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Hadamard Product Of Meromorphic p-valent Functions with positive coefficients

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Abstract

In this research we introduce the new class $A(\alpha, \beta, \lambda, p)$ of meromorphic p-valent functions with positive coefficient.The aim of the paper is to obtain coefficient inequalities, some distortion theorem, radii of starlikeness, convexity, and convolution properties for $h(z)$ in the class $A(\alpha, \beta, \lambda, p)$.

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1. Introduction

Let $\sum_p$ denoted the class of functions of the form:

$$f(z) = \frac{1}{z^p} + \sum_{n=1}^{\infty} a_{p+n-1} z^{p+n-1} \qquad (p \in N) \tag{1.1}$$

which are analytic and p-valent in the punctured disk $E = \{z : 0 < |z| < 1\}$. Further let $\sum_p^*(\alpha)$ be the class of $\sum_p$ consisting of function f which satisfies the inequality

$$Re\left(\frac{-zf'(z)}{f(z)}\right) > \rho, \qquad (0 \leq \rho < p)$$

and let $\sum_{p}^{C}(\alpha)$ be the class of $\sum_{p}$ consisting of function f which satisfies the inequality

$$Re\left(-p-\frac{zf''(z)}{f'(z)}\right)>\rho, \qquad (0\leq\rho<p)$$

Clearly, we have

$$\sum_{p}^{C}(\alpha)\Leftrightarrow zf'\in\sum_{p}^{*}(\alpha), \quad (0\leq\alpha<p,\, p\in N).$$

This condition is obviously analogous to the well-known Alexander equivalent (see for details [1]).

For $f\in\sum_{p}$ given by(1.1) and $g\in\sum_{p}$ given by

$$g(z)=\frac{1}{z^p}+\sum_{n=1}^{\infty}b_{p+n-1}z^{p+n-1} \qquad (p\in N) \tag{1.2}$$

We define the Hadamard product (convolution) of f and g by

$$h(z)=(f*g)(z)=\frac{1}{z^p}+\sum_{n=1}^{\infty}a_{p+n-1}b_{p+n-1}z^{p+n-1}=(g*f)(z) \tag{1.3}$$

Many important properties and characteristics of various interesting subclasses of the class $\sum_{p}^{*}$ of meromorphic p-valent functions, including the classes $\sum_{p}^{C}(\alpha)$ and $\sum_{p}^{*}(\alpha)$ defined above, were studied by Srivastava et al. [2], Aouf et al. [3], Mogra [4], Kulkarni et al. [5], Moa'ath et al. [6], Saibah and Maslina [7], Ghanim [8], Kamali [9] and Makinde [10].

A function h given by (1.3) is said to be a member of the class $A(\alpha,\beta,\lambda,p)$ if it satisfies

$$\left|\frac{z^{p+1}h'(z)+p}{\alpha z^{p+1}f'(z)+[p+(\lambda-\alpha)(p-\alpha)]}\right|\leq\beta. \tag{1.4}$$

Where

$$\begin{aligned}0<\beta\leq 1, 0<\alpha&<\lambda+p-\sqrt{(\lambda+p)^2-4p(1+\lambda)}\\ <\alpha&<\lambda+p+\sqrt{(\lambda+p)^2-4p(1+\lambda)}\leq p,\\ (\lambda+p)^2>4p(1+\lambda)\ \text{ and }\ \lambda&\geq\frac{p}{2}-2\ \text{ for all } z\in E.\end{aligned}$$

In this paper, sharp results concerning coefficients, distortion theorem and the radius of convexity for the class $A(\alpha,\beta,\lambda,p,c)$ are determined. Finally, we prove that the class is closed under convex linear combination.

2. Coefficient Inequalities

In this section, we provided a sufficient condtion for a function, analytic in E to be in $A(\alpha, \beta, \lambda, p)$.

