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Quantum Algorithm for 3-SAT Problem of 7, and 8 Variables by Quantum Fourier Transform with Repeat Qubits on QCEngine

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Abstract

A quantum algorithm for the 3-SAT problem of 7, and 8 variables by the quantum Fourier transform with the repeat qubits on the QCEngine, and its example are reported. When there are 3 literals with 2 'OR's in each clause, a number of clauses is m, an r-th clause ($1 \leq r \leq m$) is $C_{u,r}$ (x_1, x_2, x_3, … , x_n) [u is $2^0x_1 + 2^1x_2 + 2^2x_3 + \ldots + 2^{n-1}x_n$. x_1, x_2, …, and x_n are the variables, and the repeat qubits.], and S(u) is $\Sigma_{r=1 \to m}$ $r \times C_{u,r}$ (x_1, x_2, x_3, … , x_n), mod(S(u)$_{max}$) of S(u) [S(u)$_{max}$ is the maximum value of S(u).] is computed, next, for u, the quantum Fourier transform is done. In this time, there are 7, and 8 variables, and m = 11, and 12, respectively. The complexity of this method is able to be several times.

Keywords: Quantum algorithm, 3-SAT problem, 7, and 8 variables, quantum Fourier transform, repeat qubits, QCEngine.

AMS subject classification: Primary 81-08; Secondary 81-10, 68Q12.

1. **Introduction**

Cook discussed the complexity of the 3-SAT problem. [1] Quantum computer's example of the 3-SAT problem is reported by Johnston, Harrigan, and Gimeno-Segovia with QCEngine (free on-line quantum computer simulator). [2] Fujimura discussed a quantum algorithm for the 3-SAT problem by the Shor's Fourier transform with the RAM on the QCEngine. [3] Still more, Fujimura discussed a quantum algorithm for the 3-SAT problem of 5, and 6 variables by the Shor's Fourier transform with the repeat qubits on the QCEngine. [4]

According to my advanced study, when the quantum Fourier transform with the repeat qubits on the QCEngine for the 3-SAT problem of 7, and 8 variables is used, the complexity of the 3-SAT problem of 7, and 8 variables is able to be several times. This method is equal to the Shor's Fourier transform.

Therefore, the quantum algorithm for the 3-SAT problem of 7, and 8 variables is examined by the quantum Fourier transform with the repeat qubits on the QCEngine, and its result is reported.

2. **3-SAT Problem**

In the 3-SAT problem, it is assumed that (i) each value of *n* variables becomes "TRUE", or "FALSE", "~" is "NOT", "V" is "OR", "&" is "AND", (ii) "V", "~", and 3 different variables are included in each parentheses (= clause) that are connected by "&". If a value of logical formula by the literals and the logical connectives is "TRUE", it is decided whether there is at least one combination of values of the variables or not. [1-4]

3. **Quantum Algorithm**

The following conditions are assumed. (I) Each value of variables x_1, x_2, x_3, … , and x_n becomes "TRUE" [= 1], or "FALSE" [= 0]. "~" is "NOT". "V" is "OR". "&" is "AND". For example, it is assumed in this algorithm that (1 V 1 V 1), (1 V 1 V 0), and (1 V 0 V 0) become 1, and (0 V 0 V 0) becomes 0. (II) "V", "~", and 3 different variables in x_1, x_2, x_3, … , and x_n

are included in each clause, and then the clauses are connected by "&". In these conditions, if a value of logical formula by the literals, and the operators is "TRUE", it is searched whether there is at least one combination of values of the variables or not. It is assumed that n is number of qubits, u is $2^0x_1 + 2^1x_2 + 2^2x_3 + \dots + 2^{n-1}x_n$, a number of clauses is m, an r-th clause ($1 \le r \le m$) is $C_{u,r}$ ($x_1, x_2, x_3, \dots, x_n$), S($u$) is $\Sigma_{r=1 \to m}\, r \times C_{u,r}$ ($x_1, x_2, x_3, \dots, x_n$), and $S(u)_{max}$ is (the maximum value of S(u)) = $(m + 1)m/2 = k$.

First of all, query quantum registers $|x_i\rangle$[$1 \le i \le n$. i is an integer. n is the number of variables in the logical formula, and the repeat qubits.], and work1 quantum registers $|w_{1,j}\rangle$ [$1 \le j \le t$. j, and t are integers. t is a necessary number for $S(u)_{max} \le 2^t$.].

Step 1: The r data are introduced to the RAM [2].

Step 2: Each qubit of $|x_i\rangle$, and $|w_{1,j}\rangle$ is set $|0\rangle$.

Step 3: The Hadamard gate $\boxed{H}$ [2-9] acts on each qubit of $|x_i\rangle$. It changes them for entangled states.

Step 4: Each clause is presented by $|x_i\rangle$, $|w_{1,j}\rangle$, add gate, and quantum operators. For $|x_i\rangle$, RAM[r - 1] [RAM has r data of 0 → (m - 1).] is incremented in $|w_{1,j}\rangle$. In a function, S(u) = $\Sigma_{r=1 \to m}\, r \times C_{u,r}$ ($x_1, x_2, x_3, \dots, x_n$) is computed. This operation makes entangled data base. In this case, n is the number of variables in the logical formula, and repeat qubits.

Step 5: For $|x_i\rangle$, the quantum Fourier transform (= QFT) [2-7] is done.

Step 6: For $|x_i\rangle$, and $|w_{1,j}\rangle$, the probes are done.

Step 7: For $|x_i\rangle$, the read is done.

Step 8: A number of spikes is estimated by the function (https: //oreilly-qc. github. io? p = 12-4 [2]), where the function estimate_num_spikes (spike, range) [spike: read value, range: 2^n] is used.

Step 9: From candidates of the number of spikes, the repeat period P is obtained.

