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Effects of Induced Magnetic Field on Electrically Conducting Oldroyd-B Nanofluid Flow over a Perforated Linearly Extending Surface

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Abstract

This study investigated the effects of induced magnetic field on MHD boundary layer laminar flow for an Oldroyd-B nano fluid , on a heat producing or absorbing , perforated and elongating surface . Application of an externally magnetic field orthogonally to the extending sheet , affects the fluid flow . The parameters that describe the flow clearly are Lewis number , Magnetic prandtl number , Deborah's numbers , Brownian motion parameter and Thermophoretic parameter. The governing PDES were converted to ODES using similarity transformation . The ODES obtained were reduced to systems of first order ODES and solved by collocation method in MATLAB . It was established that as magnetic induction of the fluid increases , the momentum boundary layer thickness reduces , the temperature profile rises and the diffusion of the nanoparticles is enhanced . Effects of **B1 , B2 , Nt , Nb and Le** are also discussed . The results of findings are presented in form of graphs for velocity , temperature and concentration profiles for various flow parameters as a function of similarity variable .

ABBREVIATIONS

PDES	Partial differential equations
ODES	Ordinary differential equations
MHD	Magnetohydrodynamics
SHSF	Specified heat surface flux
SST	Specific surface temperature
BVP	Boundary Value Problem
DES	Differential equations
LHS	Left hand side
RHS	Right hand side
wrt	With respect to
N-S	Navier stokes

NOMENCLATURE

f	Dimensionless stream function
H	Dimensionless magnetic induction
Q_0	Heat generating or absorbing parameter
T	Temperature of the nanofluid in the boundary layer
u_s	slip velocity
$+v_w$, $-v_w$	Velocity for suction and injection respectively
f_w	Suction or injection parameter
P_r	Prandtl number
Nu_x	Local Nusselt number
Sh_x	Local Sherwood number
Re_x	Local Reynolds number
x,y	Cartesian coordinate system

M	Magnetic Hartmann number
B_0	Uniform magnetic field applied a long y-axis
b	Induced magnetic field
J	Current density
c_p	Specific heat for nano particles at a constant pressure
c_f	Specific heat capacity for base fluid
C	Concentration of nanoparticles
D_B	Coefficient of Brownian Diffussion
D_T	Coefficient of Thermophoretic Diffusion
u, v	Velocity components in x and y directions respectively
i, j	Unit vectors in x and y directions respectively
T_∞	Temperature of nano fluid a way from the wall
L_e	Lewis number
N_b	Parameter for Brownian motion
N_t	Parameter for thermophoresis
k	Thermal conductivity
a	Non-dimensionless parameter
S	Extra stress tensor
$\frac{D}{Dt}$	Covariant differentials
P	Pressure
I	Identity tensor
A_1	Rivlin-Ericksen first order tensor
$*$	Transpose of the matrix $\left(\nabla\vec{V}\right)$
T_1	Cauchy stress tensor
l_1	Component of the the slip velocity

GREEK SYMBOLS

μ	Dynamic co-efficient of viscosity
θ	Dimensionless temperature
ρ_f	Density of the the base fluid
ρ_p	Density of nanoparticles
η	Similarity variable
ϕ	Dimensionless concentration of nanoparticles
α	Thermal diffusivity
τ	Proportion influencing heat absorbing capacity of nanoparticles and the base fluid
σ	Electrical conductivity for the nanofluid
ε	Heat source or sink
λ_1 , λ_2	Relaxation time,retardation time respectively
β_1 , β_2	Deborah's numbers for relaxation and retardation time respectively
∇	Nabla operator
∇^2	Laplacian operator
ψ	Two dimensional stream function
ζ	Velocity slip parameter
ν	Kinematic viscosity
μ_0	Magnetic permeability of free space
δ	Magnetic diffusivity
ρ	Density of the nanofluid

