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Perfect Fuzzy Matching on Intuitionistic Fuzzy Graph

R. Seethalakshmi
Saiva Bhanu Kshatriya College,
Aruppukottai - 626 101.
Tamilnadu, India.
E-mail: seethagnanam@rediffmail.com

R. B. Gnanajothi
V.V. Vanniaperumal College,
Virudhunagar - 626 001.
Tamilnadu, India.
E-mail: gnanajothi_pcs@rediffmail.com

Abstract

The notion of matching in a fuzzy graph could be defined using the concept of effective edges [8] or fractional matching [5]. In this paper, we introduce the notion of fuzzy matching and perfect fuzzy matching on an intuitionistic fuzzy graph. We derive a necessary condition for an intuitionistic fuzzy graph on a cycle or a complete graph or a stargraph to have an intuitionistic perfect fuzzy matching.

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1. Introduction

Zadeh [9] introduced the notion of Fuzzy sets and Fuzzy relations to deal with the problems of uncertainty in real physical world. Atanasov [2] introduced the notion of Intuitionistic fuzzy sets which is a generalization of fuzzy sets.

In 1975, Rosenfeld [6] introduced the concept of fuzzy graphs. The first definition of intuitionistic fuzzy Graph(IFG) was proposed by Atanasov [1].

The notion of fractional matching given by Scheinerman [3], in 1997, involves the vertex weight, the edge weight and the incidence of edges. Using the concept of effective edges, Somasundaram [8] defined matching in a fuzzy graph. Ramakrishnan P.V and

Vaidyanathan M [5] introduced matching in a fuzzy graph using the concept of fractional matching. This motivated us to study matching in an IFG.

In this paper, we introduce the notion of intuitionistic fuzzy matching and intuitionistic perfect fuzzy matching on IFG. We derive the necessary condition for an IFG on a cycle or a complete graph or a star graph to have intuitionistic perfect fuzzy matching. Further, we prove that a strong regular intuitionistic fuzzzy graph will have no intuitionistic perfect fuzzy matching.

2. Preliminaries

Definition 2.1. A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ with $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$, $\forall\, u, v \in V$, where V is a finite nonempty set and $\wedge$ denote minimum.

Definition 2.2. [7] An intuitiionistic fuzzy graph is of the form $G = (\sigma, \mu, \gamma, \chi)$ on (V, E) where

1. $V = \{v_1, v_2, \cdots, v_n\}$ such that $\sigma : V \to [0,1]$ and $\gamma : V \to [0,1]$ denote the degree of membership and nonmembership of vertices of V respectively such that $0 \leq \sigma(v_i) + \gamma(v_i) \leq 1\ \forall\ v_i \in V$.
2. $E \subseteq V \times V$ where $\mu : V \times V \to [0,1]$ and $\chi : V \times V \to [0,1]$ are membership and non-membership functions of edges in E respectively such that $\mu(v_i, v_j) \leq min\left\{\sigma(v_i), \sigma(v_j)\right\}$, $\chi(v_i, v_j) \leq max\left\{\gamma(v_i),\ \gamma(v_j)\right\}$ and $0 \leq \mu(v_i, v_j) + \chi(v_i, v_j) \leq 1$ for every edge (v_i, v_j) in E.

Definition 2.3. [7] The intuitionistic fuzzy graph is said to be strong if

$$\mu(v_i, v_j) = min\left\{\sigma(v_i), \sigma(v_j)\right\}, \chi(v_i, v_j) = max\left\{\gamma(v_i), \gamma(v_j)\right\},\ \forall\,(v_i, v_j) \in E.$$

Definition 2.4. [7] The IFG is said to be regular if $\sum\limits_{v_j, v_i \neq v_j} \mu(v_i, v_j) = \text{constant}$ and $\sum\limits_{v_j, v_i \neq v_j} \chi(v_i, v_j) = \text{constant}, \forall\ v_i \in V$.

