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Numerical Solution of a Steady MHD Boundary Layer Flow of a Nanofluid over a Porous Exponentially Shrinking Surface with Heat and Mass Fluxes

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Abstract

This study investigates the effect of induced magnetic field on the MHD flow of a Nano fluid over an exponentially shrinking porous sheet with heat and mass transfer .The flow problem is characterized by the parameters ;Nano-particle volume fraction ,thermophoresis, Lewis number and magnetic Prandtl number .It is governed by Partial differential equations which are transformed into Ordinary differential equations to ease computation of solutions .The O.D.E.s are solved numerically by using collocation method through MATLAB functions bvp4c.A single solution was found. Induced magnetic field delayed boundary layer separation hence a low wall suction(S), was required to maintain a steady flow. Induced magnetic field increased flow velocity, increased mass flux and increased temperature flux.

Keywords: Nanofluid, Exponentially shrinking sheet, Heatflux, Massflux, Induced magnetic field.

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NOMENCLATURE:

p	Pressure
t	dimentionless time,s
T	Temperature,K
C_p	Specific heat capacity, $jKg^{-1}K^{-1}$
K	Thermal conductivity
U_1	freestream velocity ,ms^{-1}
v	fluid velocity in the y direction,ms^{-1}
u	fluid velocity in the x direction, ms^{-1}
B_O	induced magnetic fluid
U_w	velocity of shrinking surface
P_r	prandtl number
E_c	Eckert number
T_w	Wall temperature,k
R_m	magnetic reynolds number is eratio of convective to difusive term of the induction equation
t	Time(seconds)
R_e	Reynolds number,ratio of inertial forces to viscuos forces within fliud
∇	Gradient Operator
D_B	Brownian diffusion co-efficient
D_T	Thermophoretic diffusion co-efficient
C_f	Nanofluid specific heat
C_p	Nanoparticle specific heat
ρ_p	Density of nanoparticle
N_b	Brownian motion parametre

N_t	Thermophoresis parameter
T_∞	Freestream temperature
N	Nano particle concentration
P_m	Magnetic prandtl number
ρ_{bf}	Density of base fluid
i,j	Unit vectors in X and Y direction

GREEK SYMBOLS:

μ	Dynamicviscosity coeficient of fluid
ρ	Density of the fluid
σ	Electrical conductivity of the fluid
∇^2	Laplacian operator
ϕ	viscous dissipation function
η	similiratity variable
α	Ratio of the free stream velocity of the free stream velocity to the stretching|shrinking velocity.
$\theta(\eta)$	Dimensionaless temperature of the fluid
$\varphi(\eta)$	Rescaled nanoparticle volume fraction
Ψ	stream function
α_m	Thermal diffusivity of the fluid
ν	kimematic viscosity of the fluidparticle
τ	Ratio of the effective heat capacity of the nano material and the ordinary fluid.

1. INTRODUCTION AND LITERATURE REVIEW

1.1. Literature Review

The boundary layer flow due to a shrinking sheet was first discovered in a research by[13]. This study showed that shrinking sheet flows shows different physical characteristics from those of the stretching sheet. The physical structure of shrinking sheet flow is complex. The research revealed that vorticity generated due to shrinking sheet is not confined within the boundary layer , thus the steady flow over a shrinking sheet is not possible unless some opposite force is used to prevent the vorticity diffusion and to maintain the boundary layer structure. According to a study by [7],it was found that steady shrinking sheet flow is dependent on the wall mass suction at the porous sheet.In a study by [11] which considered the MHD rotating flow for viscous fluid over a shrinking sheet, an analytic solution was sought using the Homotopy Analysis Method (H.A.M).A solution was found and further the research determined that for shrinking surface the stable and convergent solutions are possible only for MHD flows.

According to [12] in a research on flow over an unsteady shrinking sheet with mass transfer. Numerical techniques were used to solve the similarity equations for different values of the mass suction parameters and the unsteadiness parameters. This study revealed that multiple solutions exist for a certain range of mass suction parameters and unsteadiness parameters. The research also discovered that different flow behavior is observed for an unsteady shrinking sheet differnt from that of an unsteady stretching sheet.

