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Mappings on Almost strongly Lindelöf and weakly strongly Lindelöf spaces

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Abstract

In this paper our purpose is to study the effect of some mappings and decompositions of continuity on some generalization of strongly Lindelöf spaces ; almost strongly Lindelöf and weakly strongly Lindelöf spaces. we show that some mappings preserved these properties such as preirresolute functions and another kind of mappings gave another results on the image of these topological properties such as precontinuous, almost precontinuous, Θ-continuous, α-continuous and almost α-continuous functions.

Keywords : strongly Lindelöf, almost strongly Lindelöf, weakly strongly Lindelöf, preirresolute, precontinuous, α-continuous, almost precontinuous, almost α-continuous, contra-continuous, Θ-continuous.

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1. INTRODUCTION

The study of covering properties of topological spaces takes a lot of interest in several works, specially covers which involves open sets. Also, an attention has been given to covering properties of topological spaces by preopen sets.

In [16] Mashhour et. al. introduced the notion of strongly Lindelöf spaces by requiring that each preopen cover has a countable subcover, where a subset A of a topological space X is called preopen if it is contained in the interior of its closure, and several papers study these property and gave more results about these spaces. Such as Ganster [11, 12], Hdeib and Sarsak [13], and Al-Omary et. al. [2].

In [1] the authors introduced a generalization of strongly Lindelöf spaces; almost strongly Lindelöf spaces and weakly strongly Lindelöf spaces, and gave some characterizations of these spaces, also some properties was proved about these spaces and their subspaces.

The effect of mappings and decompositions of continuity on topological spaces have been taken a lot of interest in many works; Singal and Singal [26], Popa and Stan [23], Noiri [20, 21], Mashhour et. al.[17, 18], Dontchev [5], Park and Ha [22], and Fawakhreh and kilicman [7, 8].

In this paper we study the effect of mappings on almost strongly Lindelöf spaces and weakly strongly Lindelöf spaces. We prove some results concerning the effect of some decompositions of continuity on these spaces. We show that some mappings preserved these topological properties such as preirresolute functions and other mappings will give another properties on the image of these properties.

Throughout this paper, a space X means a topological space (X, τ) on which no separation axioms are assumed. The interior and the closure of any subset A of X will be denoted by $Int(A)$ and $cl(A)$ respectively.

2. PRELIMINARIES

Let X be a topological space. Then a subset A of X is called preopen (resp. preclosed) if $A \subseteq Int(cl(A))$ (resp. $cl(Int(A) \subseteq A)$, A is called regular open (resp. regular closed) if $A = Int(cl(A))$ (resp. $A = cl(Int(A)))$, and A is called α-open if $A \subseteq Int(cl(int(A)))$. Note that the complement of preopen is preclosed and the complement of regular open is reular closed. Also a subset A is called semi-open if $A \subseteq cl(int(A))$. The complement of semi-open is called semi-closed.

The preclosure of a subset A of X is the intersection of all preclosed subsets containing A, denoted by $pcl(A)$, i.e. $pcl(A)$ is the smallest preclosed set containing A. The preinterior of A is the union of all preinterior subsets contained in A, denoted by $pint(A)$, i.e. $pint(A)$ is the largest preopen set contained in A. It is well known that $pcl(A) = A \cup cl(int(A))$ and $pint(A) = A \cap int(cl(A))$ [3].

Definition 2.1. [14] A subset A of a topological space (X, τ) is called pre-regular p-open if $A = pint(pcl(A))$.

One can observe that $A \subseteq X$ is pre-regular p-open if and only if A is the pre-interior of some pre-closed subset, and if $S \subseteq X$ is preopen and $A = pint(pcl(S))$ then $pcl(S) = pcl(A)$.

Theorem 2.2. *[18] Let A and B be subsets of a topological space (X, τ).*

(i) If A is preopen in X and B is semi-open in X then $A \cap B$ is preopen in B.

(ii) If A is preopen in B and B is preopen in X then also A is preopen in X.