Definition 2.5. [7] The IFG G is said to be strong regular if $G = (V, E)$ where

1. $\mu(v_i, v_j) = min\left\{\sigma(v_i), \sigma(v_j)\right\}, \chi(v_i, v_j) = max\left\{\gamma(v_i), \gamma(v_j)\right\},\ \forall (v_i, v_j) \in E.$
2. $\sum\limits_{v_j, v_i \neq v_j} \mu(v_i, v_j) = \text{constant}$ and $\sum\limits_{v_j, v_i \neq v_j} \chi(v_i, v_j) = \text{constant}.$

Definition 2.6. [5] Let $G = (\sigma, \mu)$ be a fuzzy graph on (V, E). where V is the vertex set and E is the set of edges with non-zero weights. A subset M of E is called a Fuzzy matching if for each vertex u, we have $\sum\limits_{v \in V} \mu(u, v) \leq \sigma(u)$.

Defintion 2.7. [5] A fuzzy matching M is called a Perfect Fuzzy Matching if for each vertex u, $\sum_{v \in V, (u,v) \in M} \mu(u, v) = \sigma(u)$.

3. Intuitionistic Perfect Fuzzy Matching

Definition 3.1. Let $G = (\sigma, \mu, \gamma, \chi)$ be an intuitionistic fuzzy graph on (V, E) where V is vertex set and E is the set of edges with nonzero weights. A subset M of E is called an intuitionistic fuzzy matching M if for each vertex $u \in M$, we have

$$\sum_{v \in V, (u,v) \in M} \mu(u, v) \leq \sigma(u)$$

and

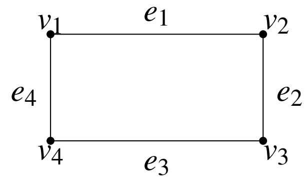
$$\sum_{v \in V, (u,v) \in M} \chi(u, v) \leq \gamma(u)$$

Example 3.2. Let $G = (\sigma, \mu, \gamma, \chi)$ be a IFG on (V, E) where $V = \{v_1, v_2, v_3, v_4\}$ and

$$E = \{e_1, e_2, e_3, e_4\}, e_1 = v_1v_2, e_2 = v_2v_3, e_3 = v_3v_4$$

and $e_4 = v_4v_1$ with

$$\sigma(v_1) = 0.6, \sigma(v_2) = 0.6, \sigma(v_3) = 0.5, \sigma(v_4) = 0.5,$$



$$\gamma(v_1) = 0.4, \gamma(v_2) = 0.3, \gamma(v_3) = 0.4, \gamma(v_4) = 0.4,$$

$$\mu(e_1) = 0.4, \mu(e_2) = 0.1, \mu(e_3) = 0.2, \mu(e_4) = 0.2$$

and

$$\chi(e_1) = 0.3, \chi(e_2) = 0.2, \chi(e_3) = 0.1, \chi(e_4) = 0.1.$$

$$\sum_{v_2 \in V, (v_1,v_2) \in M} \mu(v_1, v_2) = 0.4 + 0.2 = 0.6 = \sigma(v_1)$$

$$\sum_{v_4 \in V, (v_3,v_4) \in M} \mu(v_3, v_4) = 0.1 + 0.2 = 0.3 \leq 0.5 = \sigma(v_3)$$

$$\sum_{v_1 \in V, (v_4,v_1) \in M} \mu(v_4, v_1) = 0.2 + 0.2 = 0.4 \leq 0.5 = \sigma(v_4)$$

$$\sum_{v_2\in V,(v_1,v_2)\in M} \chi(v_1,v_2) = 0.3+0.1 = 0.4 = \gamma(v_1)$$

$$\sum_{v_3\in V,(v_3,v_4)\in M} \chi(v_3,v_4) = 0.2+0.1 = 0.3 \leq 0.4 = \gamma(v_3)$$

$$\sum_{v_1\in V,(v_4,v_1)\in M} \chi(v_4,v_1) = 0.1+0.1 = 0.2 \leq 0.4 = \gamma(v_4)$$

Thus $M = \{e_1, e_3, e_4\}$ is an intuitionistic fuzzy matching for G.

