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The Performance of the Adjusted Tukey's Control Chart under Asymmetric Distributions

J. Mekparyup[1] S. Kornpetpanee[2] and K. Saithanu[3]

[1,2]*College of research methodology and cognitive science, Burapha University*
[1,3]*Department of Mathematics, Faculty of Science, Burapha University*
169 Muang, Chonburi, Thailand
[1]*jatupat@buu.ac.th,* [2]*suchada@buu.ac.th,* [3]*ksaithan@buu.ac.th*

Abstract

The purpose of presented study was to monitor the performance of the adjusted Tukey's control chart with asymmetric distributions by average run length (ARL). Many asymmetric distributions were selected to examine the theoretical ARL with various observations assuming the process was in-control. The results found that the ARL values of the adjusted Tukey's control chart are higher than the ARL values of the traditional Tukey's control chart in every asymmetric distribution. It could summarize that the adjusted Tukey's control chart has more efficient to monitor the process when the process is in-control. For theoretical ARL values assuming the process was out-of-control, both control charts' performance are worse when the distribution is non normal probability distributed.

Keywords: ARL, MADM, Tukey's control chart, Adjusted Tukey's control chart.

Mathematics Subject Classification: 62-07, 62G35

1. INTRODUCTION

Mekparyup, Kornpetpanee, & Saithanu (2014) proposed the adjusted Tukey's control chart (ATCC) to reduce the probability of occurrence of false alarm by substitution median of absolute deviation to the median (MADM) instead of interquartile range (IQR) of the Tukey's control chart (TCC). Then they monitored both control charts's performance under symmetric distributions by simulated the in-control ARL (ARL_0) and the results showed that the ATCC's ARL_0 values are higher than the TCC's ARL_0 values in every symmetric distribution. In the presented study, the theoretical ARL values are calculated to monitor the process performance of the TCC and the ATCC under various asymmetric distributions.

2. MATERIALS AND METHODS

2.1 CONTROL CHART DESIGN

THE TUKEY'S CONTROL CHART (TCC):

Alemi (2004) proposed the TCC following Eq. (1) and (2):

$$UCL = F^{-1}(0.75) + (k \text{ x } IQR) \tag{1}$$

$$LCL = F^{-1}(0.25) - (k \text{ x } IQR) \tag{2}$$

where $IQR = F^{-1}(0.75) - F^{-1}(0.25)$. Frigge, Hoaglin, and Iglewicz (1989), Wheeler (2004), Torng, Liao, Lee, and Wu (2009) suggested to set k = 1.5 under the normal distribution. The control limits cover 99.3% of the total population (Ryan, 2000).

THE ADJUSTED TUKEY'S CONTROL CHART (ATCC):

Mekparyup, Kornpetpanee, & Saithanu (2014) suggested the ATCC following Eq. (3) and (4):

$$UCL = F^{-1}(0.75) + (k \text{ x } MADM) \tag{3}$$

$$LCL = F^{-1}(0.25) + (k \text{ x } MADM) \tag{4}$$

where $MADM = median_i \mid x_i - median_i \; x_{ij} \mid$; i^{th} sample size and j^{th} subgroup size. For selecting parameter k under the normal distribution, Mekparyup, Kornpetpanee, & Saithanu (2014) set k = 3.

2.2 ARL CALCULATION

ARL value is the expected number of points plotted on a control chart before an out of control is detected (Montgomery, 1997) and is generally used to monitor capacity of control chart (Borror et al., 1999; Quesenberry, 1993; Davis, 2004; Torng and Lee, 2008; Torng, Liao, Lee, and Wu, 2009). The in-control ARL (ARL_0) can be calculated by $1/\alpha$ where α is type I error probability and the out-of-control ARL (ARL_1) can be calculated by $1/(1-\beta)$ where β is type II error probability. For the TCC and the ATCC for individual observations under the normal distribution, ARL_0=143.34 where α is 0.00698. The ARL_0 value of 92 could be accepted (Wheeler and Chambers, 1992).

