

Time Dependent Quadratic Demand Inventory Models Under Inflation

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Abstract

The literature is replete with inventory models having linear demand function. An attempt has been made to develop an inventory model for perishable items with the assumption that the demand is quadratic function of time and the rate of deterioration is constant. Under instantaneous replenishment with zero lead-time, EOQ is determined for optimizing the total cost under inflation. At the end sensitive analysis is presented with numerical example.

Key words: Quadratic demand, perishable, deterioration, inflation

1. Introduction

It is true that the unit price and other inventory related costs are dependent on time. However, most of the inventory models in the literature have considered unit price and inventory related costs to be independent of time and constant over the period under consideration. Buzacott [1] modified the classical EOQ model incorporating constant inflation rate under different pricing policies. Misra [2], Gupta et al [3], Vrat and Padmanabhan [4] are some of the authors who have studied inventory models with special reference to inflation rate.

It is well known that the demand rate of any product is always in dynamic state. This variation is due to time or price or even with instantaneous level of inventory. In last few decades several authors have extensively studied various aspects of inventory models considering two kinds of time –varying demand so far, viz., (a) continuous-time and (b) discrete-time. Most of the continuous –time inventory models have been developed considering either linearly increasing /decreasing demand [$D(t) = a+bt$, $a>0, b> < 0$] or exponentially increasing /decreasing demand [$D(t) = Ae^{\alpha t}$, $A > 0, \alpha >< 0$]. For a comprehensive review of literature on inventory models with time-varying demand, one may refer the work of Giri and Goyal [5]. Several

authors argued that, in realistic terms, the demand need not follow either linear or exponential trend. So, it is reasonable to assume that the demand rate, in certain commodities, due to seasonal variations may follow quadratic function of time [i.e., $D(t) = a + bt + ct^2$; $a \geq 0, b \neq 0, c \neq 0$]. The functional form of time-dependent quadratic demand explains the accelerated growth/decline in the demand patterns which may arise due to seasonal demand rate (Khanra and Chaudhuri)[6]. We may explain different types of realistic demand patterns depending on the signs of b and c . Bhandari and sharma [7] have studied a Single Period Inventory Problem with Quadratic Demand Distribution under the Influence of Marketing Policies. Khanra and Chaudhuri [6] have discussed an order-level inventory problem with the demand rate represented by a continuous quadratic function of time. It is well known that the demand for spare parts of new aero planes, computer chips of advanced computer machines, etc. increase very rapidly while the demands for spares of the obsolete aero planes, computers etc. decrease very rapidly with time. This type of phenomena can well be addressed by inventory models with quadratic demand rate. Sana and Chaudhuri [8] have developed a stock-review inventory model for perishable items with uniform replenishment rate and stock-dependent demand. Recently, Ghosh and Chaudhuri [9] have developed an inventory model for a deteriorating item having an instantaneous supply, a quadratic time-varying demand and shortages in inventory. They have used a two-parameter Weibull distribution to represent the time to deterioration. Venkateswarlu and Mohan [10] have developed inventory models for deteriorating items with time dependent quadratic demand and salvage value. Recently Venkateswarlu and Mohan [11] studied inventory model for time varying deterioration and price dependent quadratic demand with salvage value.

In this paper, we try to develop an integrated model which contains both the perishability and inflation phenomena with time-dependent quadratic demand situation. The inventory deterioration is assumed to be constant. The solutions of the models are presented and also discussed the sensitivity of the models at the end.

2. Assumptions and notations

The mathematical model is developed on the following assumptions and notations:

- (i) The selling rate $D(t)$ at time t is assumed to be $D(t) = a + bt + ct^2$, $a \geq 0, b \neq 0, c \neq 0$. Here a is the initial rate of demand, b is the initial rate of change of the demand and c is the acceleration of demand rate.
- (ii) Replenishment rate is infinite and lead time is zero.
- (iii) S , the selling price per unit,
- (iv) The rate of inflation is constant
- (v) The unit cost and other inventory related cost are subjected to the same rate of inflation, say k . This implies that the ordering quantity can be determined by minimising the total system cost over the planning period.
- (vi) $A(t)$ is the ordering cost at time t .
- (vii) The deterioration rate is taken as θ , where θ is constant
- (viii) $C(t)$ denotes unit cost at time t .
- (ix) $I(t)$ is the inventory level at time t .