Definition 3.3. An intuitionistic fuzzy matching M is called an intuitionistic perfect fuzzy matching if

$$\sum_{v\in V,(u,v)\in M} \mu(u,v) = \sigma(u)$$

and

$$\sum_{v\in V,(u,v)\in M} \chi(u,v) = \gamma(u)$$

Example 3.4. Let $G = (\sigma, \mu, \gamma, \chi)$ be a IFG on (V, E) where $V = \{v_1, v_2, v_3, v_4\}$ and

$$E = \{e_1, e_2, e_3, e_4\}, e_1 = v_1v_2, e_2 = v_2v_3, e_3 = v_3v_4$$

and $e_4 = v_4v_1$ with

$$\sigma(v_1) = 0.6, \sigma(v_2) = 0.6, \sigma(v_3) = 0.4, \sigma(v_4) = 0.5,$$

$$\gamma(v_1) = 0.4, \gamma(v_2) = 0.4, \gamma(v_3) = 0.1, \gamma(v_4) = 0.2,$$

$$\mu(e_1) = 0.4, \mu(e_2) = 0.1, \mu(e_3) = 0.3, \mu(e_4) = 0.2$$

and

$$\chi(e_1) = 0.3, \chi(e_2) = 0, \chi(e_3) = 0.1, \chi(e_4) = 0.1.$$

$$\sum_{v_2\in V,(v_1,v_2)\in M} \mu(v_1,v_2) = 0.2+0.4 = 0.6 = \sigma(v_1)$$

$$\sum_{v_4\in V,(v_3,v_4)\in M} \mu(v_3,v_4) = 0.2+0.3 = 0.5 = \sigma(v_3)$$

$$\sum_{v_1\in V,(v_4,v_1)\in M} \mu(v_4,v_1) = 0.3+0.2 = 0.5 = \sigma(v_4)$$

$$\sum_{v_3\in V,(v_2,v_3)\in M} \mu(v_2,v_3) = 0.4+0.1 = 0.5 \leq 0.6 = \sigma(v_2)$$

$$\sum_{v_2\in V,(v_1,v_2)\in M} \chi(v_1,v_2) = 0.3+0.1 = 0.4 = \gamma(v_1)$$

$$\sum_{v_3 \in V, (v_2, v_3) \in M} \chi(v_2, v_3) = 0.3 + 0 \leq 0.4 = \gamma(v_2)$$

$$\sum_{v_4 \in V, (v_3, v_4) \in M} \chi(v_3, v_4) = 0 + 0.1 = 0.1 = \gamma(v_3)$$

$$\sum_{v_4 \in V, (v_4, v_1) \in M} \chi(v_4, v_1) = 0.1 + 0.1 = 0.2 = \gamma(v_4)$$

Thus $M = \{e_1, e_3, e_4\}$ is a intuitionistic perfect fuzzy matching for G.

Definition 3.5. Let $G = (\sigma, \mu, \gamma, \chi)$ be an IFG and M be an intuitionistic fuzzy matching. Then intuitionistic fuzzy matching number $\Gamma(G)$ is defined to be $\Gamma(G) = (\sum_{(u,v) \in M} \mu(u, v), \sum_{(u,v) \in M} \chi(u, v))$.

Example 3.6. In example 3.2, $\Gamma(G) = (0.8, 0.5)$ and in example 3.4. $\Gamma(G) = (0.9, 0.5)$.

Theorem 3.7. Let $G = (\sigma, \mu, \gamma, \chi)$ be a regular intuitionistic fuzzy graph on the cycle $G^* = (V, E)$. If $\sigma(v_i) = \ constant\ = k_1$, and $\gamma(v_i) = \ constant = k_2 \forall v_i \in V$ and if $\mu(v_i, v_j) = \frac{k_1}{2}, \chi(v_i, v_j) = \frac{k_2}{2}, \forall (v_i, v_j) \in E$, then E is a perfect fuzzy matching for G.

Proof. For cycles, only two edges incident wtih each vertex, we have

$$\begin{aligned}
\sum_{v \in V, (u,v) \in E} \mu(u, v) &= \mu(u, v) + \mu(u, w),\ for\ some\ v, w \in V \\
&= \frac{k_1}{2} + \frac{k_1}{2} \\
&= k_1 \\
&= \sigma(u),\ each\ vertex\ u \in V
\end{aligned}$$

and

$$\begin{aligned}
\sum_{v \in V, (u,v) \in E} \chi(u, v) &= \chi(u, v) + \chi(u, w),\ for\ some\ v, w \in V \\
&= \frac{k_2}{2} + \frac{k_2}{2} \\
&= k_2 \\
&= \gamma(u),\ each\ vertex\ u \in V
\end{aligned}$$

Then E is intuitinistic perfect fuzzy matching in G. ■

The converse of the above theorem need not be true. This can be seen from the following example.