According to [2] in the research on MHD boundary layer due to an exponentially shrinking sheet where the sheet was taken to be porous and a variable mass transfer was considered. The solutions to the governing equations were sought using the shooting method.In this study it was observed that wall mass suction required to obtain a steady flow is reduced when magnetic field is induced. This is because the magnetic field delays the boundary layer separation and maintains a steady flow. In this study it was also observed that dual solutions for the steady MHD flows are found for certain conditions and that With increasing magnetic parameters velocity was observed to increase for first solution and to decrease for second solution.

Flow over a shrinking sheet shows some unique characteristics when heat and mass transfer at the boundary are considered. In heat engineering the heat flux phenomenon plays an important role in controlling temperature. In most cases practical solutions will occur only when hot surfaces is subjected to a constant heat flux as opposed to being on a given temperature.

In a study by [6] on the mass transfer in a steady two dimensional MHD boundary layer, expression for the velocity and concentration distribution were obtained.Also in a study by [4] on two dimensional MHD flow over a porous shrinking sheet with wall mass transfer,a closed form exact solution was found.In another research by[9] on MHD viscous flow due to shrinking sheet ,a series solution was found using the Adominian decomposition method (A.D.M).

According to [3] in the analysis of the boundary flow and heat transfer over an exponentially shrinking sheet ,revealed that amount of vorticity generated due to exponential shrinking is greater than those for linear shrinking surfaces . Nano-technology has currently attracted attention of researchers due to its numerous industrial applicability.This is because materials of nanometer-size have unique physical and chemical properties .A study conducted by [10]shows that nanofluids have a strong temperature dependence of thermal conductivity and three times higher critical heat flux compared with the base fluids .Arifin in the study [1] found out that heat transfer can be changed by different techniques such as changing the surface geometry ,boundary conditions and thermal conductivity of the fluids. Compared to standard heat transfer fluids Nano-fluids have superior thermo-physical properties for which this fluid are used as coolant and used in many devices to enhance the heat transfer.

A research by [5] on heat and mass transfer in a boundary layer of a steady nano-fluid towards a shrinking sheet with heat flux and mass flux at the boundary,where combined effect of Brownian motion , thermophoresis on heat transfer and Nano-particle volume fraction were consideredjointly. It was found that solution depends on the suction parameter and that the wall mass flux through the porous sheet causes an increase in flow velocity for the first branch of solution. The research also found that the heat transfer rate becomes lower with the increase in Lewis number.Increase in thermophoresis parameter to increase both the temperature and nanoparticle volume fraction.

1.2. Statement of the Problem

It is known that presence of a magnetic field in a flow retards the flow due to formation of Lorentz force, this in turn adversely affects the flow dynamics.The study by [5] did not include effect of a magnetic field, this is a gap that the current study seeks to research on. The present study seeks to determine the effect of induced magnetic field on the steady flow of a Nano-fluid over an exponentially shrinking porous sheet with heat and mass transfers.

2. MATHEMATICAL FORMULATION

2.1. Introduction

Consider a two dimensional steady MHD flow of an incompressible Nano-fluid over an exponentially shrinking sheet .The fluid is electrically conducting in the presence of a uniform magnetic field of strength Bo ,which is applied normal to the sheet .The induced magnetic field is neglected.The X-axis is along the sheet and Y axis is perpendicular to it .The origin is fixed where v and u are velocity components along horizontal and vertical directions .