Theorem 2.1. A function h defined by (1.3) is in the class $A(\alpha, \beta, \lambda, p)$ if and only if

$$\sum_{n=1}^{\infty}(p+n-1)(1+\alpha\beta)a_{p+n-1}b_{p+n-1} \leq \beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)] \tag{2.1}$$

Where

$$\begin{aligned} 0 < \beta \leq 1, 0 < \alpha &< \lambda + p - \sqrt{(\lambda+p)^2 - 4p(1+\lambda)} \\ &< \alpha < \lambda + p + \sqrt{(\lambda+p)^2 - 4p(1+\lambda)} \leq p, \\ (\lambda+p)^2 > 4p(1+\lambda) \quad &\text{and} \quad \lambda \geq \frac{p}{2} - 2 \ \text{ for all } z \in E. \end{aligned}$$

Proof. $\Longleftarrow$ Let us suppose that

$$\sum_{n=1}^{\infty}(p+n-1)(1+\alpha\beta)a_{p+n-1}b_{p+n-1} \leq \beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)].$$

It suffices to show that

$$\left|\frac{z^{p+1}h'(z)+p}{\alpha z^{p+1}h'(z)+[p+(\lambda-\alpha)(p-\alpha)]}\right| \leq \beta, \qquad (z \in E).$$

Thus, for $0 < |z| < 1$, we have

$$\begin{aligned} &\left|\frac{z^{p+1}h'(z)+p}{\alpha z^{p+1}h'(z)+[p+(\lambda-\alpha)(p-\alpha)]}\right| \\ &= \left|\frac{\sum_{n=1}^{\infty}(p+n-1)a_{p+n-1}b_{p+n-1}z^{2p+n-1}}{[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]+[\sum_{n=1}^{\infty}\alpha(p+n-1)a_{p+n-1}b_{p+n-1}z^{2p+n-1}]}\right| \\ &\leq \frac{\sum_{n=1}^{\infty}(p+n-1)a_{p+n-1}b_{p+n-1}\left|z\right|^{2p+n-1}}{[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]-[\sum_{n=1}^{\infty}\alpha(p+n-1)a_{p+n-1}b_{p+n-1}\left|z\right|^{2p+n-1}]} \\ &\leq \frac{\sum_{n=1}^{\infty}(p+n-1)a_{p+n-1}b_{p+n-1}}{[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]-\sum_{n=1}^{\infty}\alpha(p+n-1)a_{p+n-1}b_{p+n-1}}. \end{aligned}$$

From (2.1), the last expression satiesfies,

$$\begin{aligned} \sum_{n=1}^{\infty}(p+n-1)a_{p+n-1}b_{p+n-1} &\leq \beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)] \\ &\quad - \sum_{n=1}^{\infty}\alpha\beta(p+n-1)a_{p+n-1}b_{p+n-1}, \end{aligned}$$

that is

$$\sum_{n=1}^{\infty}(p+n-1)(1+\alpha\beta)a_{p+n-1}a_{p+n-1} \leq \beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)].$$

The inquality above hold true for all $(0 < |z| < 1)$, hence it follows that (1.4) true, and so $h \in A(\alpha, \beta, \lambda, p)$.

$\Longrightarrow$ Conversely, assume that $h \in A(\alpha, \beta, \lambda, p)$, then

$$\begin{aligned}
&\left| \frac{z^{p+1}h'(z)+p}{\alpha z^{p+1}f'(z)+[p+(\lambda-\alpha)(p-\alpha)]} \right| \\
= &\left| \frac{\sum_{n=1}^{\infty}(p+n-1)a_{p+n-1}b_{p+n-1}z^{2p+n-1}}{[p(1-\alpha)+(\lambda-\alpha)p-\alpha]+[\sum_{n=1}^{\infty}\alpha(p+n-1)a_{p+n-1}b_{p+n-1}z^{2p+n-1}]} \right| \\
\leq &\frac{\sum_{n=1}^{\infty}(p+n-1)a_{p+n-1}b_{p+n-1}}{[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]-\sum_{n=1}^{\infty}\alpha(p+n-1)a_{p+n-1}b_{p+n-1}} \leq \beta
\end{aligned}$$

that is

$$\sum_{n=1}^{\infty}(p+n-1)(1+\alpha\beta)a_{p+n-1}b_{p+n-1}z^{p+n-1} \leq \beta[p+(\lambda-\alpha)(p-\alpha)]\cdot$$

