

Predicting Run off Using A Multiple Linear Regression Equation In Sakaeo, Thailand

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Abstract

The objectives of the present study were to estimate runoff in Sakaeo, Thailand, by using a multiple linear regression equation. Runoff and meteorological factors were collected from Hydrology and Water Management Center for Eastern Region since 2010 to 2013 for building equation. The result found that the regression equation for estimation the runoff was $\text{runoff} = 65.93 + 0.08919 \text{ Rainfall} + 0.07887 \text{ Rainfall}_1 + 0.926 \text{RH}_3$ with standard error of estimation 9.56705 and adjusted coefficient of determination 0.607.

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Introduction

In rural areas of Thailand, the fresh water is essential to agriculture because the water supply can not reach these areas, especially in areas that lack of civilization. One of the main occupations in Sakaeo is agriculture so irrigation system is important to farm. The mainly water that culturists use is runoff occurred by water resources such as Prasatueng Cannal, Praong River, Bangpakong River, etc. Therefore, the management of runoff is very considerable to agriculture in Sakaeo.

Materials And Methods

The data, runoff, temperature, rainfall and relative humidity, were collected from Hydrology and Water Management Center for Eastern Region since 2010 to 2013. Simple correlation coefficients (R) were firstly calculated to identify relationship among these data.

Building The Multiple Regression Equation

The multiple linear regression equation to estimate monthly runoff was generated by regression model [1][2][3] as Equation (1).

$$\begin{aligned} \text{Runoff} = & \beta_0 + \beta_{1i}T + \beta_{1i-1}T_1 + \beta_{1i-2}T_2 + \beta_{1i-3}T_3 + \beta_{2i}R + \beta_{2i-1}R_1 + \beta_{2i-2}R_2 \\ & + \beta_{2i-3}R_{i-3} + \beta_{3i}H + \beta_{3i-1}H_1 + \beta_{3i-2}H_2 + \beta_{3i-3}H_3 + \varepsilon \end{aligned} \quad (1)$$

The model is composed of one dependent variable; *Runoff*=Monthly runoff, and twelve independent variables; *T*=Maximum monthly temperature, *T_j*=Maximum monthly temperature in time lag *j*, *R*=Monthly rainfall, *R_j*=Monthly rainfall in time lag *j*, *H*=Monthly relative humidity, *H_j*=Monthly relative humidity in time lag *j*, (*j*=1, 2, 3) and ε = Error of regression model.

Checking Assumptions For Multiple Regression Analysis

After obtained the best fitted multiple regression equation, the assumptions checking for multiple regression analysis was proceeded. There are four assumptions to be tested; (I) normality of the error distribution using Anderson-Darling statistic by Equation (2) [4];

$$AD = -n - \sum_{i=1}^n \frac{2i-1}{n} [\ln F(Y_i) + \ln(1 - F(Y_{n+1-i}))] \quad (2)$$

(II) independence of the errors using Durbin-Watson statistic by Equation (3) [5];

$$DW = \frac{\sum_{i=1}^n (\varepsilon_i - \varepsilon_{i-1})^2}{\sum_{i=1}^n \varepsilon_i^2} \quad (3)$$

(III) homoscedasticity (constant variance) of the errors using Breusch-Pagan statistic by Equation (4) [6];

$$BP = \frac{SSR^*}{2} \div \left(\frac{SSE}{n} \right)^2 \quad (4)$$

where SSR^* = Sum of squares in regression between $e_j^2 = j$ th residual and x_{ij} , SSE = Sum of squares in regression error between y_j and x_{ij} ; (IV) multicollinearity among predictor variables using Variance Inflation Factor (VIF) Equation (5)

$$VIF_j = \frac{1}{1 - R_{j|others}^2} \quad (5)$$

where $R_{j|others}^2$ = Multiple coefficient of determination between x_{ij} and all x_i .

After tested all assumptions, the comparison between the real values and the estimated values of the runoff from the obtained multiple regression equation was plotted.

Validation Between Predicted Value And Observed Runoff

Finally, the comparison between the real values and the predicted values of runoff considering by the obtained multiple regression equation was then created time series plot and scatter plot when checking all assumptions of multiple regression analysis was determined.

Results And Discussion

These was highly positive correlation between *Runoff* and *R* ($R=0.561$, p-value=0.000), *Runoff* and R_1 ($R=0.645$, p-value=0.000) and *Runoff* and H ($R=0.598$, p-value=0.000) which was the same previous studies [2][7]. According to the best subsets method, *R* (rainfall), R_1 (rainfall in time lag 1) and H_3 (relative humidity in time lag 3) were selected to build the possible multiple linear regression equation for estimation runoff as Equation (6) with the Mallows C-p=6.6, $S=9.5671$, $R^2=0.641$ and $R_{adj}^2=0.607$.

$$\hat{Runoff} = -65.93 + 0.089191R + 0.07887R_1 + 0.926H_3 \quad (6)$$

After obtained the equation, (I) the test of normality was determined again and was found that hypothesis testing of normality was satisfied with $AD=0.439$ (p-value=0.277). (II) The test of independence of the errors was tested by Durbin-Watson statistic value ($DW=1.78276$, $DL=1.318$) so the errors were independent. (III) The test of homoscedasticity of error variation was tested by Breush-Pagan statistic ($BP=5.5227$, p-value=0.011) so the error variances were constant. (IV) Test of multicollinearity: the VIF values of *R*, R_1 and H_3 were calculated and the results were as 1.4, 1.2 and 1.2 consequently. All VIF values were less than 5 then there was no relationship among independent variables in multiple regression equation [8]. After all assumptions were validated, plotting between predicted runoff by Equation (6) and the real values was compared by graph of time series and scatter plot as Figure (1a) and Figure (1b) respectively. It was shown that the both graphs were closely plotted with the correlation coefficient 0.801.

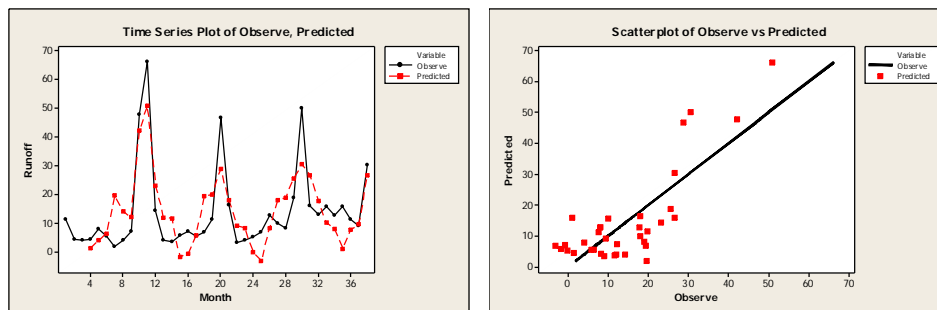


Figure 1: Comparison between observed and predicted runoff (a) Time series plot, (b) Scatter plot

Discussion

The predictors used to predict the monthly runoff in Sakaeo were Rainfall (R), Rainfall in time lag 1 (R_1) and Relative humidity in time lag 3 (H_3) with adjusted coefficient of determination (R_{adj}^2) 0.607 and the standard error of the estimation (S) 9.5671. The accuracy of estimation was shown by comparing the graph between the observe values and the predicted values from the multiple linear regression equation.

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