2.1.1. Flow Diagramm

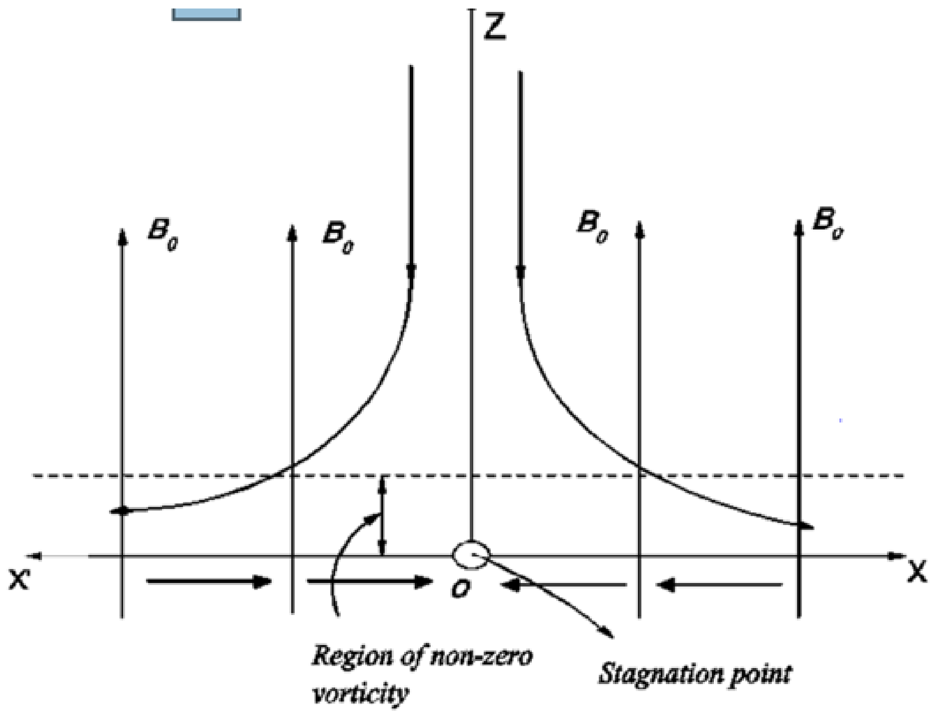

Figure 1: Flow Configuration"[8]"

2.2. Assumptions of the flow

1. The flow is steady and incompressible.
2. The pressure in the flow is constant.
3. The induced electric field is neglidible.
4. The flow is two dimensional .

2.3. Governing Equations

2.3.1. continuity equation

This equation satisfies the principle of conservation of mass,which states that the mass of a closed system remains constant over the time.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

2.3.2. Nanofluid momentum equation

Originates from Newtons second law .It describes momentum of fluid in relation to external force.where ρ is the constant fluid density.

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+\frac{\sigma}{\rho_{bf}}(-uB_0^2+vhB_0e^{\frac{X}{2L}}) \tag{2}$$

which is the specific momentum equation.

2.3.3. Nanofluid energy equation

This equation shows that energy in a closed system is conserved .Where C_P is the nanoparticle specific heat and K is the thermal conductivity.

$$\left\{u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\rho_C}{\rho_{bf}}\left[D_B\frac{\partial N}{\partial y}\frac{\partial T}{\partial y}+(\frac{D_B}{T_\infty})(\frac{\partial T}{\partial y})^2\right]\right\} \tag{3}$$

which is the specific nanofluid energy equations.

2.3.4. Nanofluid concentration equation

This equation describes the diffusion of particles within the base fluid due to temperature difference.Where D_B is the Brownian diffusion coefficient and D_T is the thermophoretic diffusion coefficient.

$$\left(u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right)=D_B\frac{\partial^2 N}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

which is the specific nanofluid concentration equation.

2.3.5. Magnetic induction equations

$$\eta_0\frac{\partial^2 h}{\partial y^2}=v\frac{\partial h}{\partial y}+h\frac{\partial v}{\partial y}-\frac{B_0}{e^{\frac{x}{2}}}(\frac{\partial u}{\partial y}) \tag{5}$$

boundary conditions

$$\begin{aligned} at y=0, u=U, v=-V(x), \frac{\partial T}{\partial y}&=-\frac{q_w(x)}{\alpha}\\ \frac{\partial N}{\partial y}=-\frac{q_{np}(x)}{D_B}, \frac{\partial h}{\partial y}&=B_0e^{\frac{X}{L}}\\ as y\longrightarrow\infty, u\longrightarrow 0, T\longrightarrow T_\infty, N\longrightarrow N_\infty, h&\longrightarrow 0\end{aligned} \tag{6}$$