The result is sharp for functions h of the form

$$h_{p+n-1}(z) = z^{-p} + \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(p+n-1)(1+\alpha\beta)}z^{p+n-1}, \quad (n \geq 1). \tag{2.2}$$

■

Corollary 2.2. Let the function h be defined by (1.3). If $h \in A(\alpha, \beta, \lambda, p)$, then

$$\left|a_{p+n-1}b_{p+n-1}\right| \leq \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(p+n-1)(1+\alpha\beta)}, \qquad (n \geq 1). \tag{2.3}$$

The result is sharp for functions $h_{p+n-1}(z)$ given by (2.2).

3. Distortion Theorem

A distortion property for functions in the class $h \in A(\alpha, \beta, \lambda, p)$, is given as follows:

Theorem 3.1. If the function h given by (1.3) is in the class $A(\alpha, \beta, \lambda, p)$, then for $0 < |z| = r < 1$, we have

$$\frac{1}{r^p} - \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{p(1+\alpha\beta)}r^p \leq \left|h(z)\right| \leq \frac{1}{r^p} + \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{p(1+\alpha\beta)}r^p$$

with equality for

$$h_p = \frac{1}{z^p} + \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{p(1+\alpha\beta)} z^p, \qquad z = (ir, r)$$

and

$$\begin{aligned} \frac{p}{r^{p+1}} - \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(1+\alpha\beta)} r^{p-1} &\leq \left|h'(z)\right| \\ &\leq \frac{p}{r^{p+1}} + \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(1+\alpha\beta)} r^{p-1}. \end{aligned}$$

With equality for,

$$h_p = \frac{1}{z^p} + \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{p(1+\alpha\beta)} z^p, \qquad z = (\pm ir, \pm r)$$

Proof. Since $h \in A(\alpha, \beta, \lambda, p)$, Theorem 2.1 yields the inequality

$$\sum_{n=1}^{\infty} \left|a_{p+n-1} b_{p+n-1}\right| \leq \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(p+n-1)(1+\alpha\beta)}. \tag{3.1}$$

Thus, for $0 < |z| = r < 1$, and making use of (2.1) we have

$$\begin{aligned} |h(z)| &\leq |z|^{-p} + \sum_{n=1}^{\infty} a_{p+n-1} b_{p+n-1} |z|^{p+n-1} \\ &\leq r^{-p} + \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(p+n-1)(1+\alpha\beta)} r^p \quad \text{(we substitute in (3.1) when } n = 1) \\ &\leq r^{-p} + \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{p(1+\alpha\beta)} r^p. \end{aligned}$$

And

$$\begin{aligned} |h(z)| &\geq |z|^{-p} - \sum_{n=1}^{\infty} a_{p+n-1} b_{p+n-1} |z|^{p+n-1} \\ &\geq r^{-p} - \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(p+n-1)(1+\alpha\beta)} r^p \quad \text{(we substitute in (3.1) when } n = 1) \\ &\geq r^{-p} - \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{p(1+\alpha\beta)} r^p. \end{aligned}$$

Also from Theorem 2.1, it follows that

$$\sum_{n=1}^{\infty} (p+n-1) \left|a_{p+n-1} b_{p+n-1}\right| \leq \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(1+\alpha\beta)}. \tag{3.2}$$

Thus

$$\begin{aligned} |h'(z)| &\leq p|z|^{-p-1} + \sum_{n=1}^{\infty}(p+n-1)a_{p+n-1}b_{p+n-1}|z|^{p+n-2}, \\ &\leq pr^{-p-1} + \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(1+\alpha\beta)}r^{p-1}, \end{aligned}$$

and

$$\begin{aligned} |h'(z)| &\geq p|z|^{-p-1} - \sum_{n=1}^{\infty}(p+n-1)a_{p+n-1}b_{p+n-1}|z|^{p+n-2}, \\ &\geq pr^{-p-1} - \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(1+\alpha\beta)}r^{p-1}. \end{aligned}$$