Step 10: From u = P = $2^0x_1 + 2^1x_2 + 2^2x_3 + \dots + 2^{n-1}x_n$, when there is $S(P)_{max}$ is $\Sigma_{r=1 \to m}\, r \times C_{P,r}$

$(x_1, x_2, x_3, \dots, x_n) = k$, it is the answer [one combination of (value of logical formula) = 1].

4. **Example of Numerical Computation**

4-1-1. **8 Variables, and 6 Repeat qubits**

For example at n = 14 [8 variables, and 6 repeat qubits], it is assumed that a logical formula : $(x_3 \lor x_4 \lor x_5)$ & $(\sim x_1 \lor x_2 \lor x_3)$ & $(\sim x_3 \lor x_4 \lor x_5)$ & $(x_3 \lor \sim x_4 \lor x_5)$ & $(\sim x_2 \lor x_3 \lor \sim x_5)$ & $(\sim x_3 \lor \sim x_4 \lor x_5)$ & $(\sim x_3 \lor x_4 \lor \sim x_5)$ & $(x_3 \lor \sim x_4 \lor \sim x_5)$ & $(\sim x_3 \lor \sim x_4 \lor \sim x_5)$ & $(x_4 \lor \sim x_5 \lor \sim x_6)$ & $(\sim x_5 \lor x_6 \lor \sim x_7)$ & $(x_6 \lor x_7 \lor \sim x_8)$, each value of $x_{1\sim8}$: $x_1 = x_2 = x_3 = x_4 = x_6 = x_7 = x_8 = 0$, $x_5 = 1$, $m = 12$, $t = 7$, and $k = (m + 1)m/2 = 78$.

An example of program on the QCEngine is the following.

```
var a = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]; // RAM_a
var query_qubits = 14;
var work1_qubits = 7;
qc.reset(query_qubits + work1_qubits);
var query = qint.new(query_qubits, 'query');
var work1 = qint.new(work1_qubits, 'work1');
qc.label('q'); // set query
query.write(0);
query.hadamard();
qc.label(' ');
qc.label('w1'); // set work1
work1.write(0);
qc.print(' RAM before increment : ' + a + '¥n');
var query16 = 16;
var work1_0 = 0;
qc.label('increment');
```

```
170 qc.not(query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
180 work1.add(a[0],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
190 qc.not(query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
200 qc.not(query.bits(0x2)|query.bits(0x4));
210 work1.add(a[1],query.bits(0x1)|query.bits(0x2)|query.bits(0x4));
220 qc.not(query.bits(0x2)|query.bits(0x4));
230 qc.not(query.bits(0x8)|query.bits(0x10));
240 work1.add(a[2],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
250 qc.not(query.bits(0x8)|query.bits(0x10));
260 qc.not(query.bits(0x4)|query.bits(0x10));
270 work1.add(a[3],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
280 qc.not(query.bits(0x4)|query.bits(0x10));
290 qc.not(query.bits(0x4));
300 work1.add(a[4],query.bits(0x2)|query.bits(0x4)|query.bits(0x10));
310 qc.not(query.bits(0x4));
320 qc.not(query.bits(0x10));
330 work1.add(a[5],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
340 qc.not(query.bits(0x10));
350 qc.not(query.bits(0x8));
360 work1.add(a[6],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
370 qc.not(query.bits(0x8));
380 qc.not(query.bits(0x4));
390 work1.add(a[7],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
400 qc.not(query.bits(0x4));
410 work1.add(a[8],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
```

```
420 qc.not(query.bits(0x8));
430 work1.add(a[9],query.bits(0x8)|query.bits(0x10)|query.bits(0x20));
440 qc.not(query.bits(0x8));
450 qc.not(query.bits(0x20));
460 work1.add(a[10],query.bits(0x10)|query.bits(0x20)|query.bits(0x40));
470 qc.not(query.bits(0x20));
480 qc.not(query.bits(0x20)|query.bits(0x40));
490 work1.add(a[11],query.bits(0x20)|query.bits(0x40)|query.bits(0x80));
500 qc.not(query.bits(0x20)|query.bits(0x40));
510 qc.label('QFT');
520 query.QFT();
530 var prob16 = 0;
540 prob16 += query.peekProbability(query16);
550 // Print output query-Prob
560 qc.print(' Prob_query16: ' + prob16);
570 var prob0 = 0;
580 prob0 += work1.peekProbability(work1_0);
590 // Print output work1-Prob
600 qc.print(' Prob_work1_0: ' + prob0);
610 //read
620 qc.label('Rq');
630 var b2 = query.read();
640 // Print output result
650 qc.print(' Read query = ' + b2 +'.');
660 // end
```

When this program is copied on Programming Quantum Computers https: //oreilly-qc. github. io/# [free on-line quantum computation simulator QCEngine] [2], you can run it. [Caution!: Please delate the line numbers.]

A result of this program is the following.

The probability probe value of $|w_{1,j}\rangle = 0$: ≈ 0.0039063 (≈ 1/256).