Where; $U=-U_0e^{\frac{X}{L}}$is the shrinking velocity $q_w(x)=q_{w0}T_0\sqrt{\frac{u_0}{2vL}}e^{\frac{X}{L}}$ is the variable heat flux ,$q_{np}(x)=q_{np0}N_0\sqrt{\frac{U_0}{2vL}}e^{\frac{X}{L}}$is the nanoparticle flux. $V(x)=V_0e^{\frac{X}{L}}$is a special type of velocity at the wall according to[3],where $V_0$is a constant where$V(X) > 0$is the velocity of suction and $V(x) < 0$is velocity of injection. U_0 , T_0 , q_{w0} , q_{np0} and N_0 are the reference velocity,temperature,heatflux,and surface nanoparticle flux.

3. METHODOLOGY

3.1. Introduction

In this chapter the PDES are transformed to ODES by method of similarity transformation.The ODES are reduced to first order by method of order reduction and finally collocation method will be used to seek a solution to the equations.

3.2. Second section

3.2.1. Similarity transformation of PDES to ODES.

given that according to the study by [5] the similarity transforms are ;

$$\begin{gathered}\Psi = \sqrt{2\nu L u_0} f(\eta) e^{\frac{X}{2l}}, \eta = y\sqrt{\frac{u_0}{2\nu L}} e^{\frac{X}{2L}}, \\ T = T_\infty + \frac{q_{w0}}{\alpha} T_0 \theta(\eta) e^{\frac{X}{2L}}, N = N_\infty + \frac{q_{np0}}{D_B} N_0 \phi(\eta) e^{\frac{X}{2L}}\end{gathered} \tag{7}$$

And taking u and v in terms of stream function to be ;

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \tag{8}$$

Where η is the similarity variable and ψ is the stream function.this leads to other non-dimentionalisation parameters as follows;

$$u = u_0 f' e^{\frac{X}{L}} \tag{9}$$

$$\frac{\partial u}{\partial x} = \frac{u_0}{2L} e^{\frac{X}{2L}} f'' \eta + \frac{u_0}{L} f' e^{\frac{X}{L}} \tag{10}$$

$$\frac{\partial u}{\partial y} = \frac{(u_0)^{\frac{3}{2}}}{(2\nu L)^{\frac{1}{2}}} f'' e^{\frac{3x}{2l}} \tag{11}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{(u_0)^2}{2\nu L} f''' e^{\frac{2X}{L}} \tag{12}$$

$$h = -u_0 H' e^{\frac{X}{L}} \tag{13}$$

$$\frac{\partial h}{\partial y} = -\frac{(u_0)\frac{3}{2}}{(2\nu L)\frac{1}{2}} H'' e^{\frac{3X}{2L}} \tag{14}$$

$$\frac{\partial^2 h}{\partial y^2} = -\frac{(u_0)^2}{2\nu L} H''' e^{\frac{2X}{L}} \tag{15}$$

$$v = -\left(\frac{\nu u_0}{2L}\right)^{\frac{1}{2}} e^{\frac{X}{L}}[\eta f' + f] \tag{16}$$

$$\frac{\partial v}{\partial y} = -\frac{u_0}{2L} e^{\frac{X}{L}}[\eta f'' + 2f'] \tag{17}$$

$$\frac{\partial T}{\partial x} = \frac{q_{w0}}{2L\alpha} T_0 e^{\frac{X}{2L}}[\theta(\eta) + \theta'(\eta)] \tag{18}$$

$$\frac{\partial T}{\partial y} = \frac{q_{w0}T_0}{\alpha}\sqrt{\frac{u_0}{2\nu L}}\theta(\eta)\, e^{\frac{X}{L}} \tag{19}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{q_{w0}T_0}{\alpha}\frac{u_0}{2\nu l}\theta''(\eta)\, e^{\frac{3X}{2L}} \tag{20}$$