Lemma 2.3. *[6] Let $B \subseteq A \subseteq X$ and A be semi-open in X. Then we have $pcl_A(B) \subseteq pcl_X(B)$.*

Definition 2.4. [17] A topological space (X, τ) is called nearly Lindelöf (resp. almost Lindelöf, weakly Lindelöf) if for every open cover $\{U_\alpha\ ;\ \alpha \in \Delta\}$ there exists a countable subfamily $\{U_{\alpha_n}\ ;\ n \in \mathbb{N}\}$ such that $X = \bigcup_{n\in\mathbb{N}} int(cl(U_{\alpha_n}))$ (resp. $X = \bigcup_{n\in\mathbb{N}} cl(U_{\alpha_n}),\ X = cl(\bigcup_{n\in\mathbb{N}} U_{\alpha_n})$).

Definition 2.5. [17] A topological space (X, τ) is called strongly Lindelöf if every preopen cover $\{A_\alpha\ ;\ \alpha \in \Delta\}$ has a countable subcover.

Definition 2.6. [1] A topological space (X, τ) is called almost strongly Lindelöf (resp. weakly strongly Lindelöf) if every preopen cover $\{U_\alpha\ ;\ \alpha \in \Delta\}$ has a countable subfamily $\{U_{\alpha_n}\ ;\ n \in \mathbb{N}\}$ such that $X = \bigcup_{n\in\mathbb{N}} pcl(A_{\alpha_n})$ (resp. $X = pcl(\bigcup_{n\in\mathbb{N}} A_{\alpha_n})$).

Note that every strongly Lindelöf space is almost strongly Lindelöf and weakly strongly Lindelöf but the converse is not true see [1].

Definition 2.7. Let (X, τ) and (Y, σ) be a topological spaces. Then a function $f : X \to Y$ is called:

1)preirresolute [25] if for each $x \in X$ and each preopen set $V \subseteq Y$ containing $f(x)$, there exists a preopen set U in X containing x such that $f(U) \subseteq V$ or equivalently, if $f^{-1}(V)$ is preopen in X for every open set $V \subseteq Y$.

2)precontinuous [17] if $f^{-1}(V)$ is preopen in X for every open set V in Y.

3)Θ-continuous [9] if for each $x \in X$ and each open set $V \subseteq Y$ containing $f(x)$ there exists an open set U in X containing x such that $f(cl(U)) \subseteq cl(V)$.

4)α-continuous [20] if $f^{-1}(V)$ is α-open in X for every open set V in Y.

5)almost α-continuous [21] if $f^{-1}(V)$ is α-open in X for every regular open set V in Y.

6)almost precontinuous [19] if for each $x \in X$ and each regular open set V in Y containing $f(x)$, there exists a preopen set U containing x such that $f(U) \subseteq V$.

7)contra-continuous [5] if $f^{-1}(V)$ is closed in X for any open set V in Y.

8)subcontra-continuous [4] if there exists an open base $\mathfrak{B}$ for the topology on Y such that $f^{-1}(V)$ is closed in X for every $V \in \mathfrak{B}$.

Theorem 2.8. *[24] A function $f : X \to Y$ is preirresolute (resp. precontinuous) if and only if for each $A \subseteq X$, $f(pcl(A)) \subseteq pcl(f(A))$ (resp. $f(pcl(A)) \subseteq cl(f(A))$).*

3. MAPPINGS ON ALMOST STRONGLY LINDELÖF SPACES

In this section we introduce the effect of mappings on almost strongly Lindelöf spaces.

Theorem 3.1. *Let $f : X \to Y$ be a preirresolute surjection. Then if X is almost strongly Lindelöf then Y is almost strongly Lindelöf.*

Proof. Let $\{U_\alpha \mid \alpha \in \Delta\}$ be a preopen cover of Y. Since f is preirresolute then $\{f^{-1}(U_\alpha) \mid \alpha \in \Delta\}$ is a preopen cover of X, so there exists a countable subfamily $\{f^{-1}(U_{\alpha_n}) \mid n \in \mathbb{N}\}$ such that $X = \bigcup_{n\in\mathbb{N}} pcl(f^{-1}(U_{\alpha_n}))$, so $Y = f(X) = f(\bigcup_{n\in\mathbb{N}} pcl(f^{-1}(U_{\alpha_n}))) = \bigcup_{n\in\mathbb{N}} f(pcl(f^{-1}(U_{\alpha_n})) = \bigcup_{n\in\mathbb{N}} pcl(f(f^{-1}(U_{\alpha_n}))$, by theorem 2.8, hence $Y = \bigcup_{n\in\mathbb{N}} pcl(U_{\alpha_n})$. Therefore Y is almost strongly Lindelöf. $\square$