The probability probe value of $|x_i\rangle = 16$: ≈ 0.0000000.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 0, 15872, 13312, 512, 14336, 7680, 11776, 1024, 384, 8832, 1024, 1408, 15872, 3072, 1024, 16320, 13312, 13824, 512, 15872, 15872, 1536, 2560, 1024, 5952, 5120, 7680, 15616, 15872, 10240, 8000, 0, 9728, 768, 1024, 15040, 1536, 16320, 16192, 15872, 1408, 14080, 1792, 15744, 15552, 12800, 11392, 4736, 15360, 640. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^{14} = 16384$]] : R_q → candidates ; 0 → nothingness ; 15872 → 32, … ; 13312 → 5, 11, 16, … ; 512 → 32, … ; 14336 → 8, 16, … ; 7680 → 2, 4, 6, 9, 11, 13, 15, 17, … ; 11776 → 4, 7, 14, 18, … ; 1024 → 16, … ; 384 → 43, … ; 8832 → 2, 4, 7, 9, 11, 13, 26, … ; 1408 → 12, 23, … ; 3072 → 5, 11, 16, ... ; 16320 → 256, … ; 13824 → 6, 13, 19, … ; 1536 → 11, 21, … ; 2560 → 6, 13, 19, … ; 5952 → 3, 6, 8, 11, 22, … ; 5120 → 3, 6, 10, 13, 16, … ; 7680 → 2, 4, 9, 11, 13, 15, 19, … ; 15616 → 21, … ; 10240 → 3, 5, 8, 16, …; 8000 → 3, 5, 8, 16, … ; 9728 → 3, 5, 10, 15, 17, … ; 768 → 21, … ; 15040 → 12, 24, … ; 16192 → 85, … ; 1408 → 12, 23, … ; 14080 → 7, 14, 21, … ; 1792 → 9, 18, … ; 15744 → 26, … ; 15552 → 20, … ; 12800 → 5, 9, 18, … ; 11392 → 3, 7, 10, 13, 23, … ; 4736 → 4, 7, 14, 21, … ; 15360 → 16, … ; 640 → 26, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 + 2^8x_9 + 2^9x_{10} + 2^{10}x_{11} + 2^{11}x_{12} + 2^{12}x_{13} + 2^{13}x_{14} = 2^0\times0 + 2^1\times0 + 2^2\times0 + 2^3\times0 + 2^4\times1 + 2^5\times0 + 2^6\times0 + 2^7\times0 + 2^8\times0 + 2^9\times0 + 2^{10}\times0 + 2^{11}\times0 + 2^{12}\times0 + 2^{13}\times0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-1-2. **8 Variables, and 5 Repeat qubits**

For example at $n = 13$ [8 variables, and 5 repeat qubits], it is assumed that the logical formula, each value of $x_{1\sim8}$, m, t, and k are same in the section 4-1-1.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle = 0$: ≈ 0.0039063 (≈ 1/256).

The probability probe value of $|x_i\rangle = 16$: ≈ 0.0000000.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 517, 768, 7680, 1152, 5632, 544, 0, 5120, 512, 7744, 736, 6912, 2816, 7680, 512, 7680, 7168, 7680, 7168, 7680, 0, 7936, 128, 256, 4736, 7808, 6848, 0, 5888, 2272, 1152, 5120, 512, 6464, 7680, 5280, 576, 7200, 7168, 3072, 1024, 7424, 5632, 5888, 6400, 7736, 512, 7616, 0, 7296. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^{13} = 8192$]] : R_q → candidates ; 512 → 16, … ; 768 → 11, 21, … ; 7680 → 16, … ; 1152 → 7, 14, 21, … ; 5632 → 3, 6, 10, 13, 16, … ; 544 → 15, 30, … ; 0 → nothingness ; 5120 → 3, 5, 8, 16, … ; 7744 → 18, … ; 736 → 11, 22, … ; 6912 → 6, 13, 19, … ; 2816 → 3, 6, 9, 12, 15, 17, … ; 7168 → 21, … ; 6848 → 6, 12, 18, … ; 5888 → 4, 7, 14, 18, … ; 2272 → 4, 7, 11, 18, … ; 1152 → 7, 14, 21, … ; 6464 → 5, 10, 14, 19, … ; 5280 → 3, 6, 8, 11, 14, 28, … ; 576 → 14, 28, … ; 7200 → 8, 17, … ; 3072 → 3, 5, 8, 16, … ; 1024 → 8, 16, … ; 7424 → 11, 21, … ; 6400 → 5, 9, 18, … ; 7936 → 32, … ; 7616 → 14, 28, … ; 7296 → 9, 18, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 + 2^8x_9 + 2^9x_{10} + 2^{10}x_{11} + 2^{11}x_{12} + 2^{12}x_{13} = 2^0×0 + 2^1×0 + 2^2×0 + 2^3×0 + 2^4×1 + 2^5×0 + 2^6×0 + 2^7×0 + 2^8×0 + 2^9×0 + 2^{10}×0 + 2^{11}×0 + 2^{12}×0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-1-3. **8 Variables, and 4 Repeat Qubits**

For example at $n = 12$ [8 variables, and 4 repeat qubits], it is assumed that the logical formula, each value of $x_{1\sim8}$, m, t, and k are same in the section 4-1-1.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle = 0$: ≈ 0.0039063 (≈ 1/256).

The probability probe value of $|x_i\rangle = 16$: ≈ 0.0032627.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 3456, 0, 128, 1056, 0, 1280, 3632, 3216, 256, 128, 128, 3968, 128, 3968, 384, 160, 2528, 3968, 2432, 208, 3456, 1888, 3712, 488, 3664, 3840, 3936, 3328, 3840, 3008, 2688, 3584, 128, 576, 128, 2624, 832, 2688, 224, 1632, 1792, 1664, 3840, 3584, 64, 208, 3456, 3456, 3328, 0. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^{12} = 4096$]] : R_q → candidates ; 3456 → 6, 13, 19, … ; 0 → nothingness ; 128 → 32, … ; 1056 → 4, 8, 12, 16, … ; 1280 → 3, 6, 10, 13, 16, … ; 3632 → 9, 18, … ; 3216 → 5, 9, 14, 28, … ; 256 → 16, … ; 3968 → 32, … ; 384 → 11, 21, …; 160 → 26, … ; 2528 → 3, 5, 8, 13, 26, … ; 2432 → 3, 5, 10, 15, 17, … ; 208 → 20, … ; 1888 → 2, 4, 7, 9, 11, 13, 26, … ; 3712 → 11, 21, … ; 448 → 9, 18, … ; 3664 → 10, 19, … ; 3840 → 16, … ; 3936 → 26, … ; 3328 → 5, 11, 16, … ; 3008 → 4, 8, 11, 15, 30, … ; 2688 → 3, 6, 9, 12, 15, 17, … ; 3584 → 8, 16, … ; 576 → 7, 14, 21, … ; 2624 → 3, 6, 8, 11, 14, 25, … ; 832 → 5, 10, 15, 20, … ; 224 → 18, … ; 1632 → 18, … ; 1792 → 2, 5, 7, 9, 16, … ; 1664 → 3, 5, 10, 15, 17, … ; 64 → 64, … ; 208 → 20, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 + 2^8x_9 + 2^9x_{10} + 2^{10}x_{11} + 2^{11}x_{12} = 2^0×0 + 2^1×0 + 2^2×0 + 2^3×0 + 2^4×1 + 2^5×0 + 2^6×0 + 2^7×0 + 2^8×0+ 2^9×0 + 2^{10}×0+ 2^{11}×0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-1-4. **8 Variables, and 3 Repeat Qubits**