$$\left(\frac{\partial T}{\partial y}\right)^2 = \frac{(q_{w0})^2 T_0^2}{\alpha^2}\sqrt{\frac{u_0}{2\nu L}}\left(\theta'\right)^2 e^{\frac{2X}{L}} \tag{21}$$

$$\frac{\partial N}{\partial x} = \frac{q_{np0N_0}}{2LD_B} e^{\frac{X}{2L}}[\phi + \eta\phi'] \tag{22}$$

$$\frac{\partial N}{\partial y} = \frac{q_{np0}N_0}{D_B}\sqrt{\frac{u_0}{2\nu L}}\phi' e^{\frac{X}{L}} \tag{23}$$

The equations (3.2.2)-(3.2.17) will be substituted to the PDES for corresponding variables to reduce them to ODES.

CONTIUTY EQUATION:
sustituiting equation(3.2.2) into the continuity equation(2.3.3)leads to;

$$\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial x}\right) = 0 \tag{24}$$

$$\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial x \partial y} = 0 \tag{25}$$

hence the continuity equation is satisfied.

MOMENTUM EQUATION:
Substituting equations (3.2.2)to (3.2.1)into the momentum equation (2.3.14)for the corresponding variables,expanding and re-arranging gives;

$$f''' = 2\left(f'\right)^2 + f'f'' + \frac{M^2}{Re}\frac{2L}{B_0\nu}f' + \sqrt{2L^2(u_0)^2}\frac{M^2}{\sqrt{Re}}[\eta f'H' + fH'] \tag{26}$$

ENERGY EQUATION:

Substituting equations (3.2.2)to (3.2.1)into the energy equation (2.3.17)for the corresponding variables,expanding and re-arranging gives;

$$\theta'' = Pr\left(f'\theta - f\theta' - N_b\theta'\phi' - N_t(\theta')^2\right) \tag{27}$$

Which is the non-dimensional form of the energy equation

CONCETRATION EQUATION:
Substituting equations (3.2.2)to (3.2.1)into the concentration equation (2.3.20)for the corresponding variables,expanding and re-arranging gives;

$$\phi'' = Le(f'\phi - f\phi') - \frac{N_t}{N_b}\phi'' \tag{28}$$

which is the non dimentional concentation equation.

Magnetic induction equation:
Substituting equations (3.2.2)to (3.2.1)into the induction equation (2.3.36)for the corresponding variables,expanding and re-arranging gives;

$$H''' = \frac{P_m}{\sqrt{Re}}\sqrt{2}L[H'' + B_0 f''] + P_m[\eta f'' H' + 2f'H'] \tag{29}$$

which is the non-dimentional magnetic induction equation.

Boundary Conditions :

$$\begin{aligned} f(\eta) &= s, f' = -1, at\eta = 0 \\ f'(\eta) &\longrightarrow 0 as\eta \longrightarrow \infty \\ \theta'(\eta) &= -1, at\eta = 0 \\ \theta(\eta) &\longrightarrow 0 as\eta \longrightarrow \infty \\ \phi'(\eta) &= -1 at\eta = 0 \\ \phi(\eta) &\longrightarrow 0 as\eta \longrightarrow \infty \\ H'(\eta) &= -1 at\eta = 0 \\ H(\eta) &\longrightarrow 0 as\eta \longrightarrow \infty \end{aligned} \tag{30}$$

where $s = -\frac{v_0}{\sqrt{\frac{\nu c}{2L}}}$ is the sunction or blowing parametre. where s>0 is the mass sunction and s<0 is the mass injection.hence in this project shrinking sheet is considered so only the mass sunction has been considered because the solution for steady boundary layer flow past a shrinking sheet occurs in the presence of sunction only.