1. LITERATURE REVIEW

Here the research work of the previous scholars on MHD boundary layer flows were examined . The chapter considered just a few and their revelations . This was done mainly to identify an area where extension could be initiated . Many researchers have explored the boundary layer fluid flow problems in regard to heat transmission on a surface that stretches , some of their findings were experimental discoveries on ways of improving thermal conductivities of various conventional fluids . According to **[3]** while documenting the outcome of his findings , used the word nanoparticles to refer to as minute solid suspensions floated in a base fluid . The discovery revealed that the presence of nanoparticles improves thermal ability of the base fluid . The research on MHD flows of Oldroyd-B fluids by **[6]** showed that electromotive force reduces the thickness of hydromagnetic boundary layer . The investigation also revealed that in the absence of relaxation and retardation time effects together with the magnetic field , makes the structure of the boundary layers identical to the classical stokes and Ekman layers . The study on the boundary layer flow on heat transfer of Oldroyd-B nanofluid towards an elongating sheet by **[9]** gave the following result ; Deborah's number for relaxation mostly retards motion and as a result velocity profile and the magnitude of boundary layer decreases with the elevation of B1 . Temperature and concentration profiles shows the reverse effect with the rise in B1 . The study also shown that for greater values of Nt and Nb appreciates temperature profile but depreciates mass fraction . The investigation also outlined that Nusselt's number decreases with the increase in Nt provided Pr is less or greater than Le . The research for three-dimensional flow of an Oldroyd-B fluid flow over a surface that stretches in two directions with PST and PHF according to **[8]**shows that ; Deborah's numbers have reverse effects on temperature profiles . The study also outlined that heat source or sink parameter increases temperarure profiles as well as their associated thermal boundary layer thickness . It was noted that the temperature profiles and thermal boundary layer thickness reduces with the hike in prandtl number . The study by **[2]** on thermophoretic effect on Oldroyd-B nanofluid unsteady flow over a surface that extends discovered that increase in Deborah's number for relation time effect leads to rise in skin friction co-efficient thereby enhancing shear stress at the surface . This research too revealed that increase in thermophoretic parameter leads to increase in thermal boundary layer thickness resulting in the reduction of heat transfer rate . The research study by **[7]** on temperature and concentration stratification effects on the mixed convection flow of an oldroyd-b fluid with thermal and chemical reaction , affirmed that temperature and concentration profiles decreases with increase in magnitude of B1 . The research also came to a conclusion that thermal boundary layer thickness decreases with the increase in Pr . The study on the contribution of the soret and Dufour effect on the mixed convection flow of an Oldroyd-B fluid with a boundary layer condition which is convective by **[1]** noted that Deborah's numbers on relaxation and retardation time effects have similar contribution on temperature and concentration profiles . It was noted that a decrease in prandtl number has a decreasing effect on temperarure , concentration as well as boundary layer thickness . Soret's number has a reverse effect on temperature and concentration profiles whereas increase in Dufour's number increases the thermal boundary thickness and temperature profiles . The study on stratified flow of an Oldroyd-B nanofluid with heat generation by **[10]** discovered that the fluid flow is impeded by larger Deborah's number B1 while the Deborar's number B2 has opposite effect . The research further revealed that larger values for Nt and Nb elevates temperature an aspect that earlier researches confirmed . The research on nonlinear convective heat and mass transfer of Oldroyd-B nanofluid on an extending sheet in the presence of uniform heat source/sink by **[5]** revealed , upon examination of various parameters on temperature , velocity and concentration profiles that ; velocity components for momentum and energy rises with the rise in Deborah's number for retardation time effect . The study also highlighted that temperature and concentration profiles elevates by raising Deborah's number for relaxation time effect . This research shows that temperature and thermal boundary layer thickness decreases with the rise in prandtl's number . The study on effect of the magnetic field on Oldroyd-B type nanofluid flow over a permeable surface that stretches with heat generating/absorbing in the presence of magnetic field with a slip velocity by **[4]** noted that , velocity profile reduces with the rise in magnetic hartmann's number , slip velocity , Deborah's number for relaxation while Deborah's number for retardation has reverse effect . It was revealed that magnetic hartmann number , velocity slip parameter , heat generating/absorbing parameter for thermophoresis have similar effect and elevates the temperature in the boundary layer . Brownian motion parameter was found to reduce the size of concentration profile while thermophoretic parameter hikes the same . Heat travel rate was noted to diminish with the rise in magnetic hartmann number , parameter for heat source , slip velocity parameter and Brownian motion parameter but the reverse is true for the heat sink parameter . Mass transfer rate was found to appreciates with the rise in magnetic harmann number , heat source parameter , slip velocity , Brownian motion parameter with the reverse true for heat absorption parameter . However , none of these researchers considered the effects of magnetic induction on Oldroyd-B type nano-fluid . According to **[4]** in their research work did not consider the contribution of the same on velocity , concentration and temperature profiles for an Oldroyd-B type nanofluid , steady , incompressible , two dimensional boundary layer flow , an area that this project investigated because the presence of induced magnetic field hinders the fluid flow and therefore has some effect which should not be overlooked . The "kalidas" model is extended by incorporating the magnetic induction and investigating its contributions to the flow fields .