For example at $n = 11$ [8 variables, and 3 repeat qubits], it is assumed that the logical formula, each value of $x_{1\sim8}$, m, t, and k are same in the section 4-1-1.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle = 0$: ≈ 0.0039063 (≈ 1/256).

The probability probe value of $|x_i\rangle = 16$: ≈ 0.0037521.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 1472, 1216, 112, 64, 480, 1984, 448, 192, 2008, 1920, 1664, 1728, 1472, 1856, 2000, 448, 0, 1816, 224, 1792, 1480, 128, 328, 1664, 152, 64, 16, 64, 1984, 144, 12000, 1312, 184, 128, 320, 704, 64, 1640, 256, 1984, 1896, 1856, 1944, 768, 1920, 112, 96, 64, 1984, 0. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^{11} = 2048$]] : R_q → candidates ; 1472 → 4, 7, 14, 18, … ; 1216 → 3, 5, 10, 17, … ; 112 → 18, … ; 64 → 32, … ; 480 → 4, 9, 13, 17, … ; 1984 → 32, … ; 448 → 5, 9, 18, … ; 192 → 11, 21, … ; 2008 → 51, … ; 1920 → 16, … ; 1664 → 5, 11, 16, … ; 1728 → 6, 13, 19, … ; 1856→ 11, 21, … ; 2000

→ 43, …; 0 → nothingness ; 1816 → 9, 18, … ; 224 → 9, 18, … ; 1792 → 8, 16, … ; 1480 → 4, 7, 11, 18, … ; 128 → 16, … ; 328 → 6, 13, 19, … ; 1152 → 2, 5, 7, 9,16, … ; 16 → 128, … ; 144 → 14, 28, … ; 1312 → 3, 6, 8, 11, 14, 25, … ; 184 → 11, 21, … ; 320 → 6, 13, 19, … ; 704 → 3, 6, 9, 12, 15, 17, … ; 1640 → 5, 10, 15, 20, … ; 256→ 8, 16, … ; 1896 → 13, 27, … ; 1944 → 20, … ; 768 → 3, 5, 8, 16, … ; 96 → 21, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 + 2^8x_9 + 2^9x_{10} + 2^{10}x_{11} = 2^0\times0 + 2^1\times0 + 2^2\times0 + 2^3\times0 + 2^4\times1 + 2^5\times0 + 2^6\times0 + 2^7\times0 + 2^8\times0 + 2^9\times0 + 2^{10}\times0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-1-5. **8 Variables, and 2 Repeat Qubits**

For example at $n = 10$ [8 variables, and 2 repeat qubits], it is assumed that the logical formula, each value of $x_{1\sim8}$, m, t, and k are same in the section 4-1-1.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle = 0$: ≈ 0.0039063 (≈ 1/256).

The probability probe value of $|x_i\rangle = 16$: ≈ 0.014788.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 704, 736, 0, 864, 992, 984, 480, 640, 0, 96, 256, 32, 976, 0, 32, 36, 32, 344, 888, 976, 416, 1000, 0, 416, 928, 0, 128, 904, 384, 0, 64, 0, 704, 928, 960, 360, 32, 984, 864, 16, 1016, 0, 0, 416, 432, 956, 992, 428, 32, 224. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^{10} = 1024$]] : R_q → candidates ; 704 → 3, 6, 10, 13, 16, … ; 736 → 4, 7, 14, 18, … ; 0 → nothingness ; 864 → 6, 13, 19, … ; 992 → 32, … ; 984 → 26, … ; 480 → 2, 4, 6, 9, 13, 15, 17, … ; 640 → 3, 5, 8, 16, … ; 96 → 11, 21, … ; 256 → 4, 8, 12, 16, … ; 32 → 32, … ; 976 → 21, … ; 36 → 28, … ; 344→ 3, 6, 9, 12, 15, 18, … ; 888 → 8, 15, …; 416 → 3, 5, 10, 15, 17, … ; 1000 → 43, … ; 928 → 11, 21, … ; 128 → 8, 16, … ; 904 → 9, 17, … ; 384 → 3, 5, 8, 16, … ; 64 → 16, … ; 960 → 16, … ; 360 → 3, 6, 9, 11, 14, 17, … ; 16 → 64, … ; 1016 → 128, … ; 432 → 2, 5, 7, 12, 19, … ; 956 →15, 30, … ; 428 → 2, 5, 7, 12, 24, … ; 224→ 5, 9, 18, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 + 2^8x_9 + 2^9x_{10} = 2^0\times0 + 2^1\times0 + 2^2\times0 + 2^3\times0 + 2^4\times1 + 2^5\times0 + 2^6\times0 + 2^7\times0 + 2^8\times0 + 2^9\times0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-1-6**. 8 Variables, and 1 Repeat Qubit**

For example at n = 9 [8 variables, and 1 repeat qubit], it is assumed that the logical formula, each value of $x_{1\sim8}$, m, t, and k are same in the section 4-1-1.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle = 0$: ≈ 0.0039063 (≈ 1/256).