3.2.2. Order Reduction of ODES.

Oder reduction is performed on the emerging ODES to linearise them to first oder.

$$
\begin{aligned}
Let f = x_1, f' = x_2, f'' = x_3, f''' = x'_3 \\
H = x_4, H' = x_5, H'' = x_6, H''' = x'_6 \\
\theta = x_7, \theta' = x_8, \theta'' = x'_8 \\
, \phi = x_9, \phi' = x_{10}, \phi'' = x'_{10}
\end{aligned} \tag{31}
$$

Hence applying equation (3.2.30) to the momentum equation (3.2.22)it leads to;

$$x'_3 = 2(x_2)^2 + x_2x_3 + \frac{M^2}{Re}\frac{2L}{B_0\nu}x_2 + \sqrt{2L^2(u_0)^2}\frac{M^2}{\sqrt{Re}}[\nu x_2x_5 + x_1x_5] \tag{32}$$

applying equation (3.2.30) on the energy equation(3.2.23) it becomes;

$$x'_8 = Pr[x_2x_7 - x_1x_8 - N_bx_2x_9 - N_t(x_8)^2] \tag{33}$$

now also applying equation(3.2.30) to the concentration equation we get;

$$x'_{10} = Le(x_2x_9 - x_1x_9) - \frac{N_t}{N_b}x'_{10} \tag{34}$$

substituting equation(3.2.30) into the magnetic induction equation (3.2.29) gives;

$$x'_6 = \frac{P_m}{\sqrt{Re}}\sqrt{2}L[x_6 + B_0x_3] + P_m[\eta x_3x_5 + 2x_2x_5] \tag{35}$$

Boundary conditions:

$$
\begin{aligned}
x_1(0) = s, x_2(0) = -1, x_2(\infty) = 0 \\
x_8(0) = -1, x_7(\infty) = 0 \\
x_{10}(0) = -1, x_9(\infty) = 0 \\
x_5(0) = -1, x_4(\infty) = 0
\end{aligned} \tag{36}
$$

3.3. Solution method to the ODES.

In solving of ODES ,collocation method was applied.The collocation method is a numerical method that yields the approximate solution of a boundary problem in the form of a function as opposed to a set of discrete points .

4. RESULTS AND DISCUSSIONS

This chapter discusses the results obtained and the effect that varying of various parameters has on the different flow profiles.To ease discussion the flow profiles have grouped for different flow profiles.

4.1. Results Validation

Using collocation method through MATLAB bvp4c function a single solution was found, this was a departure from the non-MHD flow study conducted by[5] were a dual solution was obtained. In the current study it was observed that steady MHD flow was only possible for positive parameters which agrees well with the findings of the research [3] on MHD boundary layer flow due to an exponentially shrinking sheet. In the current study it was found out that there was a steady flow with a suction parameter s=1,this is far less than the s=2.6, suggested for a similar non-MHD flow in the research [5]. This is because the magnetic field delays the boundary layer separation and maintains a steady flow even when suction is low. This agrees well with the study [3] conducted on the same type of flow.

4.2. Discussions

VELOCITY PROFILES :

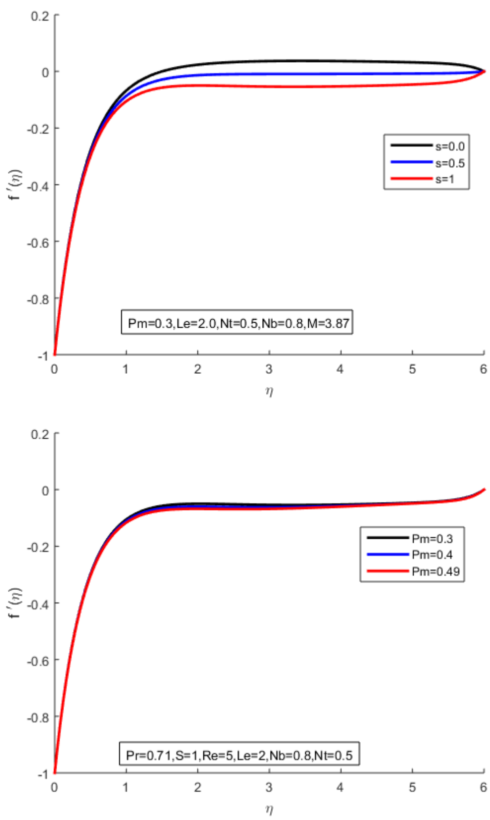

Figure 2: RIGHT:figure 2;Effect of suction paramter on velocity profile.RIGHT:figure6;Effect of prandtl number on velocity