2. MATHEMATICAL FORMULATION

Here we derive the governing equations for a two dimensional , incompressible , stable , viscous and electrically conducting Oldroyd-B Nanofluid flow , over a siphoning linearly expanding surface kept at a fixed temperature T_w and concentration C_w . The temperature T_∞ and concentration C_∞ values far a way from the wall respectively . The flow is initiated by elongating sheet which is restricted to $y > 0$. The sheet is extending with a linear velocity $u_w = ax$, where a is dimensionally constant and x is the co-ordinate along which the sheet extends . The geometrical picture of the flow is outlined below .

Two equal anti- forces are applied in the x-direction with the origin fixed . A transverse constant magnetic field is directed vertically upwards to the laminar flow that is a long y-direction . An account of induced magnetic field is factored in because magnetic Reynolds number is significant .

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\lambda_1\left(u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\partial y}\right)=\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+ \\ \nu\lambda_2\left(u\frac{\partial^3 u}{\partial x^3}+u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial x^2\partial y}+u\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}-\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x^2}-\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}\right) \\ -\frac{\sigma B_0^2 u}{\rho}+\frac{\sigma B_0 b v}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\tau\left\{D_B\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x}+\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\frac{D_T}{T_\infty}\left(\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right)\right\}+\frac{Q_0}{(\rho c_p)_f}(T-T_\infty) \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

$$\delta\frac{\partial^2 b}{\partial y^2}=v\frac{\partial b}{\partial y}+b\frac{\partial v}{\partial y}-B_0\frac{\partial u}{\partial y} \tag{5}$$

Equations 1 , 2 , 3 , 4 and 5 together with their boundary conditions ; $u = u_w + u_s$, $v = +or-(v_w)$, $T = T_w$, $C = C_w$, $b = 0$ for $y = 0$ and $u = 0$, $v = 0$, $T = T_\infty$, $C = C_\infty$ and $b = 0$ as $y \to \infty$ are the governing equations

for the flow .
u_s , stands for slip velocity , which is directly a function of local shear stress at the surface and is given by $u_s = l_1 \frac{\partial u}{\partial y}$, where l_1 is the component of slip velocity . u_w , +ve and -ve , stands for suction and injection velocity respectively . The expression of great importance to the problem are the local Sherwood and Nusselt numbers .They are expressed as ;

$$\begin{aligned} Nu_x &= -\frac{x}{T_w - T_\infty}\left(\frac{\partial T}{\partial y}\right)_{y=0} \\ Sh_x &= -\frac{x}{C_w - C_\infty}\left(\frac{\partial C}{\partial y}\right)_{y=0} \end{aligned} \tag{6}$$

The equation 6 in a dimensionless form becomes ;

$$\begin{aligned} Re_x^{-\frac{1}{2}} Nu_x &= -\theta^1(0) \\ Re_x^{-\frac{1}{2}} Sh_x &= -\phi^1(0) \end{aligned} \tag{7}$$

, where $Re_x = \frac{x u_w(x)}{\nu}$ is the local Reynold's number .