The probability probe value of $|x_i\rangle = 16$: ≈ 0.061888.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 456, 464, 256, 480, 16, 144, 48, 484, 496, 60, 298, 32, 390, 496, 510, 400, 480, 464, 160, 0, 496, 320, 428, 104, 400, 416, 312, 32, 24, 54, 0, 56, 112, 0, 16, 496, 4, 16, 0, 40, 426, 30, 496, 28, 480, 496, 480, 480, 0, 440. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^9 = 512$]] : R_q → candidates ; 456 → 9, 18, … ; 464 → 11, 21, … ; 256 → 2, 4, 6, 8, 10, 12, 14, 16, … ; 480 → 16, … ; 16 → 32, … ; 144 → 4, 7, 14, 18, … ; 48 → 11, 21, … ; 484 → 18, … ; 496→ 32, … ; 60 → 9, 17, …; 298 → 2, 5, 7, 12, 24, … ; 32 → 16, … ; 390 → 4, 8, 13, 17, … ; 510 → 256 ; 400 → 5, 9, 18, … ; 160 → 3, 6, 10, 13, 16, … ; 0 → nothingness ; 320 → 3, 5, 8, 16, … ; 428 → 6, 12, 18, … ; 104 → 5, 10, 15, 20, … ; 416 → 4, 9, 13, 22, … ; 312 → 3, 5, 10, 13, 18, … ; 24 → 21, … ; 54 → 10, 19, … ; 56 → 9, 18, … ; 112 → 5, 9, 18, … ; 4 → 128, … ; 40→ 13, 26, … ; 426 → 6, 12, 18, …; 30 → 17, … ; 28 → 18, … ; 440 → 7, 14, 21, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 + 2^8x_9 = 2^0\times0 + 2^1\times0 + 2^2\times0 + 2^3\times0 + 2^4\times1 + 2^5\times0 + 2^6\times0 + 2^7\times0 + 2^8\times0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-1-7. **8 Variables, and 0 Repeat Qubit**

For example at n = 8 [8 variables, and 0 repeat qubit], it is assumed that the logical formula, each value of $x_{1\sim8}$, m, t, and k are same in the section 4-1-1.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle = 0$: ≈ 0.0039063 (≈ 1/256).

The probability probe value of $|x_i\rangle = 16$: ≈ 0.043414.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 120, 27, 0, 78, 192, 72, 248, 248, 240, 246, 213, 225, 252, 233, 16, 208, 60, 234, 192, 12, 178, 219, 248, 232, 40, 17, 83, 29, 158, 212, 154, 36, 168, 252, 0, 10, 78, 54, 228, 48, 232, 176, 12, 224, 79, 193, 248, 118, 8, 16. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^9 = 512$]] : R_q → candidates ; 120 → 2, 4, 6, 9, 11, 13, 15, 17, … ; 27 → 10, 19, … ; 0 → nothingness ; 78 → 3, 7, 10, 13, 23, … ; 192 → 4, 8, 12, 16, … ; 72 → 4, 7, 14, 18, … ; 248 → 32, … ; 240 → 16, … ; 246 → 26, … ; 213→ 6, 12, 18, … ; 225 → 8, 17, …; 252 → 64, … ; 233 → 11, 22, … ; 16 → 16, … ; 208 → 5, 11, 16, … ; 60 → 4, 9, 13, 17, … ; 234 → 12, 23, … ; 12 → 21, … ; 178 → 3, 7, 10, 13, 23, … ; 219 → 7, 14, 21, … ; 232 → 11, 21, … ; 40 → 6, 13, 19, … ; 17 → 15, 30, … ; 83 → 3, 6, 9, 12, 15, 19, … ; 27 → 9, 18, … ; 158 → 3, 5, 8, 13, 26, … ; 212 → 6, 12, 17, … ; 154→ 3, 5, 10, 15, 20, … ; 36 → 7, 14, 21, …; 168 → 3, 6, 9, 12, 15, 17, … ; 10 → 26, … ; 78 → 3, 7, 10, 13, 23, … ; 54 → 5, 10, 14, 19, … ; 228 → 9, 18, … ; 48 → 5, 11, 16, … ; 176 → 3, 6, 10, 13, 16, … ; 224→ 8, 16, … ; 79 → 3, 7, 10, 13, 26, … ; 193 → 4, 8, 12, 16, … ; 118 → 2, 4, 7, 9, 11, 13, 26, … ; 8 → 32, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 = 2^0\times0 + 2^1\times0 + 2^2\times0 + 2^3\times0 + 2^4\times1 + 2^5\times0 + 2^6\times0 + 2^7\times0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-2-1**. 7 Variables, and 6 Repeat Qubits**

For example at $n = 13$ [7 variables, and 6 repeat qubits], it is assumed that a logical formula : (x_3 V x_4 V x_5) & (~x_1 V x_2 V x_3) & (~x_3 V x_4 V x_5) & (x_3 V ~x_4 V x_5) & (~x_2 V x_3 V ~x_5) & (~x_3 V ~x_4 V x_5) & (~x_3 V x_4 V ~x_5) & (x_3 V ~x_4 V ~x_5) & (~x_3 V ~x_4 V ~x_5) & (x_4 V ~x_5 V ~x_6) & (~x_5 V x_6 V ~x_7), each value of $x_{1\sim7}$: $x_1 = x_2 = x_3 = x_4 = x_6 = x_7 = 0$, $x_5 = 1$, $m = 11$, $t = 7$, and $k = (m + 1)m/2 = 66$.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle = 0$: 0.078125 (= 1/128).