Figure 2: Depicted that an increase in suction parameter (S) caused a decrease in velocity profile. It was also observed that an increase in magnetic Prandtl number (Pm)

decreased velocity profile as shown in figure 6. This is because when Prandtl number (Pm) increases ratio of boundary layer increases over magnetic boundary layer hence Lorentz force becomes more. From figure 14, it is depicted that an increase in Hartman number (M)increases velocity profile.

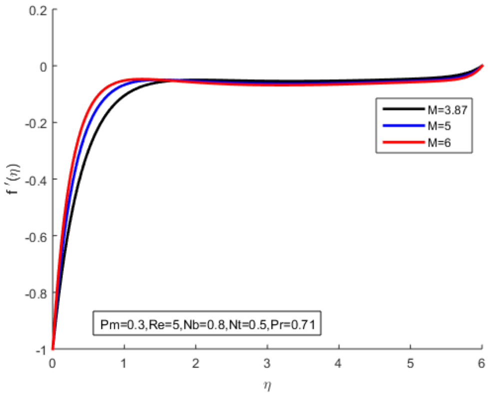

Figure 3: Fiure 14;Effect of Hartman number on volocity profile

TEMPERATURE PROFILES:

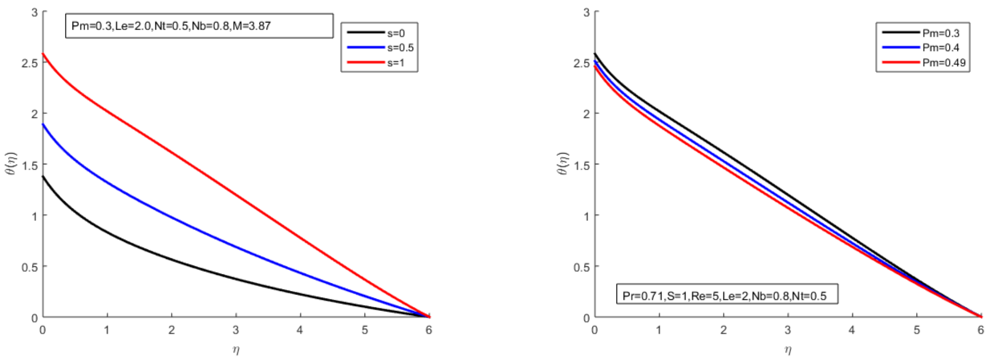

Figure 4: LEFT:Fifure4;Effect of suction on temperature profile RHITE:Figure 8;Effect of prandtl number on temperature profile

An increase in suction parameter(S) was seen to increase the temperature profile as seen in figure 4. In figure 8, it was observed that an increase in Prandtl number(Pm) caused a decrease in temperature profile. In this study an increase in Hartman number(M) was found to increase temperature profile as depicted in figure 16. when Thermophoresis parameter(Nt) increased temperature profile increases as seen in figure 20. An increase in Brownian motion parameter(Nb) reduces temperature diffusivity as seen in figure 24

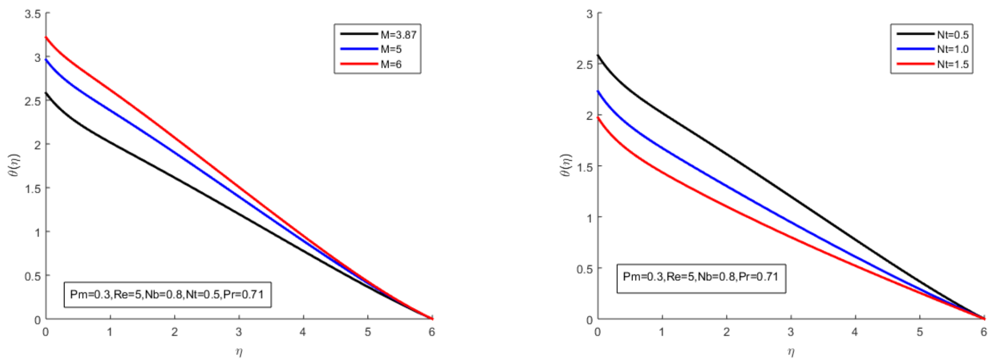

Figure 5: LEFT:Figure 16;Effect of Hartmannumber on temperature profile RIGHT:Figure 20;Effect of thermophoresis parameter on temparature profile