Using the following similarity transformation variables in equation 8 and the derivatives of u , v , θ , ϕ and b noting that $\beta_1 = \lambda_1 a$, $\beta_2 = \lambda_2 a$, $\varepsilon = \frac{Q_0}{a\rho c_p}$, $N_b = \tau D_B \frac{(C_w - C_\infty)}{\nu}$, $N_t = \tau D_T \frac{T_w - T_\infty}{T_\infty \nu}$, $p_r = \frac{\nu}{\alpha}$, $M = \frac{\sigma B_0^2}{\rho a}$, $P_m = \frac{\nu}{\delta}$ yields equation 9 , 10 , 11 , 12 and their boundary condition 13 .

$$\begin{aligned} \psi = (a\nu)^{\frac{1}{2}} x f(\eta) \quad , \quad u = \frac{\partial \psi}{\partial y} \quad , \quad V = -\frac{\partial \psi}{\partial x} \\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad , \quad \phi = \frac{C - C_\infty}{C_w - C_\infty} \\ \eta = \left(\frac{a}{\nu}\right)^{\frac{1}{2}} y \quad , \quad b = -axH'(\eta) \end{aligned} \tag{8}$$

$$\begin{aligned} f^2 - ff'' - \beta_1\left(2ff'f'' - f^2 f'''\right) = f''' + B_2\left(f''^2 - ff^{(iv)}\right) \\ -Mf' + \frac{M(a\nu)^{\frac{1}{2}}}{B_0} H'f \end{aligned} \tag{9}$$

$$\theta'' + Pr\left[f\theta' + N_b\theta'\phi' + N_t\theta'^2 + \varepsilon\theta\right] = 0 \tag{10}$$

$$\phi'' + \frac{N_t}{N_b}\theta'' + PrLe\phi' f = 0 \tag{11}$$

$$H''' - p_m\left\{H'' f - H' f' + \frac{B_0}{a} f'\right\} = 0 \tag{12}$$

The boundary conditions for the ODEs becomes ;

$$\begin{aligned} \eta \to 0 : f(0) = f_w, f'(0) = 1 + \zeta f''(0), \theta(0) = 1, H'(0) = 0 \\ : \eta \to \infty : f'(\infty) = 0, f'' = 0, \theta(\infty) = 0, \phi(\infty) = 0, H''(\infty) = 0 \end{aligned} \tag{13}$$

These emerging ODEs are further converted to a system of linear first order DEs:

$$\begin{aligned} y_1 = f \quad , \quad y_2 = f' \quad , \quad y_3 = f'' \quad , \quad y_4 = y''' \\ t_1 = \phi \quad , \quad t_2 = \phi' \\ z_1 = H \quad , \quad z_2 = H' \quad , \quad z_3 = H'' \\ r_1 = \theta \quad , \quad r_2 = \theta' \end{aligned} \tag{14}$$

$$\begin{aligned} y_1' &= y_2 \\ y_2' &= y_3 \\ y_3' &= y_4 \\ y_4' = \frac{1}{\beta_2 y_1}\left[y_4 - y_2^2 + y_1 y_3 - \beta_1\left(y_1^2 y_4 - 2y_1 y_2 y_3\right) + \beta_2 y_3^2 - M y_2 + \frac{M(a\nu)^{\frac{1}{2}}}{B_0} z_2 y_1\right] \\ r_1' &= r_2 \\ r_2' = -Pr\left[y_1 r_2 + N_b r_2 t_2 + N_t t_2^2 + \varepsilon r_1\right] \\ t_1' &= t_2 \\ t_2' + \frac{N_t}{N_b} r_2' = -PrLe y_1 t_1 \\ z_2' &= z_3 \\ z_3' = -P_m\left(z_3 y_1 - z_2 y_2 + \frac{B_0}{a} y_3\right) \\ \eta \to 0 : y_1(0) = f_w \quad , \quad y_1' = 1 + \zeta y_3(0) \quad , \\ z_2(0) = 1 \quad , \quad t_1(0) = 1 \quad , \quad r_1(0) = 1 \\ \eta \to \infty : y_2(\infty) = 0 \quad , \quad y_3(\infty) = 0 \quad , \\ z_2(\infty) = 0 \quad , \quad r_1(\infty) = 0 \quad , \quad t_1(\infty) \end{aligned} \tag{15}$$

3. RESULTS AND DISCUSSION

Here the project results are presented together with their explanations.