ARL OF THE TCC:

Assuming the process is in control, ARL_0 is computed with Eq. (5):

$$ARL_0 = \frac{1}{\alpha} = \frac{1}{1 - \int_{F^{-1}(0.25)-(k \text{ x } IQR)}^{F^{-1}(0.75)+(k \text{ x } IQR)} f(x)dx} \tag{5}$$

and ARL_1 is calculated when the process is assumed to be out-of-control or process shift occurs with Eq. (6):

$$ARL_1 = \frac{1}{1-\beta} = \frac{1}{1 - \int_{F^{-1}(0.25)-(k \text{ x } IQR)-\delta\sigma}^{F^{-1}(0.75)+(k \text{ x } IQR)-\delta\sigma} f(x)dx} \quad (6)$$

where δ is the shift size, $\delta = \frac{\mu_1 - \mu_0}{\sigma}$, μ_0 is process mean, and $P(\delta) = 1 - \int_{LCL-\delta\sigma}^{UCL-\delta\sigma} f(x)dx$.

ARL OF THE ATCC:

Assuming the process is in control, ARL_0 is computed with Eq. (7):

$$ARL_0 = \frac{1}{\alpha} = \frac{1}{1 - \int_{F^{-1}(0.25)-(k \text{ x } MADN)}^{F^{-1}(0.75)+(k \text{ x } MADM)} f(x)dx} \quad (7)$$

and ARL_1 is calculated when the process is assumed to be out-of-control with Eq. (8).

$$ARL_1 = \frac{1}{1-\beta} = \frac{1}{1 - \int_{F^{-1}(0.25)-(k \text{ x } MADM)-\delta\sigma}^{F^{-1}(0.75)+(k \text{ x } MADM)-\delta\sigma} f(x)dx} \quad (8)$$

2.3 PERFORMANCE EVALUATION

Many asymmetric distributions are selected to monitor the performance between the TCC and the ATCC using the theoretical ARL values (Borror et al., 1999; Stoumbos and Reynolds, 2000; Calzada and Scariano, 2001; Lin and Chou, 2007; Abu-Shawiesh, 2008; Torng and Lee, 2008) and δ equal to 0, 0.5, 1, 1.5, 2, 2.5 and 3 are used.

3. RESULTS

3.1 THE THEORETICAL ARL VALUES

Consequently, the theoretical ARL values of the TCC and the ATCC were shown as of Table 1 and Table 2. Regarding to ARL_0 values for both control charts, it was illustrated that the ATCC's ARL_0 are higher than the TCC's ARL_0 values in every distribution so the ATCC's performance has more efficiency than the TCC when the process is in-control or no process mean shift. For considering of ARL_1 values for both control charts, ARL_0 was adjusted to 143.34 and parameter k was set corresponding to ARL_0. From Table 3, it was explained that the performance of detecting the process of both control charts was worse when the distribution was away from a normal distribution. Obviously, it could see in case of t(4) and Lap(0,1) as of Figure 1.

Table 1: The theoretical ARL values of the TCC for k = 1.5

Distribution	Shift size (δ)						
	0	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**
N(0,1)	143.34	68.18	22.29	8.66	4.12	2.37	1.62
Gam(1,1)	20.78	12.61	7.65	4.64	2.81	1.71	1.03
Gam(2,1)	31.52	17.51	9.89	5.70	3.39	2.10	1.39
Gam(3,1)	39.56	20.72	11.14	6.20	3.60	2.22	1.49
Gam(4,1)	46.14	23.13	12.00	6.50	3.72	2.28	1.53
Chi(1)	13.22	8.50	5.35	3.24	1.76	1.00	1.00
Chi(2)	20.78	12.61	7.65	4.64	2.81	1.71	1.03
Chi(3)	26.64	15.39	8.97	5.30	3.19	1.97	1.29
Chi(4)	31.52	17.51	9.89	5.70	3.39	2.10	1.39