Hence completes the proof of Theorem 3.1. ■

4. Radii of Starlikeness and Convexity

The radii of starlikeness and convexity for the class $A(\alpha, \beta, \lambda, p)$, is given by the following theorem:

Theorem 4.1. If the function h defined by (1.3) is in the class $A(\alpha, \beta, \lambda, p)$, then h is starlike of order $\rho(0 \leq \rho < p)$, in the disk $|z| < r_1(\alpha, \beta, \lambda, p, \rho)$, where $r_1(\alpha, \beta, \lambda, p, \rho)$,is the largest value for which

$$r_1 = r_1(\alpha, \beta, \lambda, p, \rho) = \inf_{n\geq 1}\left(\frac{(p-\rho)[(p+n-1)(1+\alpha\beta)]}{(3p+n-\rho-1)\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}\right)^{\frac{1}{2p+n-1}}.$$

The result is sharp for functions $h_{p+n-1}(z)$ given by (2.2).

Proof. It suffices to show that

$$\left|\frac{zh'(z)}{h(z)} + p\right| \leq (p-\rho).$$

For $|z| \leq r_1$, we have

$$\begin{aligned} \left|\frac{zh'(z)}{h(z)} + p\right| &= \left|\frac{\sum_{n=1}^{\infty}(2p+n-1)a_{p+n-1}b_{p+n-1}z^{2p+n-1}}{1+\sum_{n=1}^{\infty}a_{p+n-1}b_{p+n-1}z^{2p+n-1}}\right| \\ &\leq \frac{\sum_{n=1}^{\infty}\frac{(2p+n-1)\beta[p(1-p)+(\lambda-\alpha)(p-\alpha)])}{(p+n-1)(1+\alpha\beta)}|z|^{2p+n-1}}{1-\sum_{n=1}^{\infty}\frac{\beta[p(1-p)+(\lambda-\alpha)(p-\alpha)])}{(p+n-1)(1+\alpha\beta)}|z|^{2p+n-1}} \\ &\leq (p-\rho). \end{aligned}$$

The inquality above holds true if

$$\sum_{n=1}^{\infty} \frac{(3p+n-\rho-1)\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(p+n-1)(1+\alpha\beta)}|z|^{2p+n-1} \leq (p-\rho),$$

and it follows that

$$|z| \leq \left(\frac{(p-\rho)[(p+n-1)(1+\alpha\beta)]}{(3p+n-\rho-1)\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}\right)^{\frac{1}{2p+n-1}}, \qquad (n \geq 1)$$

then we have,

$$r_1(\alpha, \beta, \lambda, p, \rho) = \inf_{n\geq 1}\left(\frac{(p-\rho)[(p+n-1)(1+\alpha\beta)]}{(3p+n-\rho-1)\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}\right)^{\frac{1}{2p+n-1}}.$$

as required. ■

Theorem 4.2. If the function h defined by (1.3) is in the class $A(\alpha, \beta, \lambda, p)$, then h is convex of order $\rho(0 \leq \rho < p)$, in the disk $|z| < r_2(\alpha, \beta, \lambda, p, \rho)$, where $r_2(\alpha, \beta, \lambda, p, \rho)$,is the largest value for which

$$r_2 = r_2(\alpha, \beta, \lambda, p, \rho) = \inf_{n\geq 1}\left(\frac{p(p-\rho)(1+\alpha\beta)}{(3p+n-\rho-1)\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}\right)^{\frac{1}{2p+n-1}}.$$

The result is sharp for functions h given by (2.2).