3.1 Effects of increasing magnetic prandtl number on velocity , induction , temperature and concentration profiles

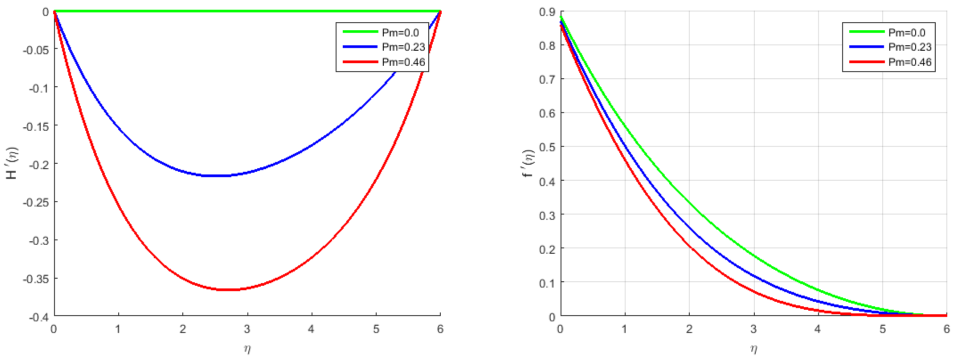

Figure 1: LEFT :Induction profile **RIGHT** : Velocity profile

Figure 1.LEFT Shows that Increase in magnetic Prandtl number increases the induction profile , this is because more Alfven's waves are generated leading to an increase in magnetic induction . At the boundary surface the induction is zero , this is because the Alfven's waves are muted since the fluid particles stick to the surface and move with the velocity of the plate and so there is no velocity gradient on the surface . **Figure 1 . RIGHT** show that velocity profile shifts downwards because induction initiates Lorentz force which opposes the motion of the fluid.

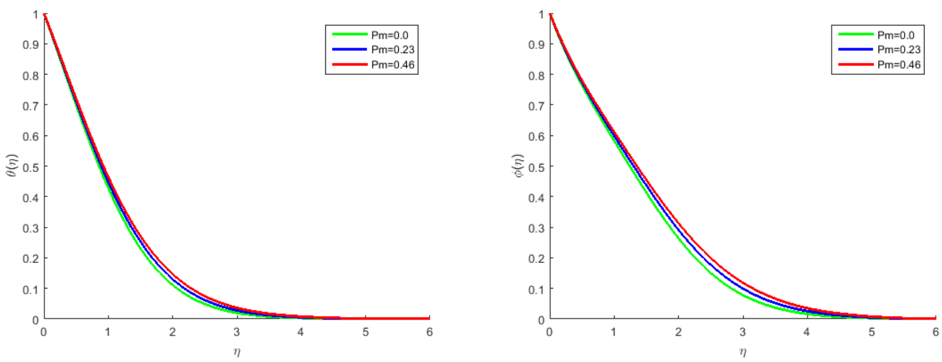

Figure 2: LEFT:Temperature profile **RIGHT**:Concentration profile

Figure 2.LEFT shows that Temperature profile increases since the extra energy is used

to do the work of dragging the nanoparticles against the Lorenz force . The increase in temperature enhances diffusion of nanoparticles which in turn increases concentration profile . **Figure 2 . RIGHT**

3.2 Profiles for increasing Deborah's numbers on induction , velocity , temperature and concentration profiles

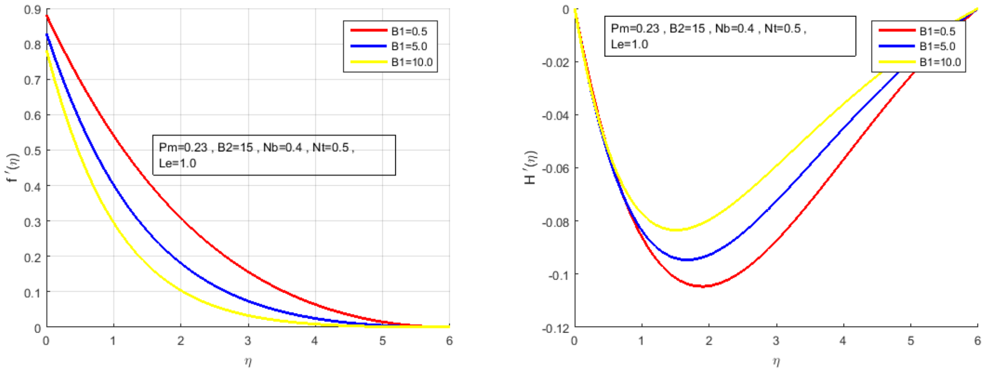

Figure 3: LEFT:Velocity profile **RIGHT**:Induction profile

Keeping magnetic prandtl number and other parameters constant and varying B1 reduces velocity . **B1** is an indication of higher relaxation time factor . Higher **B1** signifies longer time factor for relaxation which hinders the fluid motion resulting into reduction in velocity and a thinner momentum boundary layer , see **figure 3 . LEFT** . Induction profile decreases because it directly depends on velocity of the fluid , **figure 3. RIGHT** .

