Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 10, Number 5 (2014), pp. 711-718

http://www.ripublication.com

Fuzzy Assignment Problem within the Sort of Alpha Optimal Solution

Geetha.N. K

Assistant Professor (S. G),
Department of Mathematics, Saveetha School of Engineering
Saveetha University, Chennai
nkgeeth@gmail. com/matdpt@yahoo. com,

Abstract

The fuzzy assignment method works for any problems in which one to one matching is called for in the light of the given payoffs where the total payoff is sought to minimized or maximized. Throughout this paper, solving technique absolutely think about as an assignment problem with the assistance of trapezoidal membership functions and its arithmetic operations. Solving procedure has been applied from the approach of fuzzy Hungarian method using Robust Ranking technique. The fuzzified version of the problem has been mentioned with the assistance of a numerical example associated it's proving that the Robust ranking methodology offers an efficient tool for handling the assignment problem. Finally the optimum solution with in the kind of fuzzy numbers and verified its solution with in the nature of fuzzy membership functions.

Keywords: Trapezoidal Fuzzy Numbers, Fuzzy Hungarian Method, Alpha cut optimum solution, Robust Ranking Principle.

1. Introduction

In every work place, there are jobs to be done and there are people to do the job. But every individual is not equally efficient at every job. Someone may be more efficient on one job and less efficient on the other, and it might be otherwise for someone other. The relative efficiency is manifested in terms of the time taken for, or the cost associated with, performance of different jobs by different people. An obvious difficulty for a manager to handle is to assign jobs to various workers in a manner to handle is to assign jobs to various workers in a manner that they can be done in the most efficient way. Interestingly, such problems can be formulated as linear programming problems or as transportation problems and solved as such, but a method called fuzzy Hungarian assignment method provides an easy route. Assignment problem is studied by many researchers from old time. Further many efficient algorithms are developed (see the recent survey paper due to Pentico D. W, [6]). Assignment problem has been extensively studied in the literature and several polynomial algorithms are available to solve it. These are combinatorial optimization method, linear programming based method using the unimodular property and its linear programming version, maximum flow based method etc., though we do not mention each method. Geetha and Nair [2] have considered some variant of the assignment problem and proposed a fuzzy version of the assignment problem. In 2004, Makani Das and Hemanta K. Baruah had developed in solving procedure for linear programming problem in the way of triangular fuzzy membership functions [3]. Further we have applied in the same methodology for transportation problem and assignment problem in the way of trapezoidal fuzzy membership functions [9, 10]. In this paper, we have considered the whole problem in terms of trapezoidal membership functions of assignment problem by suitability of each job to each facility for finding maximum profit of the total assignment cost with the assistance of fuzzy Hungarian methodology to be finding out the optimum solution for the maximum profit cost. The fuzzified version of the problem has been mentioned with the assistance of a numerical example associated it's proving that the Robust ranking methodology offers an efficient tool for handling the assignment problem. Finally the optimal solution with in the kind of fuzzy numbers and verified its solution with in the nature of fuzzy membership functions.

2. Fuzzy Preliminaries

The concept of fuzzy mathematical programming on a general level was first proposed by the frame work of fuzzy decision of Bellman and Zadeh [1]. Now we present some necessary definitions.

2. 1. Definition

A trapezoidal fuzzy number A can be expressed as A = [a_1, a_2, a_3, a_4] and its membership function is outlined as:

$$\mu_A(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2 \\ 1 \qquad\quad , a_2 \le x \le a_3 \\ \dfrac{x-a_4}{a_3-a_4}, a_3 \le x \le a_4 \end{cases}$$

The Pictorial representation of trapezoidal membership function μ_A is given below within the figure.

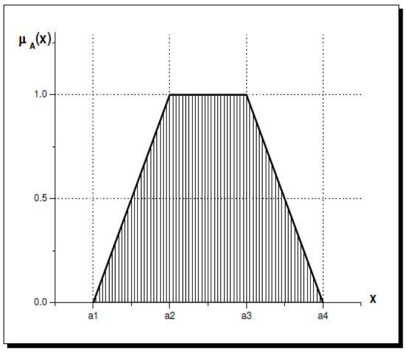

Figure: 1 Trapezoidal Membership function μ_A

2. 2. Arithmetic operations

Let A = [a_1, a_2, a_3, a_4] and B = [b_1, b_2, b_3, b_4] two trapezoidal fuzzy numbers then the arithmetic operations on A and B as follows:

Addition:

$$\mathrm{A}+\mathrm{B}=(a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4)$$

Subtraction:

$$A-B=(a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1)$$

Multiplication:

