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Mathematical Modelling of Transmission Dynamics of Hepatitis B Virus Incorporating Moderation of Human Activities In Uganda

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Abstract

Hepatitis B is among the dangerous diseases that threaten the lives of many people in the world because it causes cirrhosis and liver cancer. Moderating human activities that lead to its transmission is one way of preventing this deadly disease. In this study,Hepatitis B model for moderation of human activities that lead to transmission dynamics has been formulated and analysed. The threshold value for Hepatitis B transmission was determined using next generation matrix. Jacobian matrix was used to determine local stability of disease free equilibrium point which is stable. The study established the endemic equilibrium point and its local stability was analysed using the Routh-Hurwitz theorem, and was asymptotically stable. We determined the most sensitivity parameter contribution using sensitivity analysis on the threshold value for HBV spread. Numerical simulation was also established and it was found out that Hepatitis B transmission can be reduced from the population through moderation of human activities that contribute towards the increased transmission of Hepatitis B virus.

Keywords: Modelling, Hepatitis B Virus disease, equilibrium point, Local stability, Reproductive number,Numerical simulation.

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1. INTRODUCTION

Hepatitis B is one of the most common life-threatening diseases that affects the liver and it is caused by Hepatitis B virus(HBV)[2]. This virus is commonly known to cause acute and chronic Hepatitis B infection which also causes cirrhosis and liver cancer. HBV belongs to the family of Hepadnavirus which is DNA virus. Research shows that of the world's population, 7% is infected with Hepatitis B and it is approximated that 0.6 million people die every year[1]. According to the research in [2][9][15] it was shown that more than 250 million people had Hepatitis B virus infection, which resulted to more than 800, 000 deaths worldwide[11]. WHO recommends a great campaign to put emphasis on testing,vaccination and treating Hepatitis B especially among children and pregnant mothers so that HBV does not become chronic. Hepatitis B can become chronic to people who stay with it for many years without being treated and especially children who acquire the disease at birth,have low chances of recovery if not vaccinated at birth [1].

This disease is deadly because it starts by being acute and if not treated it develops into inactive carrier, chronical infection which increases the risk of developing liver failure, cirrhosis, liver cancer and even death[13]. As many countries all over the world have experienced Hepatitis B transmission disease, Uganda has not been spared by this problem. The ministry of Health carried out an investigation and found out that Uganda is one of the top countries greatly affected by HBV infection based on statistical data .

The data clearly indicates that 3.5 million in Uganda (10% of population) people have chronic Hepatitis B infection with Karamoja region having the highest percentage of 23.9%,while northern Uganda has 20.7%, West Nile 18.5% and western region have 10.0% [12][7] . This high incidence is attributed to the fact that they experience early sexual debut and cultural practices such as traditional tattooing or skin cutting. Such practices have been observed particularly among women in the eastern and north-western regions which can easily transmit Hepatitis B. Hepatitis B is still a threatening burden in Uganda. The Press statement on World Hepatitis day indicated that Mulago Hospital, the largest referral hospital in Uganda, 80% of liver cancer patients are related to HBV annually and it is noted that among the blood donors the trend of Hepatitis B virus has increased from 1.9% in 2012/13 to 2.3% in 2016/17 [6]. Uganda government through the ministry of health has come up to encourage people to carry out screening, testing and vaccination of Hepatitis B in order to control its spread and transmission, but the challenge is that some people do not complete the dose of hepatitis B [10] [11].

Vaccination of infants against HBV in Uganda was launched and introduced by 2002[6]. HBV spreads widely when body fluids especially blood, semen or other body fluid of a person with Hepatitis B virus enters the body of a person who is not infected [4] [5]. The rate of transmission of Hepatitis B in many countries across the world is attributed to human activities, yet many people seem not to be informed. In this study, the researcher has tried to educate the public that if human activities that are known as transmission routes of HBV are moderated they can control the rate of transmission of Hepatitis B. These transmission routes include; Having unprotected sex, Sharing of contaminated needles, sharing of house hold items like razors,toothbrushes or other personal care items with some one infected. Improper disposal of sharps and waste, Unsafe practices which lead to transmission of blood borne viruses like body tattooing,skin cutting and body piercing, Sharing of contaminated injecting equipments especially among drug addicted people, Reuse of injection equipment like syringes to administer injections to many people, Accidental needle-stick injuries in hospitals [11][16].