Example 3.8. Let $G = (\sigma, \mu, \gamma, \chi)$ be an IFG on (V, E) where $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{e_1, e_2, e_3, e_4\}, e_1 = v_1v_2, e_2 = v_2v_3, e_3 = v_3v_4 \; and \; e_4 = v_4v_1$.

Define

$$\sigma(v_1) = 0.6, \sigma(v_2) = 0.6, \sigma(v_3) = 0.5, \sigma(v_4) = 0.5$$

and

$$\gamma(v_1) = 0.3, \gamma(v_2) = 0.3, \gamma(v_3) = 0.3, \gamma(v_4) = 0.3$$

and

$$\mu(e_1) = 0.4, \mu(e_2) = 0.2, \mu(e_3) = 0.3, \mu(e_4) = 0.3.$$

and

$$\chi(e_1) = 0.2, \chi(e_2) = 0.1, \chi(e_3) = 0.2, \chi(e_4) = 0.1.$$

Then E is an intuitionistic perfect fuzzy matching in G but the conditions of the above theorem are not satisfied.

Theorem 3.9. Let $G = (\sigma, \mu, \gamma, \chi)$ be a complete intuitionistic fuzzy graph K_n on (V, E). If $\sigma(u) = \; constant \; = k_1, \gamma(u) = \; constant \; = k_2, \forall u \in V$,

$$\mu(u, v) = \left[\frac{k_1}{n}\right] = k_3$$

and

$$\chi(u, v) = \left[\frac{k_2}{n}\right] = k_4, \forall (u, v)$$

on the cycle C_n and $\mu(u, v) = \dfrac{k_1 - 2k_3}{n - 3}$ and $\chi(u, v) = \dfrac{k_2 - 2k_4}{n - 3}$ for all the edges $(u, v) \in E - C_n$, then E is an intuitionistic perfect fuzzy matching for G.

Proof. Let $G = (\sigma, \mu, \gamma, \chi)$ be a complete intuitionistic fuzzy graph K_n on (V, E). Now,

$$\begin{aligned}
\sum_{v \in V, (u,v) \in E} \mu(u, v) &= 2k_3 + (n - 3)\left(\frac{k_1 - 2k_3}{n - 3}\right) \\
&= 2k_3 + k_1 - 2k_3 \\
&= k_1 \\
&= \sigma(u), for \; each \; vertex \; u
\end{aligned}$$

and

$$\begin{aligned}
\sum_{v \in V, (u,v) \in E} \chi(u, v) &= 2k_4 + (n - 3)\left(\frac{k_2 - 2k_4}{n - 3}\right) \\
&= 2k_4 + k_2 - 2k_4 \\
&= k_2 \\
&= \gamma(u), for \; each \; vertex \; u.
\end{aligned}$$

Therefore E is an intuitionistic perfect matching for G. ■

The converse of the above theorem is not true. This can be seen from the following example.

Example 3.10. Let $G = (\sigma, \mu, \gamma, \chi)$ be an IFG on (V, E) where

$$V = \{v_1, v_2, v_3, v_4\}$$

and

$$E = \{e_1, e_2, e_3, e_4, e_5, e_6\}, e_1 = v_1v_2, e_2 = v_2v_3, e_3 = v_3v_4, e_4 = v_4v_1, e_5 = v_4v_2$$

and

$$e_6 = v_1v_3$$

Define

$$\sigma(v_1) = 0.5, \sigma(v_2) = 0.6, \sigma(v_3) = 0.5, \sigma(v_4) = 0.4$$
$$\gamma(v_1) = 0.4, \gamma(v_2) = 0.4, \gamma(v_3) = 0.4, \gamma(v_4) = 0.4$$
$$\mu(e_1) = 0.2, \mu(e_2) = 0.3, \mu(e_3) = 0.1, \mu(e_4) = 0.2, \mu(e_5) = 0.1, \mu(e_6) = 0.1.$$
$$\chi(e_1) = 0.1, \chi(e_2) = 0.2, \chi(e_3) = 0.1, \chi(e_4) = 0.2, \chi(e_5) = 0.1, \chi(e_6) = 0.1.$$

Then E is an intuitionistic perfect fuzzy matching in G but the conditions of the above theorem are not satisfied.