Proof. It suffices to show that

$$\left|\frac{zh''(z)}{h'(z)} + (1+p)\right| \leq (p-\rho).$$

for $|z| \leq r_2$, we have

$$\begin{aligned}
\left|\frac{zh''(z)}{h'(z)} + (1+p)\right| &= \left|\frac{\sum_{n=1}^{\infty}(2p+n-1)(p+n-1)a_{p+n-1}b_{p+n-1}z^{2p+n-1}}{-p+\sum_{n=1}^{\infty}(p+n-1)a_{p+n-1}b_{p+n-1}z^{2p+n-1}}\right| \\
&\leq \frac{\sum_{n=1}^{\infty}(2p+n-1)(p+n-1)\frac{\beta[p(1-p)+(\lambda-\alpha)(p-\alpha)])}{(p+n-1)(1+\alpha\beta)}|z|^{2p+n-1}}{p-\sum_{n=1}^{\infty}(p+n-1)\frac{\beta[p(1-p)+(\lambda-\alpha)(p-\alpha)])}{(p+n-1)(1+\alpha\beta)}|z|^{2p+n-1}} \\
&\leq (p-\rho).
\end{aligned}$$

The inquality above holds true if

$$\sum_{n=1}^{\infty} \frac{[(3p+n-\rho-1)]\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(1+\alpha\beta)}|z|^{2p+n-1} \leq p(p-\rho),$$

and it follows that

$$|z| \leq \left(\frac{p(p-\rho)(1+\alpha\beta)}{[(3p+n-\rho-1)]\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]} \right)^{\frac{1}{2p+n-1}}, \qquad (n \geq 1)$$

then we have,

$$r_2(\alpha, \beta, \lambda, p, \rho) = \inf_{n \geq 1} \left(\frac{p(p-\rho)(1+\alpha\beta)}{(3p+n-\rho-1)\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]} \right)^{\frac{1}{2p+n-1}}$$

as required. ■

5. Convex linear Combination

Our next result involves a linear combination of function h of the type (2.2).

Theorem 5.1. Let

$$h_z = z^{-p}, \tag{5.1}$$

and

$$h_{p+n-1}(z) = z^{-p} + \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(p+n-1)(1+\alpha\beta)} z^{p+n-1}, \qquad (n \geq 2). \tag{5.2}$$

Then $h \in A(\alpha, \beta, \lambda, p)$ if and only if it can be expressed in the form

$$h(z) = \sum_{n=1}^{\infty} \lambda_{p+n-1} h_{p+n-1}(z), \tag{5.3}$$

where

$$\lambda_{p+n-1} \geq 0 \qquad \textit{and} \qquad \sum_{n=1}^{\infty} \lambda_{p+n-1} = 1.$$

Proof. $\Leftarrow$ From (5.1),(5.2) and (5.3), it is easily seen that

$$\begin{aligned} h(z) &= \sum_{n=1}^{\infty} \lambda_{p+n-1} h_{p+n-1}(z), \\ &= z^{-p} + \sum_{n=1}^{\infty} \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]\lambda_{p+n-1}}{(p+n-1)(1+\alpha\beta)} z^{p+n-1}. \end{aligned}$$

Then it follows that

$$\sum_{n=1}^{\infty} \frac{(p+n-1)(1+\alpha\beta)}{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]} \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]\lambda_{p+n-1}}{(p+n-1)(1+\alpha\beta)}$$

and

$$\sum_{n=1}^{\infty} \lambda_{p+n-1} = 1 - \lambda_p \leq 1,$$

it follows from Theorem 2.1 that the function $h \in A(\alpha, \beta, \lambda, p)$.

$\Leftarrow$ Conversely, let us suppose that $h \in A(\alpha, \beta, \lambda, p)$. Then

$$a_{p+n-1} b_{p+n-1} \leq \frac{\beta[p(1-\alpha) + (\lambda - \alpha)(p - \alpha)]}{(p+n-1)(1+\alpha\beta)} \qquad (n \geq 1)\cdot$$

Setting

$$\lambda_{p+n-1} = \frac{(p+n-1)(1+\alpha\beta)}{\beta[p(1-\alpha) + (\lambda - \alpha)(p - \alpha)]} a_{p+n-1} \qquad (n \geq 1)$$

and $\lambda_p = 1 - \sum_{n=2}^{\infty} \lambda_{p+n-1}$,

It follows that

$$h(z) = \sum_{n=1}^{\infty} \lambda_{p+n-1} h_{p+n-1}(z)$$

this complete the proof of the theorem. ■

Theorem 5.2. The class $A(\alpha, \beta, \lambda, p)$ is closed under convex linear combinations.