In this study a mathematical model was used because models are of great importance as they help in investigations of the dynamics of transmission of infectious disease like Hepatitis B which is our major focus in this study. Also infectious disease models, help in predicting the future dynamics of diseases. In most cases these infectious epidemic models use ordinary differential equations to describe and explain the transmission mechanism of the infection [18]. Different scholars have come up with different models about transmission dynamics of Hepatitis B and these include: Qesmi came up with a mathematical model in mathematical bioscience in [14] to describe the dynamics of Hepatitis B and Hepatitis C (HCV) and their interaction with both liver and blood cells.The research was aimed at finding the extent to which reinfection of both HBV and HCV occur after liver transplantation. It was found that at times graft survival after liver transplantation can occur in some people while in other people can fail to survive. Authors in [15] proposed a mathematical model that analysed the transmission dynamics of HBV infection in China indicated that many people who are HIV infected were also HBV infected. This meant that there might be a correlation between the transmission of these two diseases since their transmission routes are similar. Edmunds et al in [17] proposed a deterministic model describing the transmission dynamics of hepatitis B virus in a high endemicity country, considering these transmission routes; horizontal, sexual and perinatal transmission. In this project we focused on investigating the transmission dynamics of Hepatitis B virus incorporating moderation of human activities.

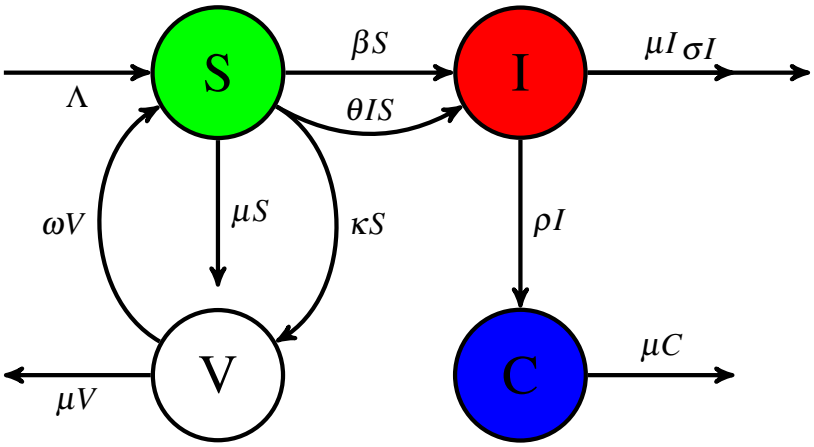
2. MODEL DESCRIPTION AND FORMULATION

In this study the research came up with a model to find out the extent to which incorporating moderating human activities can impact the transmission of Hepatitis B virus. The total population represented by N(t) is divided into four epidemiological compartments at any time t.
These are:Susceptible (S), the Infected (I), Vaccinated (V) and the Inactive carriers (C). The total population becomes:

$$N(t) = S(t) + I(t) + C(t) + V(t). \tag{2.1}$$

The Susceptible consists of the people at risk of being infected by HBV, the Infected consists of the infectious people with the signs and symptoms of the HBV, the Inactive carriers compartment contains the people who have acquired the virus,they are infectious but the virus has not advanced to cirrhosis, and after sometime HBV can become inactive or non progressive, the Vaccinated are those that have been immunized from Hepatitis B. The human behaviours consists of those activities that contribute towards the spread of HBV that need to be moderated. The susceptible are recruited into the population at the rate of Λ, β is the rate of contact between the susceptible and human activities,the rate of contact between the susceptible and infected is θ, the rate at which one moves from the infected to Inactive carrier is ρ, the vaccination rate is κ, the vaccination failure rate is ω, where as μ represents natural death of humans and σ represents death as a result of HBV.

