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Some Identities of Multiplicative Fibonacci Like Sequences

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Abstract

Sequence have been fascinating topic for mathematician for centuries. It is well known that the Fibonacci numbers and Lucas numbers are closely related. In this paper, we present some Identities of Multiplicative Fibonacci Like Sequence.

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1. Introduction:

The Fibonacci sequence is a source of many nice and interesting identities. These numbers are of great importance in the study of many subjects such as Algebra Geometry and number theory itself.

TheFibonacci Sequence, Lucas Sequence Pell Sequence, Pell Lucas Sequence, jacobsthal_Lucas sequence are most prominent examples of recursive sequences.

The Fibonacci sequence is defined by the recursive relation

$F_n = F_{n-1} + F_{n-2}$, n≥2with$F_0=0$, $F_1=1$ (1. 1)

The Lucas sequence is defined by the recurrence relation

$L_n = L_{n-1} + L_{n-2}$, n≥2 with $L_o=2$, $L_1=1$ (1. 2)

In [1, 2], B. Singh, O. Sikhwal, and S. Bhatnagar, defined Fibonacci Like sequence. The Fibonacci Like Sequence is defined by recurrence relation

$S_k = S_{k-1} + S_{k-2}$ with $S_0=2$, $S_1=2$. (1. 3)

Initial conditions S_0 and S_1 are the sum of the Fibonacci and Lucas sequences respectively, i. e, $S_0 = F_0 + L_0, S_1 = F_1 + L_1$

In [3], V. H. Badshah, M. S. Teeth, and M. M. Dar, defined the associated Generalized Fibonacci Like Sequenceis defined by

$M_n = M_{n-1} + M_{n-2}$, for all n≥2 (1. 4) with M_0=2m and M_1=1+m, m being a fixed positive integer Here M_0=F_0+mL_0, M_1=F_1+mL_1

The few terms of he sequence{ M_n }are 2m, 1+m,, 3m+1, 7m+3. . . . and so on.

2. Multiplicative Fibonacci Like Sequence:

Multipllicative Fibonacci Like Seqquence is defined by

M_n=$M_{n-1}.M_{n-2}$, for all n≥2

With M_0=2m, M_1=1+m, m being fixed positive integer.

Here the initial condition $M_0$and $M_1$are the sum of initial conditions of Fibonacci sequence and m times the initial condition of Lucas sequence respectively.

i. e., M_0=F_0+mL_0, M_1=F_1+mL_1

The few terms of the sequence $\{M_n\}$ are

2m, 1+m, 2m. (1+m), 2m.... .

Now we present some identities of Multiplicative Fibonacci like sequence.

3. Identities of Multiplicative Fibonacci like sequence:

Some identities of (1. 4)are discussed below.

Theorem 3. 1 For every integer n≥0,

$$\frac{M_{n+2}}{M_{n+1}} = M_n$$

Theorem 3. 2 Product of the first n terms with odd indices

$$M_1.M_3.M_5...M_{2n-1} = \prod_{i=1}^{n} M_{2i-1} = \frac{M_{2n}}{2m}$$

Theorem 3. 3 Product of the first n terms with even indices is

$$M_2.M_4 \ ... \mathrm{M}_{2n} = \prod_{i=1}^{n} M_{2i} = \frac{M_{2n+1}}{1+m}$$

Theorem 3. 4 For every integer n≥ 0,

(a) $\frac{M_{3n+5}}{M_{3n+2}} = M_0^{2F_{3n+2}} M_1^{2F_{3n+3}}$

(b) $\frac{M_{3n+6}}{M_{3n+3}} = M_0^{2F_{3n+3}} M_1^{2F_{3n+4}}$

(c) $\frac{M_{3n+7}}{M_{3n+4}} = M_0^{2F_{3n+4}} M_1^{2F_{3n+5}}$

Theorem 3. 5 For every integer n≥0,

(a) $M_{3n+4=}M_{n+2}.M_{n+1}^2.M_n$

(b) $\dfrac{M_{n+3}}{M_n} = M_0^{2F_n}.M_1^{2F_{n+1}}$

(c) $\prod_{k=0}^{n} M_k = M_0^{F_{n+1}}.M_1^{F_{n+2}-1}$

Theorem 3. 6 For every n≥0,

(a) $M_{3n+3} = M_1.\prod_{k=0}^{3n+1} M_k$.

(b) $M_{3n+4} = M_1 \prod_{k=0}^{3n+2} M_k$

(c) $M_{3n+5} = M_1 \prod_{k=0}^{3n+3} M_k$

4. Result:

1. For k=0,

$$M_{4k+3} = \prod_{i=0}^{4k+2} M_i$$

2. For k=1

$$M_{k+2} = \prod_{i=0}^{2k} M_i$$

5. Some Determinantal Identities:

Theorem 5. 1 Let n be a positive integer, then

$$\begin{vmatrix} 1 & M_n & M_{n-2} \\ 1 & M_{n-1} & M_{n-2} \\ 1 & M_{n-2} & M_{n-1} \end{vmatrix} = \left(M_{n-1} - M_{n-2}\right)\left(M_{n-2} - M_n\right)\left(M_{n-1} - M_n\right)$$

Theorem 5. 2 Let n be a positive integer, then

$$\begin{vmatrix} 1 & M_{n-1.}M_{n-2} & M_n(M_{n-1} + M_{n-2}) \\ 1 & M_{n-1}.M_{n-2} & M_{n-1\,(M_n+M_{n-2})} \\ 1 & M_{n.}M_{n-1} & M_{n-2}(M_n + M_{n-1}) \end{vmatrix} = 0$$

6. Conclusion

This paper describe identities of Multiplicative Fibonacci like sequence of second order. many similar identities can be developed for higher order multiplicative Fibonacci like sequence.

7. Acnowledgement
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8. References:

[1] B. Singh, O. Sikhwal and S. Bhatnagar, Fibonacci Like sequence and its Properties, Int. J. Contemp Math. Sciences Vol. 5, 2010, No. 18, 857-868.
[2] B. Singh, S. Bhatnagar, and O. Sikhwal, Some Identities Of Multiplicative Coupled Fibonacci Sequences, Int. J. Computer Application, Vol. 63, 2013, No. 12.
[3] V. H. Badshah, M. S. Teeth, M. Dar, Generalized Fibonacci Like Sequence and Its Properties, Int. J. Contem. Math. Sciences, Vol. 7, 2012, No. 24, 1155-1164.