$(i)\ \ if\ \ \ f(B)>0$

$$A\bullet B=\left(\frac{a_1}{4}(b_1+b_2+b_3+b_4), \frac{a_2}{4}(b_1+b_2+b_3+b_4), \frac{a_3}{4}(b_1+b_2+b_3+b_4), \frac{a_4}{4}(b_1+b_2+b_3+b_4)\right)$$

$(ii)\ \ if\ \ \ f(B)<0$

$$A\bullet B=\left(\frac{a_4}{4}(b_1+b_2+b_3+b_4), \frac{a_3}{4}(b_1+b_2+b_3+b_4), \frac{a_2}{4}(b_1+b_2+b_3+b_4), \frac{a_1}{4}(b_1+b_2+b_3+b_4)\right)$$

2. 3. Remark

(i). If $R(A)>R(B)$ then A is called fuzzy *maximum* then B.

(ii). If $R(A)<R(B)$ then A is called fuzzy *minimum* then B.

(iii). Let us take [-2∂, -∂, ∂, 2∂] has fuzzy zero denoted by fuzzy zero and ∂ is a positive scalar.

(iv). Let us take [-∂, 0, ∂, 4∂] has fuzzy zero denoted by fuzzy zero and ∂ is a positive scalar.

2. 4. Definition (*α* cut)

Given a fuzzy set *A* in *X* and any real number α in [0, 1], then the α -cut or α -level or cut worthy set of *A*, denoted by ${}^{\alpha}A$ is that the crisp set ${}^{\alpha}A=\{x\in X:\mu_A(x)\geq\alpha\}$ The strong *a* cut, denoted by ${}^{\alpha+}A$ is that the crisp set ${}^{\alpha+}A=\{x\in X:\mu_A(x)>\alpha\}$ For instance, let A be a fuzzy set whose membership function is given as

$$\mu_A(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2 \\ 1 \qquad , a_2 \le x \le a_3 \\ \dfrac{x-a_4}{a_3-a_4}, a_3 \le x \le a_4 \end{cases}$$

To find the α-cut of A, allow us to set $\alpha \in [0,1]$ to each left and right reference functions of A.

That is,

$$\alpha = \frac{x-a_1}{a_2-a_1} \text{ and } \alpha = \frac{x-a_4}{a_3-a_4}.$$

Expressing x in terms of α we have $x = (a_2 - a_1)\alpha + a_1$ and $x = a_4 + (a_3 - a_4)\alpha$ and which gives the α-cut of A is ${}^{\alpha}A = [(a_2 - a_1)\alpha + a_1, x = a_4 + (a_3 - a_4)\alpha]$.

2. 5. Example

In the case of the trapezoidal fuzzy number A = (2, 4, 6, 8) and also the membership function value will be

$$\mu_A(x) = \begin{cases} \dfrac{x-2}{2} & ,2 \le x \le 4 \\ 1 & ,4 \le x \le 6 \\ \dfrac{x-8}{-2} & ,6 \le x \le 8 \\ 0 & ,x > 8 \end{cases}$$

The Pictorial representation of trapezoidal membership function μ_A (A =[2, 4, 6, 8]) is given below within the figure.

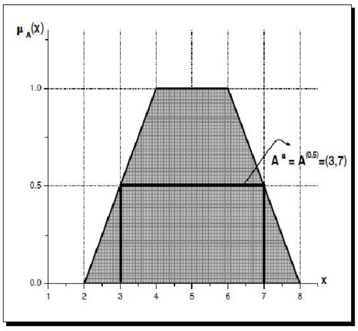

Figure: 2 α = 0. 5 cut of trapezoidal fuzzy number A = (2, 4, 6, 8)

Here, α - cut interval from the fuzzy number is

$$\frac{x-2}{2}=\alpha \Rightarrow x=2\alpha+2$$
$$\frac{x-8}{-2}=\alpha \Rightarrow x=-2\alpha+8$$
$$^{\alpha}A=[x_1^{\alpha},x_2^{\alpha}]=[2\alpha+2,-2\alpha+8]$$

If α = 0. 5, substituting 0.5 for α, we get $^{\alpha}A$
i. e., $^{\alpha}A=[x_1^{(0.5)},x_2^{(0.5)}]=[3,7]$.

2. 6. Robust Ranking Technique:
Robust ranking technique that satisfies linearity, compensation and additively properties provides results which are consist human intuition. If A could be a fuzzy number then the Robust Ranking is outlined by

$$\text{R(A)} = \int_0^1 (0.5)(A_\alpha^L, A_\alpha^U)\, d\alpha$$

where $(A_\alpha^L, A_\alpha^U)=[(a_2-a_1)\alpha+a_1, a_4+(a_3-a_4)\alpha]$ is the α level cut of the fuzzy number A.