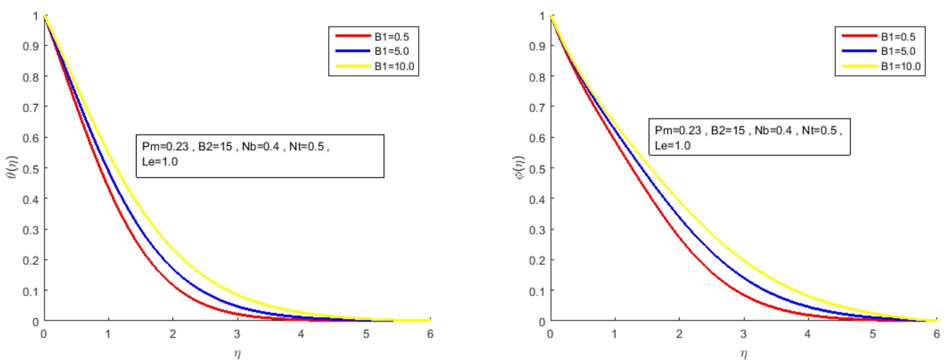

Figure 4: LEFT:Temperature profile **RIGHT**:Concentration profile

A higher value of Deborah's relaxation time is an indication that the temperature and concentration of the fluid is higher . This explains why the temperature and concentration of nanofluid increases evident by **figure 4. LEFT** and **figure 4. RIGHT** respectively . Nusselt and sherwood's numbers increases with the increase in temperature and concentration respectively . Increase in **B2** has the reverse effect as **B1** .

3.3 Increase in Thermophoretic and Brownian Motion Parameters on temperature and concentration profiles

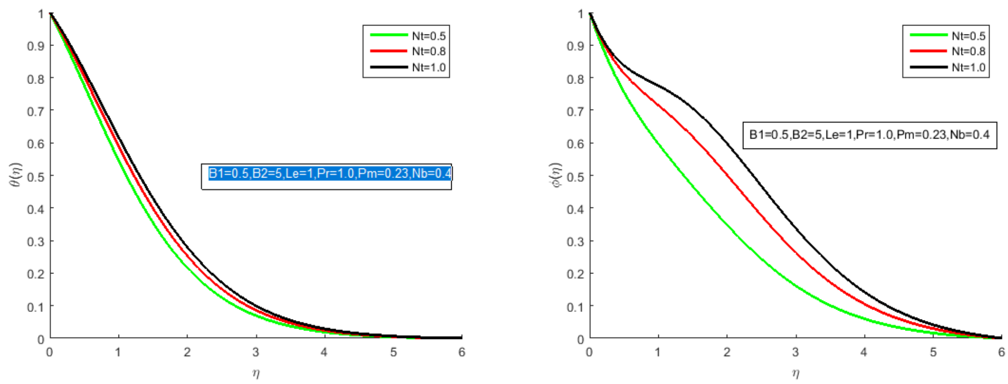

Figure 5: LEFT:Temperature profile **RIGHT**:Concentration profile

An increase in **Nt** on temperature is revealed by **figure 5. LEFT** . This is because the rise in **Nt** enhances the liquid temperature .This takes place due to the continuous build up in the percentage of nanoparticles with the surge in **Nt** . Larger values of **Nt** enhances thermophoretic force which initiates movement of nanoparticles from the regions of higher temperatures to regions of lower temperatures , this brings about the rise in nanoparticle concentration evident in **figure 5. RIGHT**. The rise in **Nb** produces more heat as a result of disorderly movement of nanoparticles which is activated for greater values of **Nb** . This explains why the fluid experiences the rise in temperature . Increase in the size of **Nb** favours the rate at which the nanoparticles move from one place to another with definite velocities due to Brownian motion impact . Therefore higher **Nb** leads to a decrease in nanoparticle concentration as well as decline in concentration boundary thickness .