Theorem 3.2. *Let $f : X \to Y$ be an almost precontinuous surjection. Then if X is almost strongly Lindelöf then Y is almost Lindelöf.*

Proof. Let $\{U_\alpha \mid \alpha \in \Delta\}$ be a open cover of Y. Then $\{int(cl(U_\alpha)) \mid \alpha \in \Delta\}$ is a regular open cover of Y. Since f is almost precontinuous then $f^{-1}(int(cl(U_\alpha)))$ is preopen in X for every $\alpha \in \Delta$, so $\{f^{-1}(int(cl(U_\alpha))) \mid \alpha \in \Delta\}$ is a preopen cover of X, hence there exists a countable subfamily $\{f^{-1}(int(cl(U_{\alpha_n}))) \mid n \in \mathbb{N}\}$ such that

$$X = \bigcup_{n\in\mathbb{N}} cl(f^{-1}(cl(U_{\alpha_n}))) = \bigcup_{n\in\mathbb{N}} pcl(f^{-1}(int(cl(U_{\alpha_n})))) \subseteq \bigcup_{n\in\mathbb{N}} pcl(f^{-1}(cl(U_{\alpha_n}))).$$

Since f is almost precontinuous and $cl(U_{\alpha_n})$ is regularly closed then $f^{-1}(cl(U_{\alpha_n}))$ is preclosed in X, so

$$X = \bigcup_{n\in\mathbb{N}} pcl(f^{-1}(cl(U_{\alpha_n}))) = \bigcup_{n\in\mathbb{N}} f^{-1}(cl(U_{\alpha_n})) = f^{-1}(\bigcup_{n\in\mathbb{N}} cl(U_{\alpha_n})).$$

Thus $Y = f(X) = f(f^{-1}(\bigcup_{n\in\mathbb{N}} cl(U_{\alpha_n}))) = \bigcup_{n\in\mathbb{N}} cl(U_{\alpha_n})$. Therefore Y is almost Lindelöf. $\square$

Theorem 3.3. *Let $f : X \to Y$ be an almost precontinuous and subcontracontinuous surjection. Then if X is almost strongly Lindelöf then Y is Lindelöf.*

Proof. Let $\mathfrak{B}$ be an open base for the topology on Y such that $f^{-1}(V)$ is closed in X for every $V \in \mathfrak{B}$. Let $\mathfrak{U} = \{U_\alpha \mid \alpha \in \Delta\}$ be a open cover of Y. For each $x \in X$ let $U_{\alpha_x} \in \mathfrak{U}$ such that $f(x) \in U_{\alpha_x}$, then there exists $V_{\alpha_x} \in \mathfrak{B}$ such that $f(x) \in V_{\alpha_x} \in U_{\alpha_x}$. Since f is precontinuous then $f^{-1}(V_{\alpha_x})$ is preopen in X, so $\{f^{-1}(V_{\alpha_x}) \mid x \in X\}$ is a preopen cover of X, hence there exists a countable subfamily $\{f^{-1}(V_{x_n}) \mid n \in \mathbb{N}\}$ such that

$$X = \bigcup_{n\in\mathbb{N}} pcl(f^{-1}(V_{x_n}) = \bigcup_{n\in\mathbb{N}} (f^{-1}(V_{x_n}),$$

so

$$Y = f(X) = f(\bigcup_{n\in\mathbb{N}} f^{-1}(V_{x_n})) = \bigcup_{n\in\mathbb{N}} f(f^{-1}(V_{x_n})) = \bigcup_{n\in\mathbb{N}} V_{x_n} \subseteq \bigcup_{n\in\mathbb{N}} U_{\alpha_{x_n}}.$$

Therefore, Y is Lindelöf. □

Theorem 3.4. *Let $f : X \to Y$ be a Θ-continuous surjection. Then if X is almost strongly Lindelöf, Y is almost Lindelöf.*