Flow diagram for Hepatitis B model



The model is given by four ordinary differential equations:

$$\left.\begin{aligned} \frac{dS}{dt} &= \Lambda + \omega V - \kappa S - \mu S - \beta S - \theta SI \\ \frac{dV}{dt} &= \kappa S - \omega V - \mu V \\ \frac{dI}{dt} &= \beta S + \theta SI - \mu I - \sigma I - \rho I \\ \frac{dC}{dt} &= \rho I - \mu C \end{aligned}\right\} \tag{2.2}$$

3. THE DISEASE-FREE EQUILIBRIUM (DFE)

under the disease free equilibrium conditions there is no infection of the disease.
The disease free equilibrium is denoted by (E^0).
At DFE let

$$S = S^0, V = V^0, I = I^0, C = C^0$$

and

$$I^0 = 0, C^0 = 0$$

So $E_0 = (S^0, V,^0 I^0, C^0)$. To find the equilibrium point, we solve the following set of equations:-

$$\left.\begin{aligned} \Lambda + V\omega - (\kappa + \mu + \beta)S - SI\theta &= 0 \qquad (i) \\ \kappa S - (\omega + \mu)V &= 0 \qquad (ii) \\ \beta S + \theta SI - (\mu + \sigma + \rho)I &= 0 \qquad (iii) \\ \rho I - \mu C &= 0 \qquad (iv) \end{aligned}\right\} \tag{3.1}$$

From eqn (ii)

$$V = \frac{\kappa S}{\mu + \omega} \qquad (v)$$

Substituting eqn (v) in eqn (i) we get

$$S^0 = \frac{\Lambda(\omega+\mu)}{\kappa\mu+(\mu+\beta)(\omega+\mu)}$$

$$V^0 = \frac{\kappa\Lambda}{\kappa\mu+(\mu+\beta)(\omega+\mu)}$$

Therefore the DFE is given by:

$$(S^0,V^0,0,0) = \left(\frac{\Lambda(\omega+\mu)}{\kappa\mu+(\mu+\beta)(\omega+\mu)}, \frac{\kappa\Lambda}{\kappa\mu+(\mu+\beta)(\omega+\mu)}, 0, 0\right)$$

4. THE BASIC REPRODUCTION NUMBER (R_0)

Considering the system of equations in (3.1) at disease Free equilibrium. The basic reproduction number is obtained using next generation matrix method.In this method we consider the infectious compartments only

$$\frac{dI}{dt} = \beta S + \theta SI - (\mu+\sigma+\rho)I$$

We let f be the number of new infections coming into the HBV system and v be the number of new infections that are going out of the HBV system. Then,

$$f = [\beta S + \theta SI]$$

$$\frac{df}{dI} = F$$

therefore

$$F = [\theta S]$$

Again,

$$v = [(\mu+\sigma+\rho)I]$$

then

$$\frac{dv}{dI} = V$$

therefore

$$V = [\mu+\sigma+\rho]$$

$R_0 = \rho(FV^{-1})$ where ρ is the spectral radius. Thus

$$V^{-1} = \left[\frac{1}{\mu+\sigma+\rho}\right]$$

and

$$FV^{-1} = \frac{\theta S}{\mu+\sigma+\rho}$$

thus

$$R_0 = \frac{\theta S}{\mu+\rho+\sigma}$$

But

$$S = \frac{\Lambda(\omega+\mu)}{\kappa\mu+(\mu+\beta)(\omega+\mu)}$$

Therefore, the basic reproduction number is given by:

$$R_0 = \frac{\theta\Lambda(\omega+\mu)}{(\mu+\rho+\sigma)[\kappa\mu+(\mu+\beta)(\omega+\mu)]} \tag{4.1}$$