3. Formulation and solution of the model

The objective of the model is to determine the optimum profit for items having time dependent quadratic demand and constant rate of deterioration with no shortages.

The inventory level depletes as the time passes due to demand only during (0, t₁) and due to demand and deterioration during the period (t₁, T).

If I(t) be the inventory level at time t, the differential equations which describes the inventory level at time t are given by

$$\frac{dI(t)}{dt} + \theta I(t) = -R(t), \quad 0 \leq t \leq t_1 \tag{1}$$

$$\frac{dI(t)}{dt} = -R(t), \quad t_1 \leq t \leq T \tag{2}$$

where $R(t) = (a + bt + ct^2)$.

Further we assume that $I(t) = I(0)$ at time $t = 0$ and $I(T) = 0$. (3)

The solution of equation (1) and (2) are

$$I(t) = -e^{-\theta t} \left[at + (a\theta + b)\frac{t^2}{2} + (b\theta + c)\frac{t^3}{3} + c\theta\frac{t^4}{4} \right] + c_1 e^{-\theta t} \quad 0 \leq t \leq t_1 \tag{4}$$

$$I(t) = -\int_{t_1}^T (a + bt + ct^2) dt \quad t_1 \leq t \leq T \tag{5}$$

where c_1 is a constant of integration.

Now expanding $e^{-\theta t}$ and ignoring higher order terms involving θ as θ is small, and also using the conditions given in equation (3), equations (4) and (5) can be expressed as

$$I(t) = a(t_1 - t) + (a\theta + b)\left(\frac{t_1^2}{2} - \frac{t^2}{2}\right) + (b\theta + c)\left(\frac{t_1^3}{3} - \frac{t^3}{3}\right) + c\theta\left(\frac{t_1^4}{4} - \frac{t^4}{4}\right) - (\theta) \left[a(t_1 - t) + b\left(\frac{t_1^2}{2} - \frac{t^2}{2}\right) + c\left(\frac{t_1^3}{3} - \frac{t^3}{3}\right) \right]; \quad 0 \leq t \leq t_1 \tag{6}$$

$$I(t) = a(t_1 - t) + b\left(\frac{t_1^2}{2} - \frac{t^2}{2}\right) + c\left(\frac{t_1^3}{3} - \frac{t^3}{3}\right); \quad t_1 \leq t \leq T \tag{7}$$

As the replenishment is instantaneous,

$$Q = I(0) = aT + (a\theta + b)\left(\frac{T^2}{2}\right) + (b\theta + c)\left(\frac{T^3}{3}\right) + c\theta\left(\frac{T^4}{4}\right) \tag{8}$$

Total system cost during the planning period ‘ τ ’ is the sum of the Material cost, ordering cost and Carrying cost. Assume that $\tau = m \cdot T$, where ‘ m ’ is an integer for the number of replenishments to make during the period ‘ τ ’, and ‘ T ’ is time between replenishments.

Denote $C(t) = C_0 e^{kt}$ where C_0 is the unit cost at time zero and $A(t) = A_0 e^{kt}$ where A_0 is the ordering cost at time zero.

The Ordering cost during the period (0, τ) is

$$A(0) + A(T) + A(2T) + A(3T) + \dots + A(m-1)T]$$

$$= A_0 \left(\frac{e^{k\tau} - 1}{e^{kT} - 1} \right) \quad (9)$$

The Material cost during the period $(0, \tau)$ is

$$= QC_0 \left(\frac{e^{k\tau} - 1}{e^{kT} - 1} \right) \quad (10)$$

The Carrying Cost/holding cost during the period $(0, \tau)$ is

$$\begin{aligned} & C(t) i \int_0^T I(t) dt \\ &= C_0 i \left(\frac{e^{k\tau} - 1}{e^{kT} - 1} \right) \left(\int_0^{t_1} I(t) dt + \int_{t_1}^T I(t) dt \right) \end{aligned} \quad (11)$$

$$\text{Here } \int_0^{t_1} I(t) dt = \frac{at_1^2}{2} + \frac{bt_1^3}{3} + \frac{ct_1^4}{4} + \frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8} + \frac{c\theta t_1^5}{10}$$

$$\text{and } \int_{t_1}^T I(t) dt = -(T - t_1)^2 \left(\frac{3a + 3bT + 6bt_1 + 2cT^2 + 4cTt_1 + 6ct_1^2}{12} \right)$$