The probability probe value of $|x_i\rangle = 16$: ≈ 0.0000000.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 6400, 0, 7680, 7360, 7680, 512, 320, 7936, 5888, 5888, 1024, 7680, 0, 4864, 1024, 3584, 7296, 320, 7936, 256, 6656, 7936, 7936, 0, 7936, 256, 7680, 1152, 896, 3072, 7936, 256, 2560, 7488, 5376, 512, 448, 256, 7680, 7552, 448, 0, 7936, 5376, 6656, 1216, 5376, 4096, 1792, 1536. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^{13} = 8192$]] : R_q → candidates ; 6400 → 5, 9, 18, … ; 0 → nothingness ; 7680 → 16, … ; 7360 → 10, 20, … ; 512 → 16, … ; 320 → 26, … ; 7936 → 32, … ; 5888 → 4, 7, 14, 18, … ; 1024 → 8, 16, … ; 4864 → 3, 5, 10, 15, 17, … ; 3584 → 2, 5, 7, 9, 16, … ; 7296 → 9, 18, ... ; 256 → 32, … ; 6656 → 5, 11, 16, … ; 1152 → 7, 14, 21, … ; 896 → 9, 18, … ; 3072 → 3, 5, 8, 16, … ; 2560 → 3, 6, 10, 13, 16, … ; 7488 → 12, 23, … ; 5376 → 3, 6, 9, 12, 15, 17, … ; 448 → 17, … ; 7552 → 13, 26, … ; 1216 → 7, 13, 20, … ; 4096 → 2, 4, 6, 8, 10, 12, 14, 16, … ; 1792 → 5, 9, 18, … ; 1536 → 5, 11, 16, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 + 2^8x_9 + 2^9x_{10} + 2^{10}x_{11} + 2^{11}x_{12} + 2^{12}x_{13} = 2^0\times0 + 2^1\times0 + 2^2\times0 + 2^3\times0 + 2^4\times1 + 2^5\times0 + 2^6\times0 + 2^7\times0 + 2^8\times0 + 2^9\times0 + 2^{10}\times0 + 2^{11}\times0 + 2^{12}\times0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-2-2. **7 Variables, and 5 Repeat Qubits**

For example at $n = 12$ [7 variables, and 5 repeat qubits], it is assumed that the logical formula, each value of $x_{1\sim7}$, m, t, and k are same in the section 4-2-1.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle = 0$: 0.0078125 (= 1/128).

The probability probe value of $|x_i\rangle = 16$: ≈ 0.0000000.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 384, 3456, 3136, 256, 32, 1280, 0, 1152, 2496, 3968, 3840, 3200, 3200, 256, 3168, 128, 1792, 1536, 256, 128, 128, 1024, 3936, 3456, 2496, 3808, 3968, 2560, 64, 256, 3744, 384, 3840, 3904, 288, 3936, 0, 3968, 1280, 896, 1024, 3712, 1536, 3104, 2816, 2208, 1152, 3968, 0, 3712. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^{12} = 4096$]] : R_q → candidates ; 384 → 11, 21, … ; 3456 → 6, 13, 19, … ; 3136 → 4, 9, 13, 17, … ; 256 → 16, … ; 32 → 128 ; 1280 → 3, 6, 10, 13, 16, … ; 0 → nothingness ; 1152 → 4, 7, 14, 18, ... ; 2496 → 3, 5, 10, 13,18, …; 3968 → 32, … ; 3840 → 16, … ; 3200 → 5, 9, 18, … ; 3168 → 4, 9, 13, 22, … ; 128 → 32, … ; 1792 → 2, 5, 7, 9, 16, … ; 1536 → 3, 5, 8, 16, … ; 1024 → 4, 8, 12, 16, … ; 3936 → 26, … ; 3808 → 14, 28, … ; 2560 → 3, 5, 8, 16, … ; 64 → 64, … ; 3744 → 12, 23, … ; 3904 → 21, … ; 288 → 14, 28, … ; 896 → 5, 9, 18, … ; 3712 → 11,12, … ; 3104 → 4, 8, 12, 17, … ; 2816 → 3, 6, 10, 13, 16, … ; 2208 → 2, 4, 7, 9, 11, 13, 26, … ; 1152 → 4, 7, 14, 18, … ; 3968 → 32, … ; 3712 → 11, 21, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 + 2^8x_9 + 2^9x_{10} + 2^{10}x_{11} + 2^{11}x_{12} = 2^0\times0 + 2^1\times0 + 2^2\times0 + 2^3\times0 + 2^4\times1 + 2^5\times0 + 2^6\times0 + 2^7\times0 + 2^8\times0 + 2^9\times0 + 2^{10}\times0 + 2^{11}\times0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-2-3. **7 Variables, and 4 Repeat qubits**

For example at $n = 11$ [7 variables, and 4 repeat qubits], it is assumed that the logical formula, each value of $x_{1\sim7}$, m, t, and k are same in the section 4-2-1.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle = 0$: 0.0078125 (= 1/128).

The probability probe value of $|x_i\rangle = 16$: ≈ 0.0051407.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 320, 1952, 1536, 768, 1280, 1856, 1856, 96, 0, 960, 96, 96, 832, 1920, 1600, 640, 128, 1792, 64, 144, 1408, 768, 768, 832, 640, 192, 1344, 128, 1984, 576, 1472, 1968, 1984, 16, 192, 1920, 1920, 1968, 64, 832, 1280, 2016, 1920, 128, 64, 64, 1984, 1424, 2032, 1920. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^{11} = 2048$]] : R_q → candidates ; 320 → 6, 13, 19, … ; 1952 → 21, … ; 1536 → 4, 8, 12, 16, … ; 768 → 3, 5, 8, 16, … ; 1280 → 3, 5, 8, 16, … ; 1856 → 11, 21, … ; 96 → 21, … ; 0 → nothingness ; 960 → 2, 4, 6, 9, 11, 13, 15, 17, … ; 832 → 3, 5, 10, 15, 17, … ; 1920 → 16, … ; 1600 → 5, 9, 18, … ; 640 → 3, 6, 10, 13, 16, … ; 128 → 16, … ; 1792 → 8, 16, … ; 64 → 32, … ; 144 → 14, 28, … ; 1408 → 3, 6, 10, 13, 16, … ; 192 → 11, 21, … ; 1344 → 3, 6, 9, 12, 15, 17, … ; 1984 → 32, … ; 576 → 4, 7, 14, 18, … ; 1472 → 4, 7, 14, 18, … ; 1968 → 26, … ; 16 → 128, … ; 2016 → 64, … ; 1984 → 32, … ; 1424 → 3, 7, 10, 13, 23, … ; 2032 → 128, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 + 2^8x_9 + 2^9x_{10} + 2^{10}x_{11} = 2^0\times0 + 2^1\times0 + 2^2\times0 + 2^3\times0 + 2^4\times1 + 2^5\times0 + 2^6\times0 + 2^7\times0 + 2^8\times0 + 2^9\times0 + 2^{10}\times0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-2-4. **7 Variables, and 3 Repeat Qubit**