5. LOCAL STABILITY OF THE DISEASE-FREE EQUILIBRIUM

Theorem 1 The Disease Free Equilibrium Point (E^0) is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$. We use the Jacobian matrix approach to obtain the eigenvalues below. The jacobian matrix is given by:

$$J = \begin{bmatrix} -(\kappa+\mu+\beta) & \omega & -S\theta & 0 \\ \kappa & -(\omega+\mu) & 0 & 0 \\ \beta+\theta I & 0 & \theta S-(\mu+\sigma+\rho) & 0 \\ 0 & 0 & \rho & -\mu \end{bmatrix}$$

At DFE

$$J(S^0,V^0,0,0) = \left(\frac{\Lambda(\omega+\mu)}{\kappa\mu+(\mu+\beta)(\omega+\mu)}, \frac{\kappa\Lambda}{\kappa\mu+(\mu+\beta)(\omega+\mu)}, 0, 0\right)$$

$$= \begin{pmatrix} -(\kappa+\mu+\beta)-A & \omega & -\dfrac{\theta\Lambda(\omega+\mu)}{\kappa\mu+(\mu+\beta)(\omega+\mu)} & 0 \\ \kappa & -(\omega+\mu)-A & 0 & 0 \\ \beta & 0 & \dfrac{\theta\Lambda(\omega+\mu)}{\kappa\mu+(\mu+\beta)(\omega+\mu)}-(\mu+\sigma+\rho)-A & 0 \\ 0 & 0 & \rho & -\mu-A \end{pmatrix}$$

The eigenvalues which are obtained are as follows;

$$\begin{aligned} A_1 &= -\mu \\ A_2 &= -(\omega+\mu) \\ A_3 &= -(\kappa+\mu+\beta) \\ A_4 &= \frac{\theta\Lambda(\omega+\mu)}{\kappa\mu+(\mu+\beta)(\omega+\mu)} - (\mu+\sigma+\rho) \end{aligned}$$

Then the disease free equilibrium is globally asymptotically stable when all eigenvalues have a negative real part for $0 \leq t < \infty$. Since A_1, A_2, A_3 are negative we need to make A_4 negative also.

Theorem 2:
The disease Free equilibrium is globally Asymptotically stable.
Proof:
For D.F.E to be Asymptotically stable ,we have to make $A_4 < 0$.
This means

$$\frac{\theta\Lambda(\omega+\mu)}{\kappa\mu+(\mu+\beta)(\omega+\mu)} - (\mu+\sigma+\rho) < 0$$

or

$$\frac{\theta\Lambda(\omega+\mu)(\mu+\rho+\sigma)}{(\mu+\rho+\sigma)[\kappa\mu+(\mu+\beta)(\omega+\mu)]} - (\mu+\rho+\sigma) < 0$$

But from eqn 4.1

$$R_0 = \frac{\theta\Lambda(\omega+\mu)}{(\mu+\rho+\sigma)[\kappa\mu+(\mu+\beta)(\omega+\mu)]}$$

Thus

$$R_0(\mu+\rho+\sigma) - (\mu+\rho+\sigma) < 0$$

meaning

$$R_0(\mu+\rho+\sigma) < (\mu+\rho+\sigma)$$

or

$$R_0 < 1$$

Since $R_0 < 1$, then D.F.E is globally asymptotically stable.

6. ENDEMIC EQUILIBRIUM

The endemic equilibrium points is expressed in the form; $(E^*) = (S^*, I^*, V^*, C^*)$ considering this system of equation:

$$\left.\begin{aligned} \Lambda + V\omega - (\kappa+\mu+\beta)S - SI\theta &= 0 \quad (1) \\ \kappa S - (\omega+\mu)V &= 0 \quad (2) \\ \beta S + \theta SI - (\mu+\sigma+\rho)I &= 0 \quad (3) \\ \rho I - \mu C &= 0 \quad (4) \end{aligned}\right\}$$

from equation (4)

$$C^* = \frac{\rho I^*}{\mu} \tag{5}$$

$$V^* = \frac{\kappa S^*}{\mu+\omega} \tag{6}$$

replacing eqn (6) in equation (1) we obtain

$$S^* = \frac{\Lambda(\mu+\omega)}{\kappa\mu + (\mu+\beta+\theta I^*)(\mu+\omega)} \tag{7}$$

$$V^* = \frac{\kappa\Lambda}{\kappa\mu + (\mu+\beta+\theta I^*)(\mu+\omega)} \tag{8}$$

