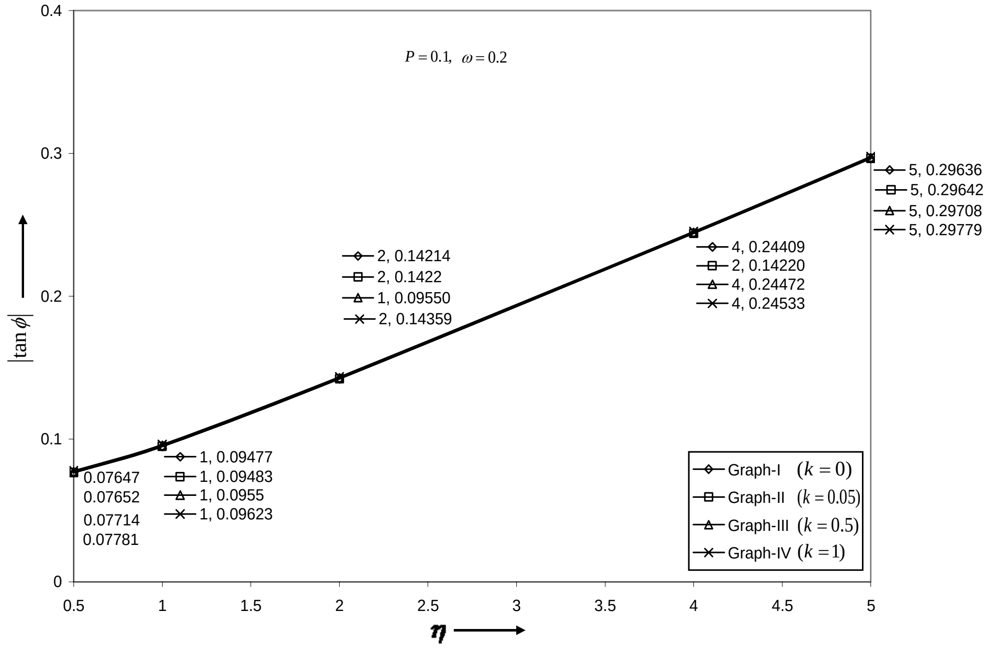

Figure 4

Table-1: Real part M_r when $P = 0.1$, $\omega = 0.2$

k	Graph-I	Graph-II	Graph-II	Graph-I-V
η	0	0.05	0.5	1
0	0	0	0	0
0.5	0.22792	0.22791	0.22788	0.22784
1	0.35114	0.35114	0.35108	0.35101
2	0.35998	0.35997	0.3599	0.35982
4	0.27366	0.27365	0.27358	0.2735
5	0.24449	0.24448	0.24441	0.24433

Table-2: Imaginary part M_i when $P = 0.1$, $\omega = 0.2$

k	Graph-I	Graph-II	Graph-II	Graph-I-V
η	0	0.05	0.5	1
0	0	0	0	0
0.5	-0.01743	-0.01744	-0.01758	-0.01773
1	-0.03328	-0.03330	-0.03353	-0.03378
2	-0.05117	-0.05119	-0.05142	-0.05167
4	-0.06680	-0.06681	-0.06695	-0.06710
5	-0.07246	-0.07247	-0.07261	-0.07276

Table-3: $|M_r + iM_i| = \sqrt{M_r^2 + M_i^2} = M$ when $P = 0.1$, $\omega = 0.2$

k	Graph-I	Graph-II	Graph-II	Graph-I-V
η	0	0.05	0.5	1
0	0	0	0	0
0.5	0.22858	0.22856	0.22753	0.22851
1	0.35268	0.35270	0.35266	0.35261
2	0.36359	0.36358	0.36357	0.35349
4	0.28169	0.28167	0.28163	0.28160
5	0.25500	0.25499	0.25495	0.25491

Table-4: Value of $|\tan\phi|$ for $P = 0.1$, $\omega = 0.2$

k	Graph-I	Graph-II	Graph-II	Graph-I-V
η	0	0.05	0.5	1
0.5	0.07647	0.07652	0.07714	0.07781
1	0.09477	0.09483	0.09550	0.09623
2	0.14214	0.14220	0.14287	0.14359
4	0.24409	0.24414	0.24472	0.24533
5	0.29636	0.29642	0.29708	0.29779
0.5	0.07647	0.07652	0.07714	0.07781

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