Theorem 3.11. Let $G = (\sigma, \mu, \gamma, \chi)$ be a strong IFG on the star graph $G* = (V, E)$ with $V = \{v, v_1, v_2, \cdots, v_{n-1}\}$.

If $\sigma(v_i) = constant = k_1$ *and* $\gamma(v_i) = constant = k_2, \forall i = 1, 2, \cdots, (n-1)$ and if $\sigma(v) = (n-1)k_1, \gamma(v) = (n-1)k_2$, then E is an intuitionistic perfect fuzzy matching in G.

Proof.

$$\begin{aligned} Now,\ for\ all\ \ v_i \in V, \quad \mu(v, v_i) &= min\{\sigma(v), \sigma(v_i)\} \\ &= min\{(n-1)k_1, k_1\} \\ &= k_1 \end{aligned}$$

$$\begin{aligned} \chi(v, v_i) &= max\{\gamma(v), \gamma(v_i)\} \\ &= max\{(n-1)k_2, k_2\} \\ &= (n-1)k_2 \end{aligned}$$

$$\begin{aligned} For\ \ v \in V, \quad \sum_{v_i \in V, (v, v_i) \in E} \mu(v, v_i) &= \sum_{(v, v_i) \in E} k_1 \\ &= (n-1)k_1,\ since\ (n-1)\ edges\ incident\ with\ v. \\ &= \sigma(v) \end{aligned}$$

$$\begin{aligned}\sum_{v_i \in V, (v, v_i) \in E} \chi(v, v_i) &= \sum_{(v, v_i) \in E} k_2 \\ &= (n-1)k_2, \ since\ (n-1)\ edges\ incident\ with\ v. \\ &= \gamma(v)\end{aligned}$$

Therefore, E is an intuitionistic perfect fuzzy matching for G. ■

The converse of the above theorem is not true. This can be seen from the following example.

Example 3.12. Let $G = (\sigma, \mu, \gamma, \chi)$ be an IFG on (V, E) where

$$V = \{v_1, v_2, v_3, v_4, v_5\}$$

and

$$E = \{e_1, e_2, e_3, e_4, e_5\}, e_1 = v_1 v, e_2 = v_2 v, e_3 = v_3 v, e_4 = v_4 v$$

and

$$e_5 = v_5 v.$$

Define

$$\sigma(v_1) = 0.1, \sigma(v_2) = 0.2, \sigma(v_3) = 0.1, \sigma(v_4) = 0.2, \sigma(v_5) = 0.2, \sigma(v) = 0.8$$

$$\gamma(v_1) = 0.2, \gamma(v_2) = 0.2, \gamma(v_3) = 0.2, \gamma(v_4) = 0.1, \gamma(v_5) = 0.3, \gamma(v) = 1$$

$$\mu(e_1) = 0.1, \mu(e_2) = 0.2, \mu(e_3) = 0.1, \mu(e_4) = 0.2, \mu(e_5) = 0.2.$$

$$\chi(e_1) = 0.2, \chi(e_2) = 0.3, \chi(e_3) = 0.2, \chi(e_4) = 0.1, \chi(e_5) = 0.3.$$

Then E is an intuitionistic perfect fuzzy matching in G but the conditions of the above theorem are not satisfied.

The following theorem establishes that a strong regular IFG need not have an intuitionistic perfect fuzzy matching in G.

Theorem 3.13. If $G = (\sigma, \mu, \gamma, \chi)$ is a strong regular IFG on (V, E) with each vertex is of degree atleast two, then E is not an intuitionistic perfect fuzzy matching in G.

Proof. If possible, let E be a fuzzy perfect matching for G. Then

$$\sum_{v \in V, (u, v) \in E} \mu(u, v) = \sigma(u). \tag{1}$$

and

$$\sum_{v \in V, (u, v) \in E} \chi(u, v) = \gamma(u).$$

As G is regular,

$$\sum_{v \in V, u \neq v} \mu(u, v) = constant = k_1, \forall u \in V, \tag{2}$$

which imply $\sigma(u) = k_1, \forall u \in V$. (Using (1)). Since each vertex is of degree atleast two, from (2), $\mu(u, v) < k_1$.

As G is strong, $\sigma(u) \wedge \sigma(v) < k_1 \Rightarrow k_1 \wedge k_1 < k_1 \Rightarrow k_1 < k_1$, a contradiction. Therefore, E is not perfect matching for G. ■

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