The total cost over the period $(0, \tau)$ is

$K(T, \tau) = \text{Ordering cost} + \text{Material cost} + \text{Carrying cost}$

$$\begin{aligned} K(T, \tau) &= A_0 \left(\frac{e^{k\tau} - 1}{e^{kT} - 1} \right) + QC_0 \left(\frac{e^{k\tau} - 1}{e^{kT} - 1} \right) + C_0 i \left(\frac{e^{k\tau} - 1}{e^{kT} - 1} \right) \left(\int_0^{t_1} I(t) dt + \int_{t_1}^T I(t) dt \right) \\ &= \left(A_0 + QC_0 + C_0 i \left(\int_0^{t_1} I(t) dt + \int_{t_1}^T I(t) dt \right) \right) \left(\frac{e^{k\tau} - 1}{e^{kT} - 1} \right) \\ &= \left(A_0 + C_0 \left(aT + (a\theta + b) \left(\frac{T^2}{2} \right) + (b\theta + c) \left(\frac{T^3}{3} \right) + c\theta \left(\frac{T^4}{4} \right) \right) \right. \\ &\quad \left. + C_0 i \left(\int_0^{t_1} I(t) dt + \int_{t_1}^T I(t) dt \right) \right) \left(\frac{e^{k\tau} - 1}{e^{kT} - 1} \right) \end{aligned} \quad (12)$$

Our object is to minimize the total cost $K(T, \tau)$ per unit time. The necessary condition for the total cost $K(T, \tau)$ to be minimum is $\frac{\partial}{\partial T} K(T, \tau) = 0$ i.e.,

$$\left\{ \begin{array}{l} (e^{kT} - 1) \left(\begin{array}{l} C_0(c\theta T^3 + (c + b\theta)T^2 + (b + a\theta)T + a) \\ - C_0 i \left(\frac{(2T - 2t_1)(cT^2 + 2cTt_1 + 2bT + 3ct_1^2 + 4bt_1 + 6a)}{12} \right) \\ + \frac{(T - t_1)^2(2b + 2Tc + 2ct_1)}{12} \end{array} \right) \\ - (ke^{Tk}) \left(\begin{array}{l} A_0 + C_0 \left(aT + \frac{(b + a\theta)T^2}{2} + \frac{(c + b\theta)T^3}{3} + \frac{c\theta T^4}{4} \right) \\ + C_0 i \left(\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \frac{ct_1^4}{4} - \frac{(T - t_1)^2(cT^2 + 2cTt_1 + 2bT + 3ct_1^2 + 6a)}{12} \right) \\ + \frac{a\theta t_1^3}{6} + \frac{bt_1^4}{8} + \frac{ct_1^5}{10} \end{array} \right) \end{array} \right\} = 0$$

The sufficient condition for the total cost $K(T, \tau)$ per unit time to be minimum is $\frac{d^2(TC)}{dT^2} > 0$.

It is found that the optimality conditions are satisfied for all T in the following two cases viz.,

- (i) $c > 0$ and $b > 0$ which gives accelerated growth in demand model
- (ii) $c < 0$ and $b > 0$ gives retarded decline in demand model

3.1. Numerical Example

To demonstrate the effectiveness of the models developed, a numerical example is taken with the following values for the parameters.

$$a = 100, b = 15, c = 0.05, A_0 = 100, \\ C_0 = 6, \theta = 0.05, i = 0.05, k = 0.04$$

The MATHCAD output is presented in Table-1 and Table-2 which contains the optimum values of time (T), ordering quantity (Q) and total cost of the system (K) for various values of inflation parameter (k) and deterioration parameter (θ). These tables provide certain important insights about the problem under study. Some observations are as follows:

The behaviour of both the models developed here is almost similar in nature but the rate of change is slightly different. The optimal values of cycle time, ordering quantity and total cost increases with an increase in the inflation rate parameter ' k '. For some particular values of θ , when the inflation rate k increases from 0.04 to 0.08, the cycle time and ordering quantity increases while the total cost decreases in both the models.