Proof. Suppose that the functions h_1 and h_2 defined by,

$$h_i(z) = z^{-p} + \sum_{n=1}^{\infty} a_{p+n-1,i} b_{p+n-1,i} z^{p+n-1} \qquad (i = 1, 2; z \in E) \tag{5.4}$$

are in the class $A(\alpha, \beta, \lambda, p)$.

Setting $h(z) = \mu h_1(z) + (1-\mu) h_2(z)$ we want to show that $h \in A(\alpha, \beta, \lambda, p)$.

For $0 \leq \mu \leq 1$, we can write

$$h(z) = z^{-p} + \sum_{n=1}^{\infty} \left\{ \mu a_{p+n-1,1} b_{p+n-1,1} + (1-\mu) a_{p+n-1,2} b_{p+n-1,2} \right\} z^{p+n-1}, \qquad (z \in E)$$

In view of theorem 2.1, we have

$$\begin{aligned}
&\sum_{n=1}^{\infty}[(p+n-1)(1+\alpha\beta)]\Big\{\mu a_{p+n-1,1}b_{p+n-1,1}+(1-\mu)a_{p+n-1,2}b_{p+n-1,2}\Big\},\\
&\quad=\mu\sum_{n=1}^{\infty}[(p+n-1)(1+\alpha\beta)]a_{p+n-1,1}b_{p+n-1,1}\\
&\qquad+(1-\mu)\sum_{n=1}^{\infty}[(p+n-1)(1+\alpha\beta)]a_{p+n-1,2}b_{p+n-1,2}\\
&\quad\leq\mu\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]+(1-\mu)\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)],\\
&\quad=\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)].
\end{aligned}$$

which show that $h\in A(\alpha,\beta,\lambda,p)$. Hence the theorem. ■

References

[1] P.L. Duren, Univalent functions. In Grundlehren der Mathematischen Wissenschaften, 259, Springer-Verlag, New York, (1983).

[2] H. M. Srivastava, H. M. Hossen and M. K. Aouf., A unified presentation of some classes meromorphically multivalent functions, Comp. Math. with App. 38(1999), 63–700.

[3] M. K. Aouf, On a class meromorphic multivalent functions with positive coefficients, Math. Japan. 35, (1990), 603–608.

[4] M. L. Mogra, Meromorphic multivalent functions with positive coefficients I, Math. Japan. 35, (1990), 1–11.

[5] S. R. Kulkarni, U. H. Naik and H. M. Srivastava, A certain class of meromorphically p-valent, quasi-convex functions, Pan Amer. Math. J. 8 (1998), 57–64.

[6] Moa'ath and Darus, Certain class of meromorphic p-valent functions with positive coefficients, Tamkang Jornal of Mathematics. 37, (2006), 251–260.

[7] Saibah and Maslina, On subclasses of p-valent functions with fixed second negative coefficients, Journal of Analysis and Applications. 3(1), (2006), 19–30.

[8] F. Ghanim and M. Darus On a certain subclass of meromorphic univalent functions wih fixed second positive coefficients, surveys in mathematics and applications. 5, (2010), 5 49–60.

[9] M. Kamali, K. Suchithra, B.A. Stephen and A. Gangadharan, On certain subclass of meromorphic p-valent functions with negative coefficients. General Mathematics. (2011), 19(1), 109–122.

[10] D.O Makinde *On a Certin Family of Meromorphic p-valent Functions with Negative Coefficients*, Mathematical Theory and Modeling. 2, (2012).