Here allow us to thought of this methodology for ranking the objective values. The Robust ranking index R(A) offers the representative value of fuzzy number A.

3. Fuzzy Assignment Problem
Fuzzy assignment problem may be regarded as a special case of fuzzy transportation problem. The fuzzy assignment problem can be expressed as follows;

Jobs and Facilities

	1	2 …..	n	Supply
1	C_{11} X_{11}	C_{12}…….. X_{12}…..	C_{1n} X_{1n}	a_1
2 ⋮	C_{21} ⋮ X_{21} ⋮	C_{22} …….. ⋮ X_{22}……	C_{2n} ⋮ X_{2n} ⋮	a_2 ⋮
m	C_{m1} X_{m1}	C_{m2}….. X_{m2}…..	C_{mn} X_{mn}	a_m
Demand	b_1	b_2……	b_n	

Here facilities represent the ‘sources’ while the jobs represent the ‘destinations’. The supply available at each source is [-∂, 0, ∂, 4∂] where ∂ is small positive number, i. e., $a_i = [a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}] = [-\partial, 0, \partial, 4\partial]$ for all i. Similarly, the demand at each destinations is [-∂, 0, ∂, 4∂]i. e., $b_j= [b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}] = [-\partial, 0, \partial, 4\partial]$ for all j. The fuzzy cost of transporting (assigning) facility i to job j is $C_{ij} = [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}]$.

The resulting fuzzy assignment problem can be represented as in the above table (hereX_{ij}= [$x_{ij}^{(1)}$, $x_{ij}^{(2)}$, $x_{ij}^{(3)}$, $x_{ij}^{(4)}$]).

Let

[-2∂, -∂, ∂, 2∂] if the i^{th}

[$x_{ij}^{(1)}$, $x_{ij}^{(2)}$, $x_{ij}^{(3)}$, $x_{ij}^{(4)}$, $x_{ij}^{(5)}$] =facilityis not assigned to j^{th} job

[-∂, 0, ∂, 4∂] if the i^{th} facility is assigned toj^{th} job

Then the problem is given by

Maximize Z = [$z^{(1)}$, $z^{(2)}$, $z^{(3)}$, $z^{(4)}$] =

$$\sum_{j=1}^{m}\sum_{i=1}^{n} [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}]\,[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}]$$

Subject to the constraints

$$\sum_{j=1}^{m} [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}] = [-\partial, 0, \partial, 4\partial], \text{ for } i = 1, 2, \ldots, n$$

$$\sum_{i=1}^{n} [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}] = [-\partial, 0, \partial, 4\partial], \text{ for } j = 1, 2, \ldots, m$$

and

$$[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}] = [-\partial, 0, \partial, 4\partial] \text{ or } [-2\partial, -\partial, \partial, 2\partial]$$

we see that if the last condition is replaced by[$x_{ij}^{(1)}$, $x_{ij}^{(2)}$, $x_{ij}^{(3)}$, $x_{ij}^{(4)}$] ≥ [-2∂, -∂, ∂, 2∂], we have fuzzy assignment problem with all fuzzy requirements and available fuzzy resources equal to [-∂, 0, ∂, 4∂].

However, fuzzy transportation technique cannot be used to solve this problem because of fuzzy degeneracy. Whenever we make a fuzzy assignment, we automatically satisfy row and column fuzzy requirements simultaneously (rim requirements being equal to [-∂, 0, ∂, 4∂]), resulting in fuzzy degeneracy. This special structure of fuzzy assignment problem allows a more convenient method of solution.

4. Formulation and Solution of a Fuzzy Assignment problem

Sometimes the fuzzy assignment problem may deal with maximization of an objective function. For example, the problem may be to assign person to jobs in such a way that the expected profit is maximized in terms of fuzziness. The maximization problem has to be reduced to minimization problem before fuzzy Hungarian method may be applied. It is done by subtracting form the highest element, all the elements of the matrix.

We shall consider a numerical example which will make clear the techniques of formulation and solution of fuzzy assignment problems.

5. Results and Discussion

Using the projected technique the overall fuzzy assignment maximum cost is [39, 47, 51, 59], which may be physically understood as follows:

(1) The smallest amount of cost is 39.

(2) The foremost attainable quantity of cost lies between 47and 51.
(3) The greatest quantity of cost is 59.