For some particular values of k , when θ increases from 0.05 to 0.20, the cycle time and ordering quantity decreases whereas total cost increases in both the models.

3.2 SENSITIVE ANALYSIS

We now study sensitivity of the models developed to examine the implications of

underestimating and overestimating the parameters individually and all together on optimal value of cycle time, ordering quantity and total system cost. The Sensitive analysis is performed by changing each of the parameter by -20%, -10%, +10% and +20% taking one parameter at a time and keeping the remaining parameters are unchanged and finally all parameters are considered. The results are shown in Table-3 and Table-4. The following observations are made from these two tables:

- (i) The cycle time (T), the ordering quantity (Q) and the total system cost (TC) decreases (increases) with the decrease (increase) in the values of the parameter 'a' but the rate of change is more pronounced in case of ordering quantity and total cost.
- (ii) Decrease (increase) in parameters b , c and θ increases (decreases) the cycle time and ordering quantity while the total cost also decreases (increases) however the rate of increase/decrease is marginal in case of total cost.
- (iii) The effect of the parameters A_0 and C_0 on the optimum value of the total cost is similar but the rate of change is significant in case of C_0 . The cycle time and ordering quantity increase (decrease) with an increase (decrease) in A_0 but the effect is quite opposite in case of C_0 .
- (iv) The total cost is more sensitive than the cycle time and ordering quantity when the values of all parameters are under-estimated or over-estimated by 15%.

Table-1: Accelerated Growth Model ($a>0$, $b>0$ and $c>0$)

S.No	i	θ	a	b	c	K=0.04			K=0.06			K=0.08		
						T	K	Q	T	K	Q	T	K	Q
1	0.05	0.05	100	15	0.05	1.74	433.127	205.687	1.974	427.64	238.427	2.36	420.86	295.221
2		0.10	100	15	0.05	1.397	448.922	165.509	1.509	445.054	181.145	1.655	440.61	202.089
3		0.15	100	15	0.05	1.207	461.905	143.905	1.279	458.905	154.047	1.365	455.537	166.405
4		0.20	100	15	0.05	1.081	473.22	129.837	1.132	470.797	137.104	1.192	468.106	145.792
5	0.10	0.05	100	15	0.05	2.503	413.561	317.156	3.566	403.777	495.957	--	--	--
6		0.10	100	15	0.05	1.68	434.596	205.74	--	--	--	2.178	423.199	282.462
7		0.15	100	15	0.05	1.371	449.951	167.277	2.835	408.082	395.819	1.608	441.937	202.785
8		0.20	100	15	0.05	1.193	462.726	145.938	1.261	459.795	155.972	1.342	456.515	168.182
9	0.15	0.05	100	15	0.05	--	--	--	--	--	--	--	--	--
10		0.10	100	15	0.05	2.273	416.213	297.982	2.835	408.082	395.819	4.475	396.078	744.623
11		0.15	100	15	0.05	1.628	435.983	205.877	1.802	431.074	233.414	2.049	425.266	274.504
12		0.20	100	15	0.05	1.348	450.951	169.098	1.445	447.323	184.119	1.566	443.204	203.436
13	0.20	0.05	100	15	0.05	--	--	--	--	--	--	43.689	146.084	4797.0
14		0.10	100	15	0.05	8.386	382.002	2029.0	--	--	--	1038.0	0	2043.0
15		0.15	100	15	0.05	2.119	418.496	286.585	2.514	411.27	358.459	3.253	401.804	510.631
16		0.20	100	15	0.05	1.584	437.303	206.365	1.738	432.616	232.021	1.949	427.143	268.938

Table -2: Retarded Decline Model (i.e., $a>0$, $b>0$ and $c<0$)