Table 2: The theoretical ARL values of the ATCC for k = 3

Distribution	Shift size (δ)						
	0	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**
N(0,1)	143.34	68.18	22.29	8.66	4.12	2.37	1.62
Gam(1,1)	30.26	18.35	11.13	6.75	4.09	2.48	1.51
Gam(2,1)	39.41	21.78	12.22	6.99	4.10	2.49	1.61
Gam(3,1)	46.61	24.27	12.96	7.14	4.10	2.49	1.63
Gam(4,1)	52.63	26.24	13.52	7.26	4.10	2.48	1.63
Chi(1)	24.52	16.08	10.42	6.63	4.10	2.38	1.00
Chi(2)	30.26	18.35	11.13	6.75	4.09	2.48	1.51
Chi(3)	35.16	20.23	11.73	6.88	4.10	2.50	1.58
Chi(4)	39.41	21.78	12.22	6.99	4.10	2.49	1.61

Table 3: The theoretical ARL values of the TCC and the ATCC

Control Charts	Distribution	k	Shift size (δ)						
			0	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**
TCC	N(0,1)	1.5	143.34	68.18	22.29	8.66	4.12	2.37	1.62
	Gam(1,1)	3.2577	143.34	86.94	52.73	31.98	19.40	11.77	7.14
	Gam(2,1)	2.5173	143.34	77.48	42.27	23.33	13.06	7.45	4.35
	Gam(3,1)	2.2519	143.34	72.31	37.14	19.50	10.52	5.87	3.43
	Gam(4,1)	2.1068	143.34	68.89	33.97	17.27	9.12	5.05	2.97
	Chi(1)	4.8746	143.34	96.52	64.74	43.21	28.66	18.86	12.28
	Chi(2)	3.2577	143.34	86.94	52.73	31.98	19.40	11.77	7.14
	Chi(3)	2.7674	143.34	81.34	46.37	26.59	15.36	8.95	5.29
	Chi(4)	2.5173	143.34	77.48	42.27	23.33	13.06	7.45	4.35
ATCC	N(0,1)	3	143.34	68.18	22.29	8.66	4.12	2.37	1.62
	Gam(1,1)	5.3061	143.34	86.94	52.73	31.98	19.40	11.77	7.14

Gam(2,1)	4.5691	143.34	77.48	42.27	23.33	13.06	7.45	4.35
Gam(3,1)	4.2274	143.34	72.31	37.14	19.50	10.52	5.87	3.43
Gam(4,1)	4.0202	143.34	68.89	33.97	17.27	9.12	5.05	2.97
Chi(1)	6.2437	143.34	96.52	64.74	43.21	28.66	18.86	12.28
Chi(2)	5.3061	143.34	86.94	52.73	31.98	19.40	11.77	7.14
Chi(3)	4.8505	143.34	81.34	46.37	26.59	15.36	8.95	5.29
Chi(4)	4.5691	143.34	77.48	42.27	23.33	13.06	7.45	4.35

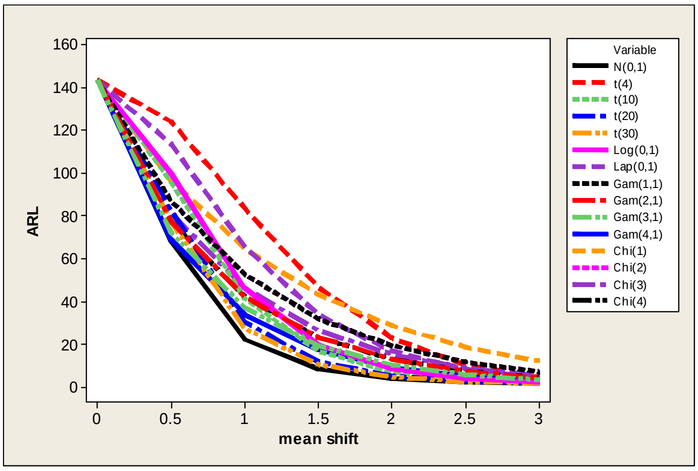

Figure 1. The theoretical ARL values of the TCC with various distributions.

4. DISCUSSION

In this presented study, many asymmetric distributions were selected to examine the performance by comparing the theoretical ARL values of the TCC and the ATCC. The results found that the ARL_0 values of the ATCC are higher than the TCC in every asymmetric distribution so the ATCC has more efficient to detect the process under the in-control state. For the out-of-control situation, ARL_1 values of the TCC and the ATCC are worse when the distribution is non normal probability distributed. For further studies, the ATCC can be adjusted to reduce type I error probability by using a robustness scale parameter having more efficient than the MADM.

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