For example at n = 10 [7 variables, and 3 repeat qubit], it is assumed that the logical formula, each value of $x_{1\sim7}$, m, t, and k are same in the section 4-2-1.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle = 0$: 0.0078125 (= 1/128).

The probability probe value of $|x_i\rangle = 16$: ≈ 0.010653.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 960, 992, 32, 616, 0, 64, 224, 992, 32, 0, 960, 864, 976, 48, 704, 0, 0, 128, 992, 16, 968, 56, 832, 928, 864, 792, 0, 136, 992, 992, 992, 992, 96, 824, 832, 832, 640, 744, 992, 0, 64, 32, 64, 64, 120, 320, 704, 64, 344, 32. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^{10} = 1024$]] : R_q → candidates ; 960 → 16, … ; 992 → 32, … ; 32 → 32 ; 616 → 3, 5, 10, 15, 20, … ; 0 → nothingness ; 64 → 16, … ; 224 → 5, 9, 18, … ; 864 → 6, 13, 19, … ; 976 → 21, … ; 48 → 21, … ; 704 → 3, 6, 10, 13, 16, … ; 128 → 8, 16, … ; 16 → 64, … ; 968 → 18, … ; 56 → 18, … ; 832 → 5, 11, 16, … ; 928 → 11, 21, … ; 792 → 4, 9, 13, 22, … ; 136 → 8, 15, … ; 96 → 11, 21, … ; 824 → 5, 10, 15, 20, … ; 640 → 3, 5, 8, 16, … ; 744 → 4, 7, 11, 22, … ; 120 → 9, 17, … ; 320 → 3, 6, 10, 13, 16, … ; 344 → 3, 6, 9, 12, 15, 18, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 + 2^8x_9 + 2^9x_{10} = 2^0\times0 + 2^1\times0 + 2^2\times0 + 2^3\times0 + 2^4\times1 + 2^5\times0 + 2^6\times0 + 2^7\times0 + 2^8\times0 + 2^9\times0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-2-5. **7 Variables, and 2 Repeat Qubits**

For example at n = 9 [7 variables, and 2 repeat qubits], it is assumed that the logical formula, each value of $x_{1\sim7}$, m, t, and k are same in the section 4-2-1.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle$ = 0 : 0.0078125 (= 1/128).

The probability probe value of $|x_i\rangle$ = 16 : ≈ 0.078301.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 128, 476, 392, 32, 16, 352, 0, 480, 32, 16, 496, 436, 48, 496, 16, 208, 0, 16, 128, 484, 488, 84, 16, 32, 400, 496, 104, 72, 0, 176, 436, 168, 256, 208, 496, 352, 496, 16, 216, 316, 80, 56, 8, 132, 288, 16, 48, 288, 36, 176. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^9 = 512$]] : R_q → candidates ; 128 → 4, 8, 12, 16, … ; 476 → 14, 28, … ; 392 → 4, 9, 13, 17, … ; 32 → 16, … ; 16 → 32, … ; 352 → 3, 6, 10, 13, 16, … ; 0 → nothingness ; 480 → 16, … ; 496 → 32, … ; 436 → 7, 13, 20, … ; 48 → 11, 21, … ; 208 → 3, 5, 10, 15, 17, … ; 484 → 18, … ; 488 → 21, … ; 84 → 6, 12, 18, … ; 400 → 5, 9, 18, … ; 104 → 5, 10, 15, 20, … ; 72 → 7, 14, 21, … ; 176 → 3, 6, 9, 12, 15, 17, … ; 168 → 3, 6, 9, 12, 15, 18, … ; 256 → 2, 4, 6, 8, 10, 12, 14, 16, … ; 216 → 2, 5, 7, 12, 19, … ; 316 → 3, 5, 8, 13, 26, … ; 80 → 6, 13, 19, … ; 56 → 9, 18, … ; 8 → 64, … ; 132 → 4, 8, 12, 16, … ; 288 → 2, 5, 7, 9, 16, … ; 36 → 14, 28, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 + 2^8x_9 = 2^0\times0 + 2^1\times0 + 2^2\times0 + 2^3\times0 + 2^4\times1 + 2^5\times0 + 2^6\times0 + 2^7\times0 + 2^8\times0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-2-6. **7 Variables, and 1 Repeat Qubit**

For example at n = 8 [7 variables, and 1 repeat qubit], it is assumed that the logical formula, each value of $x_{1\sim7}$, m, t, and k are same in the section 4-2-1.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle$ = 0 : 0.0078125 (= 1/128).