3.4 Increase in Lewis's Number on concentration profiles

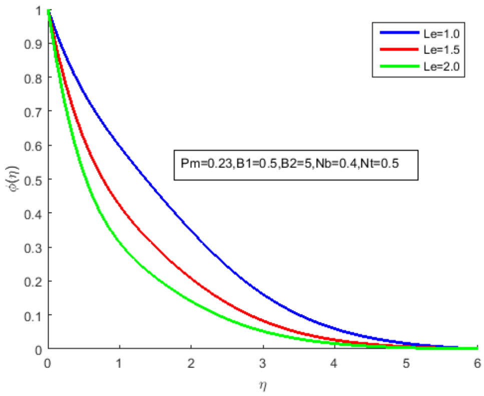

Figure 6: Concentration profile

Figure 6 shows a decrease in profile concentration . As the Lewis's number increases , the concentration profile decreases . This is because the mass transport rate increases with the rise in Lewis's number .

4. CONCLUSION AND RECOMMENDATION

- Increase in magnetic induction reduces velocity profile but increases temperature and concentration profiles . Nusselt and Sherwood's number also increases due to increase in temperature and concentration respectively .
- Higher **B1** results into reduction in velocity and a thinner momentum boundary layer , a decrease in Induction profile and an increase in Temperature and concentration profiles of nanofluid . Reverse effects is observed for higher values of **B2** .
- Increase in **Nt** enhances the temperature and concentration of the fluid . The rise in **Nb** lead to increase in temperature and a decline in nanoparticle concentration
- Increase in **Le** reduces concentration of nanoparticles .

5. RECOMMENDATIONS

We recommend that for further research , the application of varying magnetic field to the flow and investigating its effects on concentration , velocity and temperature profiles .

REFERENCES

[1] Ashraf, M. B., Hayat, T., Alsaedi, A., and Shehzad, S. (2016). Soret and dufour effects on the mixed convection flow of an oldroyd-b fluid with convective boundary conditions. *Results in Physics*, 6:917–924.

[2] Awad, F. G., Ahamed, S. M., Sibanda, P., and Khumalo, M. (2015). The effect of thermophoresis on unsteady oldroyd-b nanofluid flow over stretching surface. *PloS one*, 10(8):e0135914.

[3] Choi, S. U. and Eastman, J. A. (1995). Enhancing thermal conductivity of fluids with nanoparticles. Technical report, Argonne National Lab., IL (United States).

[4] Das, K., Chakraborty, T., and Kundu, P. K. (2018). Effect of magnetic field on oldroyd-b type nanofluid flow over a permeable stretching surface. *Propulsion and Power Research*, 7(3):238–246.

[5] Gireesha, B., Kumar, K. G., Ramesh, G., and Prasannakumara, B. (2018). Nonlinear convective heat and mass transfer of oldroyd-b nanofluid over a stretching sheet in the presence of uniform heat source/sink. *Results in Physics*, 9:1555–1563.

[6] Hayat, T., Hutter, K., Asghar, S., and Siddiqui, A. (2002). Mhd flows of an oldroyd-b fluid. *Mathematical and Computer Modelling*, 36(9-10):987–995.

[7] Hayat, T., Muhammad, T., Shehzad, S. A., and Alsaedi, A. (2015). Temperature and concentration stratification effects in mixed convection flow of an oldroyd-b fluid with thermal radiation and chemical reaction. *PloS one*, 10(6):e0127646.

[8] Hayat, T., Shehzad, S. A., and Alsaedi, A. (2014). Three-dimensional flow of an oldroyd-b fluid over a bidirectional stretching surface with prescribed surface temperature and prescribed surface heat flux. *Journal of Hydrology and Hydromechanics*, 62(2):117–125.

[9] Nadeem, S., Haq, R. U., Akbar, N. S., Lee, C., and Khan, Z. H. (2013). Numerical study of boundary layer flow and heat transfer of oldroyd-b nanofluid towards a stretching sheet. *PloS one*, 8(8):e69811.

[10] Waqas, M., Khan, M. I., Hayat, T., and Alsaedi, A. (2017). Stratified flow of an oldroyd-b nanoliquid with heat generation. *Results in physics*, 7:2489–2496.