S.No	i	θ	a	b	c	K=0.04			K=0.06			K=0.08		
						T	K	Q	T	K	Q	T	K	Q
1	0.05	0.05	100	15	-0.05	1.759	432.753	208.105	2.006	427.162	242.714	2.429	420.183	305.223
2		0.10	100	15	-0.05	1.405	448.67	166.511	1.52	444.761	182.571	1.671	440.259	204.248
3		0.15	100	15	-0.05	1.212	461.711	144.536	1.284	458.687	154.677	1.373	455.29	167.468
4		0.20	100	15	-0.05	1.084	473.06	130.213	1.136	470.621	137.621	1.197	467.911	146.456
5	0.10	0.05	100	15	-0.05	2.592	412.789	330.42	4.183	402.11	610.162	--	--	--
6		0.10	100	15	-0.05	1.697	434.233	208.049	--	--	--	2.227	422.594	290.002
7		0.15	100	15	-0.05	1.379	449.701	168.341	1.486	445.92	184.12	1.622	441.594	204.78
8		0.20	100	15	-0.05	1.197	462.531	146.456	1.267	459.578	156.786	1.35	456.269	169.305
9	0.15	0.05	100	15	-0.05	--	--	--	--	--	--	--	--	--
10		0.10	100	15	-0.05	2.331	415.545	307.104	2.983	407.035	422.259	--	--	--
11		0.15	100	15	-0.05	1.644	435.63	208.186	1.825	430.642	236.896	2.087	424.708	280.664
12		0.20	100	15	-0.05	1.355	450.703	170.068	1.454	447.037	185.409	1.58	442.868	205.55
13	0.20	0.05	100	15	-0.05	--	--	--	--	--	--	42.415	158.045	3801.0
14		0.10	100	15	-0.05	--	--	--	--	--	--	--	--	--
15		0.15	100	15	-0.05	2.162	417.892	293.684	3.511	400.327	567.461	3.511	400.327	567.461
16		0.20	100	15	-0.05	1.598	436.958	208.484	1.981	426.619	274.382	1.981	426.619	274.382

Table – 3: Accelerated Growth Model ($a>0$, $b>0$ and $c>0$)

Parameters	% change	Change in T (%)	Change in K (%)	Change in Q (%)
a	-20%	-3.3908	-16.2636	-20.9498
	-10%	-1.7241	-8.1293	-10.6657
	10%	1.8391	8.1244	11.1373
	20%	3.7356	16.2435	22.7029
b	-20%	16.8966	-2.3501	16.8766
	-10%	7.4713	-1.1297	7.4127
	10%	-6.1494	1.0565	-6.0509
	20%	-11.2644	2.0523	-11.0376
c	-20%	0.1149	-0.0085	0.1240
	-10%	0.0575	-0.0042	0.0617
	10%	-0.0575	0.0044	-0.0622
	20%	-0.1149	0.0085	-0.1245
θ	-20%	6.3793	-0.8586	6.4866
	-10%	2.9885	-0.4218	3.0235
	10%	-2.7586	0.4087	-2.7829
	20%	-5.2299	0.8053	-5.2556
A_0	-20%	-10.5747	-1.6459	-12.0897
	-10%	-5.1149	-0.7993	-5.8944
	10%	4.8851	0.7589	5.7111
	20%	9.5402	1.4829	11.2287
C_0	-20%	11.7816	-18.5336	13.9114
	-10%	5.4023	-9.2430	6.3208
	10%	-4.6552	9.2028	-5.3684
	20%	-8.7356	18.3704	-10.0142
All parameters	-20%	19.1379	-35.4503	-2.7401
	-10%	8.6207	-18.6814	-1.3452
	10%	-6.5517	18.9046	2.1231
	20%	-13.4483	43.0931	2.3886

Table – 4: Retarded Decline Model (i.e., $a>0$, $b>0$ and $c<0$)