Proof. Let $\mathcal{V} = \{V_\alpha \mid \alpha \in \Delta\}$ be a open cover of Y. For each $x \in X$ let $V_{\alpha_x} \in \mathcal{V}$ such that $f(x) \in V_{\alpha_x}$, then there exists an open set U_x in X such that $x \in U_x$ and $f(cl(U_x)) \subseteq cl(V_{\alpha_x})$ so $\{U_x \mid x \in X\}$ is an open, so preopen cover of X, hence there exists a countable subfamily $\{U_{x_n} \mid n \in \mathbb{N}\}$ such that $X = \bigcup_{n\in\mathbb{N}} pcl(U_{x_n})$. Thus

$$Y = f(X) = f(\bigcup_{n\in\mathbb{N}} pcl(U_{x_n}) = \bigcup_{n\in\mathbb{N}} f(pcl(U_{x_n}) \subseteq \bigcup_{n\in\mathbb{N}} f(cl(U_{x_n}) \subseteq \bigcup_{n\in\mathbb{N}} cl(V_{\alpha_{x_n}}).$$

Therefore Y is almost Lindelöf. □

Theorem 3.5. *Let $f : X \to Y$ be an almost precontinuous and contra-continuous surjection. Then if X is almost strongly Lindelöf, Y is nearly Lindelöf.*

Proof. Let $\{U_\alpha \mid \alpha \in \Delta\}$ be an open cover of Y. For each $x \in X$ let $V_{\alpha_x} \in \mathcal{V}$ such that $f(x) \in V_{\alpha_x} \subseteq Int(cl(V_{\alpha_x}))$. Since f is almost precontinuous and $Int(cl(V_{\alpha_x}))$ is regular open in Y then there exists a preopen set U_x in X containing x such that $f(U_x) \subseteq Int(cl(V_{\alpha_x}))$, so $\{U_x \mid x \in X\}$ is a preopen cover of X, hence there exists a countable subfamily $\{U_{x_n} \mid n \in \mathbb{N}\}$ such that $X = \bigcup_{n\in\mathbb{N}} pcl(U_{x_n}) = \bigcup_{n\in\mathbb{N}} pcl(f^{-1}(Int(cl(V_{\alpha_{x_n}})))$. Since f is contra-continuous then $f^{-1}(Int(cl(V_{\alpha_{x_n}})))$ is closed in X, so $X = \bigcup_{n\in\mathbb{N}} f^{-1}(Int(cl(V_{\alpha_{x_n}}))) = f^{-1}(\bigcup_{n\in\mathbb{N}} Int(cl(V_{\alpha_{x_n}})))$. Thus

$$Y = f(X) = f(f^{-1}(\bigcup_{n\in\mathbb{N}} Int(cl(V_{\alpha_{x_n}}))) = \bigcup_{n\in\mathbb{N}} Int(cl(V_{\alpha_{x_n}})).$$

Therefore Y is nearly Lindelöf. $\square$

Theorem 3.6. *Let $f : X \to Y$ be an α-continuous and subcontra-continuous surjection. Then if X is almost strongly Lindelöf then Y is Lindelöf.*

Proof. Similar to the proof of theorem 3.3. $\square$

Theorem 3.7. *Let $f : X \to Y$ be an almost α-continuous and contra-continuous surjection. Then if X is almost strongly Lindelöf then Y is nearly Lindelöf.*

Proof. Similar to the proof of theorem 3.5. $\square$

4. MAPPINGS ON WEAKLY STRONGLY LINDELÖF SPACES

Theorem 4.1. *Let $f : X \to Y$ be a preirresolute surjection. Then if X is weakly strongly Lindelöf then Y is weakly strongly Lindelöf.*

Proof. Let $\{U_\alpha \mid \alpha \in \Delta\}$ be a preopen cover of Y. Since f is preirresolute then $f^{-1}(U_\alpha)$ is preopen in X for every $\alpha \in \Delta$, so $\{f^{-1}(U_\alpha) \mid \alpha \in \Delta\}$ is a preopen cover of X, hence there exists a countable subfamily $\{f^{-1}(U_{\alpha_n}) \mid n \in \mathbb{N}\}$ such that $X = pcl(\bigcup_{n\in\mathbb{N}} f^{-1}(U_{\alpha_n})) = pcl(f^{-1}(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}))$, so

$$Y = f(X) = f(pcl(f^{-1}(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}))) \subseteq pcl(\bigcup_{n\in\mathbb{N}} f(f^{-1}(U_{\alpha_n}))),$$

by theorem 2.8, thus $Y = pcl(\bigcup_{n\in\mathbb{N}} U_{\alpha_n})$. Therefore Y is weakly strongly Lindelöf. $\square$