Substituting the value of S^* in equation (3)

$$\beta\Lambda(\mu+\omega) + \theta\Lambda(\mu+\omega)I^* - \kappa\mu(\mu+\sigma+\rho)I^* - [(\mu+\sigma+\rho)(\mu+\beta)(\mu+\omega)]I^* \\ -(\mu+\sigma+\rho)(\mu+\omega)\theta I^{*2} = 0 \tag{9}$$

Equation (9) reduces to the following quadratic equation.

$$AI^{*2} + BI^* + C = 0 \tag{10}$$

where

$$A = (\mu+\sigma+\rho)(\mu+\omega)\theta \tag{11}$$

$$B = [(\mu+\sigma+\rho)(\mu+\beta)(\mu+\omega)+\kappa\mu(\mu+\sigma+\rho)-\theta\Lambda(\mu+\omega)]$$

$$C = -\beta\Lambda(\mu+\omega) \tag{13}$$

writing B in terms of R_0

$$B = [(\mu+\sigma+\rho)(\mu+\beta)(\mu+\omega)+\kappa\mu]1-R_0 \tag{12}$$

I^* is obtained by solving the quadratic equation 10. Substituting the value of I^* in the endemic equation S^*, V^*, C^* the endemic equilibrium is obtained.

To determine the existence and number of positive solutions of the equation (2.2) of HBV model depends on the sign of the coefficients of A,B and C. We use Descartes' rule of signs to obtain number of endemic equilibria [8] . From equation (10), the coefficient of A is always positive and C is negative, the number of endemic equilibria depend on the sign of B, which is established by the value of R_0 in B whether it is greater than one,less than one,or equal to one .

Theorem

The system represented by equation (2.2) possesses one endemic positive equilibrium point that is locally stable when $R_0 = 1$,$R_0 < 1$, and $R_0 > 1$ in eqn (12)

Proof

We start by showing the existence of the equilibrium point by applying Descartes' rule to prove the above theorem. For the case when $R_0 = 1$,B=0, there is one change of sign from positive to negative.The quadratic equation has one positive root.

For the case when $R_0 < 1$,B is positive,the signs for A,B and C are (+ + -), which according to Descartes' rule there is only one change of sign, and so there is only one unique positive endemic equilibrium point. Finally for the case when $R_0 > 1$ B is negative. The sign for A,B and C includes (+ - -) and this again indicates one change of sign.This means there is existence of one positive root which is in this case corresponds to an endemic equilibrium point shown in the theorem.

Therefore the endemic equilibrium of the system of equation of hepatitis B exists and has real positive roots.

7. LOCAL STABILITY OF ENDEMIC EQUILIBRIUM

To prove the local stability of the (2.2) system, we apply the Routh-Hurwitz Criterion as follows: If $R_0 > 1$ the endemic equilibrium of HBV system (2.2) is locally asymptotically stable otherwise it is unstable.
The Jacobian matrix at the endemic equilibrium is evaluated to determine the local stability.

$$J = \begin{pmatrix} -(\kappa+\mu+\beta+\theta I^*) & \omega & -S^*\theta & 0 \\ \kappa & -(\omega+\mu) & 0 & 0 \\ \beta+\theta I^* & 0 & \theta S^*-(\mu+\sigma+\rho) & 0 \\ 0 & 0 & \rho & -\mu \end{pmatrix}$$

let

$$\begin{aligned} H_1 &= \kappa+\mu+\beta+\theta I^* \\ H_2 &= \omega+\mu \\ H_3 &= S\theta-(\mu+\sigma+\rho) \\ H_4 &= \beta+\theta I^* \end{aligned}$$

then

$$[J-I\lambda] = \begin{pmatrix} -(H_1+\lambda) & \omega & -S^*\theta & 0 \\ \kappa & -(H_2+\lambda) & 0 & 0 \\ H_4 & 0 & -(H_3+\lambda) & 0 \\ 0 & 0 & \rho & -\mu-\lambda \end{pmatrix}$$

$$(-\mu-\lambda)\begin{bmatrix} -(H_1+\lambda) & \omega & -S^*\theta \\ \kappa & -(H_2+\lambda) & 0 \\ H_4 & 0 & -(H_3+\lambda) \end{bmatrix} = 0$$