Parameters	% change	Change in T (%)	Change in K (%)	Change in Q (%)
a	-20%	-3.4679	-16.2719	-21.0168
	-10%	-1.8192	-8.1335	-10.7633
	10%	1.8761	8.1282	11.1804
	20%	3.8658	16.2506	22.8726
b	-20%	17.6805	-2.3850	17.7463
	-10%	7.7885	-1.1445	7.7648
	10%	-6.3104	1.0678	-6.2262
	20%	-11.5406	2.0728	-11.3395
c	-20%	0.1137	-0.0088	0.1225
	-10%	0.0569	-0.0044	0.0610
	10%	-0.0569	0.0044	-0.0615
	20%	-0.1137	0.0088	-0.1230
θ	-20%	6.5947	-0.8698	6.7255
	-10%	3.1268	-0.4270	3.1792
	10%	-2.8425	0.4132	-2.8755
	20%	-5.4008	0.8139	-5.4453
A_0	-20%	-10.6879	-1.6300	-12.2169
	-10%	-5.1734	-0.7914	-5.9609
	10%	4.9460	0.7508	5.7822
	20%	9.6646	1.4667	11.3750
C_0	-20%	11.9386	-18.5498	14.0967
	-10%	5.4576	-9.2512	6.3848
	10%	-4.7186	9.2106	-5.4405
	20%	-8.8118	18.3862	-10.0997
All parameters	-20%	20.0682	-35.4921	-1.8923
	-10%	8.9824	-18.7026	-0.9798
	10%	-7.4474	20.6141	1.0062
	20%	-13.7578	43.1385	1.9884

4. Conclusions

In this paper we have developed inventory models having quadratic time dependent demand rate with constant rate of deterioration under constant inflation rate. The models are illustrated with a numerical example to study the effect of inflation rate and deterioration rate on optimal policies. Sensitivity analysis of the models is performed. It is observed that, in both these models, the effect of inflation rate and deterioration is quite opposite on ordering policies.

REFERENCES

- [1] Buzacott, J.A., 1975. Economic order quantities with inflation. *Operat. Res. Q.*, 26: 553-558.

- [2] Misra, R.B., 1979. A note on optimal inventory management under inflation, *Naval Res. Logist.*, 26: 161-165.
- [3] 3. Gupta, R., Vrat, P. and Swarup, K. (1985), "Inventory management development – A review with special reference to perishability, inflation and stock-dependent consumption", *Towards continuing Education*, Vol. 6, Oct. – Nov., 232-237.
- [4] Padmanabhan, G. and Vrat, P. 1990. An EOQ model for items with stock dependent consumption rate and exponential decay. *Engineering Costs and Production Economics*, 18(3), 241-246.
- [5] S. K. Goyal and B. C. Giri, "Recent trends in modeling of deteriorating inventory [J], " *European Journal of Operational Research*, Vol. 134, No. 1, pp. 1–16, 2001.
- [6] S.Kharna and K.S.Chaudhuri.(2003) 'A note on order-level inventory model for a deteriorating item with time-dependent quadratic demand' *Computers and Operations research*, Vol.30, pp.1901-1916.
- [7] R.M.Bhandari and P.K.Sharma (2000) A single period inventory problem with quadratic demand distribution under the influence of Market policies', *Eng. Science* Vol. 12. No.2, pp. 117-127.
- [8] Shibshankar Sana and K.S.Chaudhary (2004) A Stock-Review EOQ Model with Stock- Dependent Demand, Quadratic Deterioration Rate', *Advanced Modeling and Optimization*, vol.6, No.2, pp 25-32.
- [9] S.K.Ghosh and K.S.Chaudhuri (2004) 'An order- level inventory model for a deteriorating item with Weibull Deterioration, Time-quadratic demand and shortages", *Advanced Modeling and Optimization*, Vol.6, No.1, pp.21-35.
- [10] R.Venkateswarlu and R. Mohan (2011) Inventory Models for Deteriorating Items with Time Dependent Quadratic Demand and Salvage Value, *International Journal of Applied Mathematical Sciences*, Vol.5, No.1-2, pp11-18.
- [11] R.Venkateswarlu and R. Mohan (2013) An Inventory Model for Time Varying Deterioration and Price Dependent Quadratic Demand with Salvage Value, *Journal of Computational and Applied Mathematics*, 07/2013; 1(1):21-27.