Theorem 4.2. *Let $f : X \to Y$ be an almost precontinuous surjection. Then if X is weakly strongly Lindelöf then Y is weakly Lindelöf.*

Proof. Let $\{U_\alpha \mid \alpha \in \Delta\}$ be a open cover of Y. Then $\{int(cl(U_\alpha)) \mid \alpha \in \Delta\}$ is a regular open cover of Y. Since f is almost precontinuous then $f^{-1}(int(cl(U_\alpha)))$ is preopen in X for every $\alpha \in \Delta$, so $\{f^{-1}(int(cl(U_\alpha))) \mid \alpha \in \Delta\}$ is a preopen cover of X, so there exists a countable subfamily $\{f^{-1}(int(cl(U_{\alpha_n}))) \mid n \in \mathbb{N}\}$ such that $X = pcl(\bigcup_{n\in\mathbb{N}} f^{-1}(int(cl(U_{\alpha_n})))) \subseteq pcl(\bigcup_{n\in\mathbb{N}} f^{-1}(cl(U_{\alpha_n}))) = pcl(f^{-1}(\bigcup_{n\in\mathbb{N}} cl(U_{\alpha_n}))) \subseteq pcl(f^{-1}(cl(\bigcup_{n\in\mathbb{N}} U_{\alpha_n})))$. Since f is almost precontinuous and $cl(U_{\alpha_n})$ is regularly closed then $f^{-1}(cl(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}))$ is preclosed in X, so $X = pcl(f^{-1}(cl(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}))) = f^{-1}(cl(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}))$. Thus $Y = f(X) = f(f^{-1}(cl(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}))) = cl(\bigcup_{n\in\mathbb{N}} U_{\alpha_n})$. Therefore Y is weakly Lindelöf. $\square$

Since every precontinuous function is almost precontinuous, we have :

Theorem 4.3. *Let $f : X \to Y$ be a precontinuous surjection. Then if X is weakly strongly Lindelöf then Y is weakly Lindelöf.*

Since every almost α-continuous function is almost precontinuous, we have :

Theorem 4.4. *Let $f : X \to Y$ be an almost α-continuous surjection. Then if X is weakly strongly Lindelöf then Y is weakly Lindelöf.*

Theorem 4.5. *Let $f : X \to Y$ be an almost precontinuous and contra-continuous surjection. Then if X is weakly strongly Lindelöf, Y is nearly Lindelöf.*

Proof. Let $\mathcal{V} = \{V_\alpha \mid \alpha \in \Delta\}$ be a regularly open cover of Y. For each $x \in X$ let $V_{\alpha_x} \in \mathcal{V}$ such that $f(x) \in V_{\alpha_x}$. Since f is almost precontinuous then there exists a preopen subset U_x of X containing x such that $f(U_x) \subseteq V_{\alpha_x}$, so $\{U_x \mid x \in X\}$ is a preopen cover of X, hence there exists a countable subfamily $\{U_{x_n} \mid n \in \mathbb{N}\}$ such that $X = pcl(\bigcup_{n\in\mathbb{N}} U_{x_n}) \subseteq pcl(\bigcup_{n\in\mathbb{N}} f^{-1}(V_{\alpha_{x_n}})) = pcl(f^{-1}(\bigcup_{n\in\mathbb{N}} V_{\alpha_{x_n}}))$. Since f is contra-continuous and $\bigcup_{n\in\mathbb{N}} V_{\alpha_{x_n}}$ is open, then $f^{-1}(\bigcup_{n\in\mathbb{N}} V_{\alpha_{x_n}})$ is closed in X, so preclosed, hence $X = f^{-1}(\bigcup_{n\in\mathbb{N}} V_{\alpha_{x_n}})$. Thus

$$Y = f(X) = f(f^{-1}(\bigcup_{n\in\mathbb{N}} V_{\alpha_{x_n}})) = \bigcup_{n\in\mathbb{N}} V_{\alpha_{x_n}}.$$

Therefore Y is nearly Lindelöf. $\square$

Theorem 4.6. *Let $f : X \to Y$ be a precontinuous and contracontinuous surjection. Then if X is weakly strongly Lindelöf then Y is Lindelöf.*

Proof. Similar to the proof of theorem 4.5. $\square$

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