The characteristic equation obtained from above matrix is

$$P(\lambda) = \lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0 \tag{7.1}$$

where

$$\left.\begin{aligned} b_1 &= -(H_1+H_2-H_3) \\ b_2 &= -(H_1H_2+H_4\theta S^*-H_1H_3-H_2H_3-\omega\kappa) \\ b_3 &= H_2H_4\theta S^*+H_3\omega\kappa-H_1H_2H_3 \end{aligned}\right\} \tag{7.2}$$

To determine local stability of endemic equilibrium of hepatitis B system (2.2) using Routh Hurwitz criterion, it is required that all roots of the equation have negative real roots. So the coefficients of the characteristic polynomial(7.1) have to be positive. In

other-words $b_1 > 0$, $b_2 > 0$, $b_3 > 0$, but in (7.1) $b_1 < 0, b_2 < 0, b_3 < 0$.
We need to obtain an inequality that will make $b_1, b_2, b_3 > 0$
But $H_1 = \kappa + \mu + \beta + \theta I^*$, $H_2 = \omega + \mu$, $H_3 = S\theta - (\mu + \sigma + \rho)$, $H_4 = \beta + \theta I^*$
therefore,

$$b_1 = \theta(S^* - I^*) - (3\mu + \sigma + \rho + \beta + \omega + \kappa)$$

An inequality making $b_1 > 0$ is formulated, thus

$$\theta(S^* - I^*) - (3\mu + \sigma + \rho + \beta + \omega + \kappa) > 0$$

if $\theta S^* > \theta I^* + 3\mu + \sigma + \rho + \beta + \omega + \kappa$ so the condition holds.
To make $b_2 > 0$ we introduce an inequality and get a condition that will make $b_2 > 0$ i.e
$b_2 = (\omega + \mu)(S^*\theta - (\mu + \sigma + \rho)) + (\kappa + \mu + \beta + \theta I^*)(S^*\theta - (\mu + \sigma + \rho)) + \omega\kappa - S^*\theta(\beta + I^*\theta) - (\kappa + \mu + \beta + \theta I^*)(\omega + \mu) > 0$

$(\omega + \mu) S^*\theta + (\kappa + \mu + \beta + \theta I^*)(S^*\theta) + \omega\kappa >$
$(\omega + \mu)(\mu + \sigma + \rho) + (\kappa + \mu + \beta + \theta I^*)(\mu + \sigma + \rho) + S^*\theta(\beta + I^*\theta)$

Therefore, $b_2 > 0$ holds. Making $b_3 > 0$ using inequalities
$b_3 = (\mu + \omega)(\beta + \theta I^*)\theta S^* + \omega\kappa(\theta S^* - (\mu + \sigma + \rho)) - (\kappa + \mu + \beta + \theta I^*)(\mu + \omega)(\theta S^* - (\mu + \sigma + \rho)) > 0$

$(\mu + \omega)(\beta + \theta I^*)\theta S^* + \omega\kappa S^*\theta + (\mu + \sigma + \rho)(\kappa + \mu + \beta + \theta I^*)(\mu + \omega) >$
$\omega\kappa(\mu + \sigma + \rho) + (\theta S^*)(\kappa + \mu + \beta + \theta I^*)(\mu + \omega).$

consequently $b_3 > 0$ if the conditions above hold
Having shown that $b_1 > 0$, $b_2 > 0$, $b_3 > 0$ which is the condition to give negative real roots of the equation (7.1), this implies $R_0 > 1$ and the local stability of endemic equilibrium is asymptotically stable.