The optimum fuzzy assignment cost are going to be continuously bigger than 39 and fewer than 59 and most likelihood is that the cost are going to be between 47 and 51. The variations in cost with relevancy likelihood is shown within the below figure. Similarly they obtained fuzzy optimum solutions x_{ij}could also be physically understood.

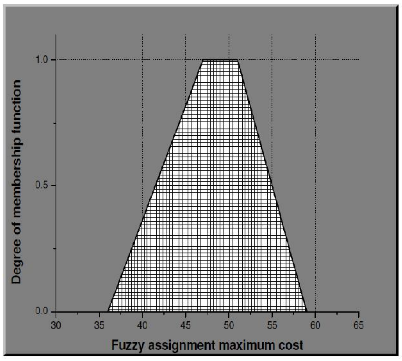

Figure: 5. Membership function of the fuzzy number representing the overall fuzzy assignment cost

(i) According to decision maker the overall fuzzy travelling minimum cost are going to be larger than Rs 39 and fewer than Rs 59.
(ii) Decision maker in favour of that the overall fuzzy travelling minimum cost are going to larger than or adequate to Rs 47 and fewer than or equal to Rs 51.
(iii) The proportion of the favourness of the decision maker for the remaining values of overall fuzzy assignment cost is often obtained as follows:

Let x represent the value of the overall fuzzy assignment cost then the proportion of the favourness of the decision maker for

$x = \mu_{max.cost.}(x)$.

$$\text{i.e., } x = \mu_{Max.\ cost}(x) = \left| \begin{array}{ll} [X-39]/8 & , 39 \le X \le 47 \\ 1 & , 47 \le X \le 51 \\ [59-X]/8 & , 51 \le X \le 59 \\ 0, \text{otherwise} & \end{array} \right|$$

6. Conclusion

In this paper, the assignment costs square measure thought about as imprecise numbers delineated by fuzzy numbers that square measure additional realistic and general in nature. Moreover, Numerical example shows an assignment problem of the optimum solution obtained from the manner of fuzzy fuzzy Hungarian technique with the assistance of Robust ranking technique [15, 18]. Finally the optimum solution for the maximum cost of the assignment connected between four different facilities and jobs and additionally this cost is verified within the manner of trapezoidal membership functions, which might be a brand new try in solving the assignment in fuzzy environment. An equivalent approach for solving the fuzzy issues may be utilized in future studies of operational analysis.

References

[1] Bellman R. E., Zadeh L. A., Decision making in a fuzzy environment, Management Sci. 17(1970), pp. 141-164.

[2] Geetha. S and Nair K. P. K, A Variation Of The Assignment Problem, European Journal of Operations Research, 68(1993), pp. 422-426.

[3] Makani Das and Hemanta K. Baruah, Solution of A Linear Programming Problem with Fuzzy Data, The Journal of Fuzzy Mathematics, 12, (4), 2004, pp. 793-811.

[4] Munkres. J, Algorithm for The Assignment and Transportation Problems, SIAM 15(1962), pp. 196-210.

[5] NagoorGani. A and Stephan Dinagar.D, A Note On Linear Programming In Fuzzy Environment, Proc. Nat. Acad. Sci. India, Sect. A, Vol. 79, Pt. I(2009).

[6] Pentico. D. W, Assignment Problem, Golden Anniversary Survey European Journal of Operations Research, 176(2007), pp. 774-793.

[7] Premkumar Gupta and Hira.D. S, Operations Research, S. Chand Company Ltd., New Delhi (2004).

[8] ShuganiPoonam, Abbas S. H. and Gupta V. K., Fuzzy Transportation Problem of Triangular Numbers with $\boldsymbol{\alpha} -$ Cut and Ranking Technique, IOSR Journal of Engineering, 2(5) (2012)pp. 1162-1164.

[9] Stephen Dinagar D. and Palanivel K., SolvingAssignment Problem in fuzzy Environment, Int. Journal of Fuzzy Mathematics, 19, No. 3 (2011), pp. 541-550.

[10] Stephen Dinagar D. and Palanivel K.., Solving a Travelling Salesman Problem in Fuzzy Environment, Int. Journal of Fuzzy Mathematics, 21, No. 1 (2013), pp. 69-76.

[11] Yager R. R., A procedure for ordering fuzzy subsets of the unit interval, Information Sciences, 24(1981), pp. 143 - 161.

[12] Geetha.N.K., Combinatorial Theory of a Complete Graph K5, International Organization of Scientific Research-Journal of Mathematics, 10, No. 5 (2014), pp. 10-12.

[13] Geetha.N.K., The Intrinsic Parity Phase Factors of Regular Graphs, International Journal of Advanced Research, 2, No.10 (2014), pp. 1087 – 1090.