The probability probe value of $|x_i\rangle$ = 16 : ≈ 0.055841.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 0, 0, 224, 240, 24, 8, 232, 16, 0, 0, 8, 252, 0, 8, 6, 62, 168, 192, 128, 232, 168, 84, 208, 232, 88, 216, 0, 0, 224, 8, 114, 16, 56, 232, 24, 52, 8, 0, 0, 168, 8, 0, 232, 0, 60, 16, 76, 0, 242, 248. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^8 = 256$]] : R_q → candidates ; 0 → nothingness ; 224 → 8, 16, … ; 240 → 16, … ; 24 → 11, 21, … ; 8 → 32, … ; 232 → 11, 21, … ; 16 → 16, … ; 252 → 64, … ; 6 → 43 ; 62 → 4, 8, 12, 17, … ; 168 → 3, 6, 9, 12, 15, 17, … ; 192 → 4, 8, 12, 16, … ; 128 → 2, 4, 6, 8, 10, 12, 14, 16, … ; 84 → 3, 6, 9, 12, 15, 18, … ; 208 → 5, 11, 16, … ; 88 → 3, 6, 9, 12, 15, 17, … ; 216 → 6, 13, 19, … ; 84 → 3, 6, 9, 12, 15, 18, … ; 208 → 5, 11, 16, … ; 88 → 3, 6, 9, 12, 15, 17, … ; 216 → 6, 13, 19, … ; 114 → 2, 5, 7, 9, 18, … ; 56 → 5, 9, 18, … ; 60 → 4, 9, 13, 17, … ; 76 → 3, 7, 10, 17, … ; 242 → 18, … ; 248 → 32, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 = 2^0\times0 + 2^1\times0 + 2^2\times0 + 2^3\times0 + 2^4\times1 + 2^5\times0 + 2^6\times0 + 2^7\times0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-2-7. **7 Variables, and 0 Repeat Qubit**

For example at n = 7 [7 variables, and 0 repeat qubit], it is assumed that the logical formula, each value of $x_{1\sim7}$, m, t, and k are same in the section 4-2-1.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle$ = 0 : 0.0078125 (= 1/128).

The probability probe value of $|x_i\rangle$ = 16 : ≈ 0.024451.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 0, 4, 12, 113, 44, 48, 2, 51, 100, 0, 46, 80, 8, 52, 113, 1, 0, 8, 115, 8, 116, 4, 120, 80, 108, 36, 120, 123, 24, 20, 127, 120, 124, 112, 20, 4, 120, 16, 119, 88, 118, 115, 119, 118, 104, 0, 48, 19, 100, 120. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^7 = 128$]] : R_q → candidates ; 0 → nothingness ; 4 → 32, … ; 12 → 11, 21, … ; 113 → 9, 17, … ; 44 → 3, 6, 9, 12, 15, 17, … ; 48 → 3, 5, 8, 16, … ; 2 → 64 ; 51 → 3, 5, 10, 15, 20, … ; 100 → 5, 9, 18,

… ; 46 → 3, 6, 8, 11, 14, 25, … ; 80 → 3, 5, 8, 16, … ; 8 → 16, … ; 52 → 3, 5, 10, 15, 17, … ; 1 → nothingness ; 115 → 10, 20, … ; 116 → 11, 21, … ; 120 → 16, … ; 108 → 6, 13, 19, … ; 36 → 4, 7, 14, 18, … ; 123 → 26, … ; 24 → 5, 11, 16, … ; 20 → 6, 13, 19, … ; 127 → nothingness ; 124 → 32, … ; 112 → 8, 16, … ; 16 → 8, 16, … ; 119 → 14, 28, … ; 88 → 3, 6, 10, 13, 16, … ; 118 → 13, 26, … ; 104 → 5, 11, 16, … ; 19 → 7, 13, 20, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 = 2^0×0 + 2^1×0 + 2^2×0 + 2^3×0 + 2^4×1 + 2^5×0 + 2^6×0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

5. **Discussion**

In the 3-SAT problem, when [(the logical formula) = 1] is obtained, there is only one combination.

5-1. **8 Variables, and Repeat Qubits from 0 to 6**

In the section 4-1, there are 8 variables. And then, in 8 variables, [(the logical formula) = 1] of combination of variables is selected, and the computation of repeats is done. When N is 2^8, in the Grover's method, the complexity is $N^{1/2} = 2^{8/2} \approx 16$, in the new method, for (variables, repeat qubits) = (8, 0), it is 50/10 ≈ 5, for (8, 1), it is 50/10 ≈ 5, for (8, 2), it is 50/8 ≈ 6, for (8, 3), it is 50/10 ≈ 5, for (8, 4), it is 50/11 ≈ 5, for (8, 5), it is 50/20 ≈ 3, and for (8, 6), it is 50/13 ≈ 4.

In this range, the new method is less than the complexity of the Grover's method, and then, in for (8, 5), the probability is the maximum value 40%.

5-2. **7 Variables, and Repeat Qubits from 0 to 6**

In the section 4-2, there are 7 variables. And then, in 7 variables, [(the logical formula) = 1] of combination of variables is selected, and the computation of repeats is done. When N is 2^7, in the Grover's method, the complexity is $N^{1/2} = 2^{7/2} \approx 11$, in the new method, for (variables, repeat qubits) = (7, 0), it is 50/17 ≈ 3, for (7, 1), it is 50/9 ≈ 6, for (7, 2), it is 50/12 ≈ 4, for (7, 3), it is 50/15 ≈ 3, for (7, 4), it is 50/18 ≈ 3, for (7, 5), it is 50/15 ≈ 3, and for (7, 6), it is 50/16 ≈ 3.

In this range, the new method is less than the complexity of the Grover's method, and then, in for (7, 4), the probability is the maximum value 36%.

6. **Summary**

The quantum algorithm for the 3-SAT problem of 7, and 8 variables by the quantum Fourier transform with the repeat qubits on the QCEngine, and its example are reported.

The complexity of this method is several times, and then, in for (7, 4), the probability is the maximum value 36%, and in for (8, 5), the probability is the maximum value 40%.

I will apply this method for other problems.

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