8. NUMERICAL SIMULATIONS

Numerical simulation of the HBV model was performed to investigate the role of human activities on the transmission dynamics of Hepatitis B virus disease. Parameter values used in numerical simulations were got from published literature as follows Recruitment rate (Λ) =1500, Natural mortality rate (μ) =0.0143, HBV related mortality rate (σ)=0.2, Transmission rate directly from the infected person(θ)=0.8,Transmission rate through

the human activities (β) = 0.04,
Rate moving from infected to carrier (ρ) = 0.4, Vaccination Rate (κ) =0.7 Vaccination failure rate(ω)= 0.1 [19][20][3] and some were estimated.
The results obtained after carrying out simulations are presented in figures below.

8.1. Population Dynamics

The figure 1 illustrates the dynamics of the population of the susceptible, vaccinated, the infected and the inactive carrier aiming at finding out the changes in compartments of various populations with time. The population of susceptible decreases steadily because of the increased number of vaccinated and the infected reducing the number of susceptible. The infected also increase at a certain rate and remain stable because as more people are vaccinated, it reduces the the number of people that join the infected class and this also leads to the increase in the number of carrier as people leave the infected class to inactive carrier. The number of inactive carrier continues to increase as they do not leave that compartment to any of the three other compartments.

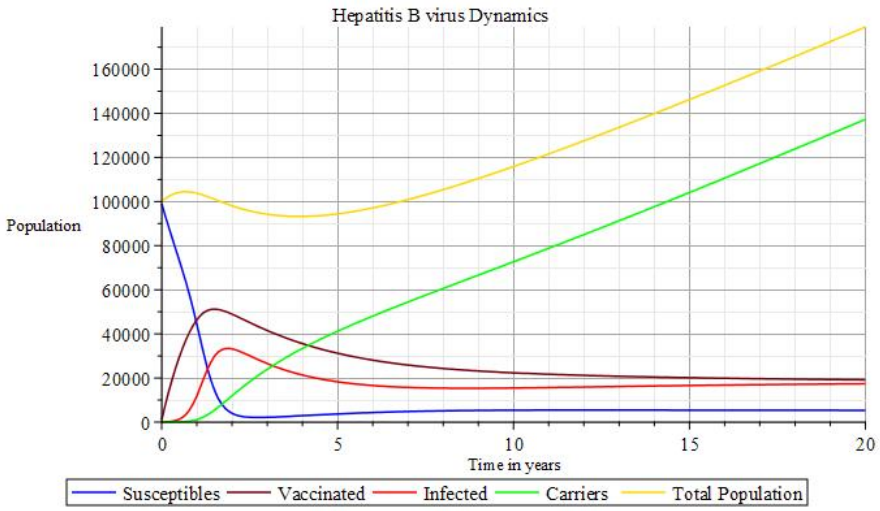


Figure 1: *Population Dynamics*

8.2. The effect of vaccination on the rate of Infection

The figure 2 shows effect of vaccination rate on the rate of infection of hepatitis B disease in the population. It is observed that as the vaccination rate increases the transmission rate of HBV will reduce hence reducing infection rate. This is because Hepatitis B vaccine is one of the major control factors of HBV disease. People acquire hard immunity when they are vaccinated.

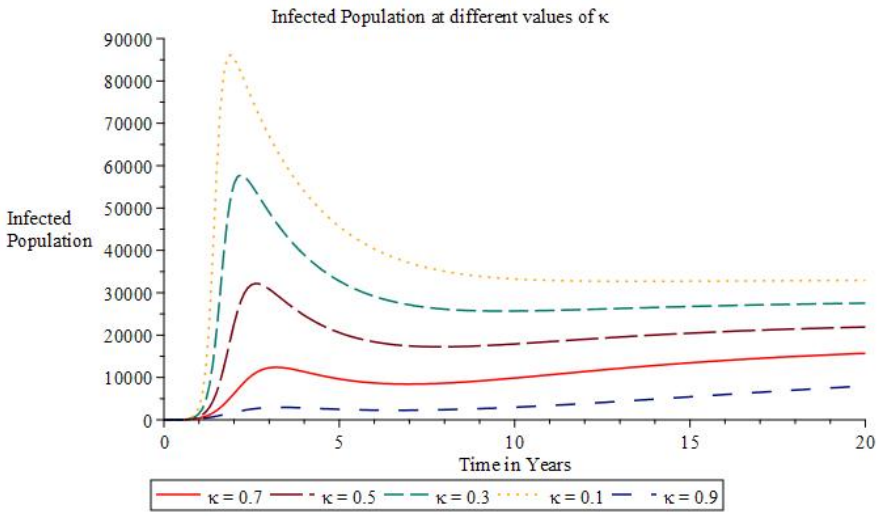


Figure 2: *The effect of vaccination on the rate of Infection*

8.3. The Effect of Human Activities on Infectious Populations

We performed numerical analysis to find out the effect of human activities on the spread of Hepatitis B disease as indicated in the figure 3 below.
It was confirmed that if people continue to carry out the human activities that lead to the spread of HBV,the rate of transmission of HBV also increases but if the human activities are moderated that lead to spread of HBV,the rate of transmission is reduced.

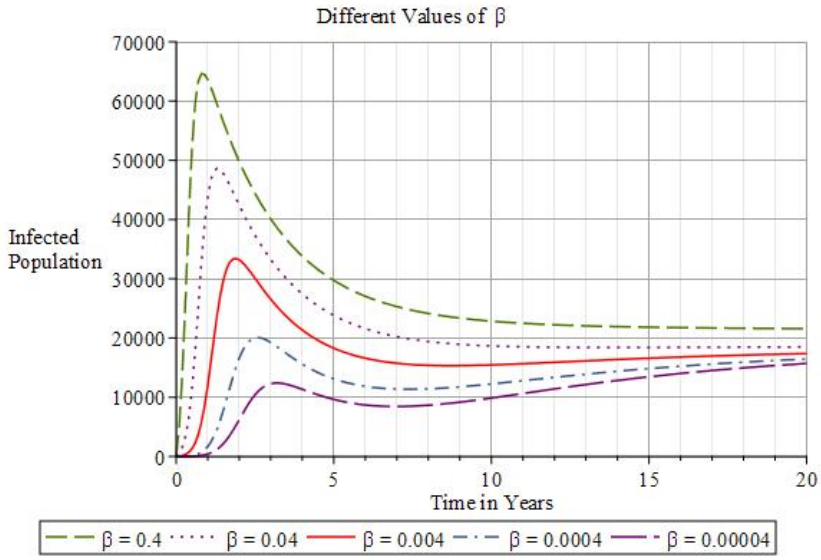


Figure 3: *The effect of human activities on the infectious population*

9. CONCLUSIONS AND RECOMMENDATIONS

9.1. Conclusions

A model for transmission dynamics of HBV moderation of human activities was formulated. We computed threshold value using next generation matrix approach.
The results of R_0 demonstrates that moderation of human behaviour reduces the average number of secondary infections when put into consideration. In this study we carried out local stability of both Disease Free Equilibrium point and Endemic Equilibrium point. From the findings local stability of disease free equilibrium was noted to be stable wherever R_0 was less than unity, whereas the local stability of the endemic equilibrium was asymptotically stable wherever R_0 was greater than unity.
Numerical simulations from the HBV were also carried out. It was established that as people continue to interact with the population carrying out activities that lead to spread of HBV the more Hepatitis B disease will be spread in the population. But moderation of human activities that are transmission route of HBV will reduces the spread of Hepatitis B in the population.

9.2. Recommendations

From this study it is recommended that further research to be carried out on HBV model including recovery and treatment compartments.
The health personnel should intensify those activities they carry out not to transmit HBV disease like ensuring that they disinfect all medical instruments and syringes after being used.
The infected people of HBV should be careful in their behaviours and activities they carry out so that they do not transmit HBV disease to the community. This is because when the contact rate of the infectious increases it leads to an increase to infected population.
The population should be more sensitised about the activities that leads to transmission of HBV diseases, so that they moderate them in order to prevent transmission of HBV disease.

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