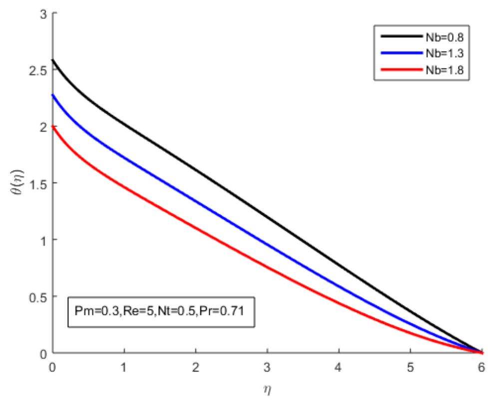

Figure 6: Figure 24:Effect of brownian motion on temperature profile.

CONCENTRATION PROFILES:

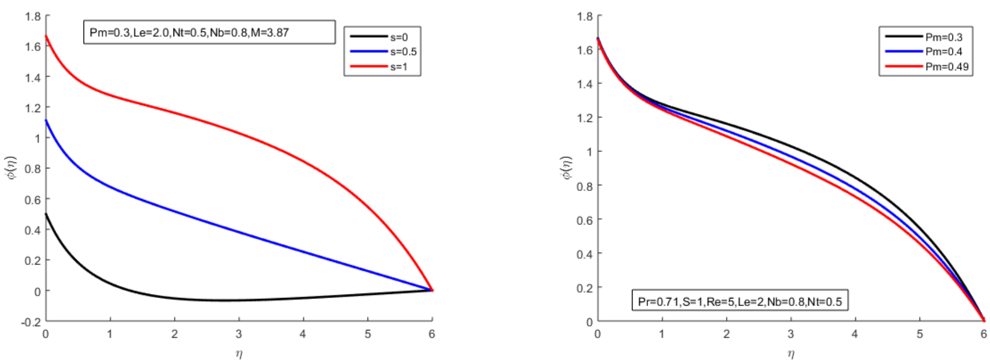

Figure 7: LEFT:FIGURE 5;Effect of suction on concentration RIGHT:FIGURE 9;Effect of magnetic prandtl number on concentration profile

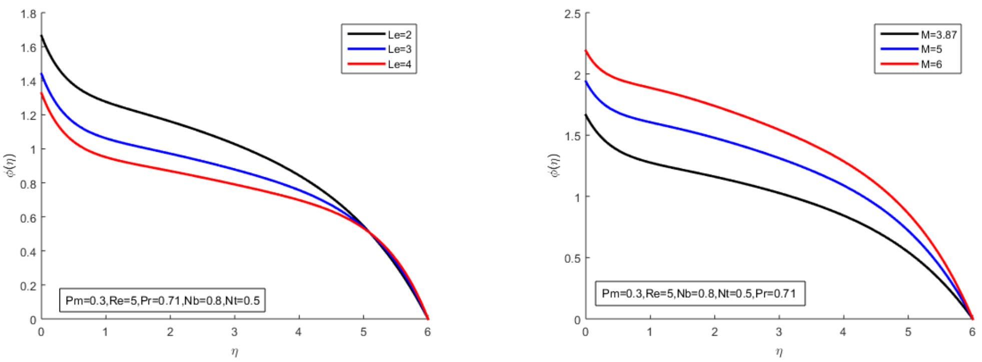

Figure 8: LEFT:FIGURE 13;Effect of Lewis number on concetration profile RIGHT:FIGURE 17;Effect of Hartman number(M) on concentration

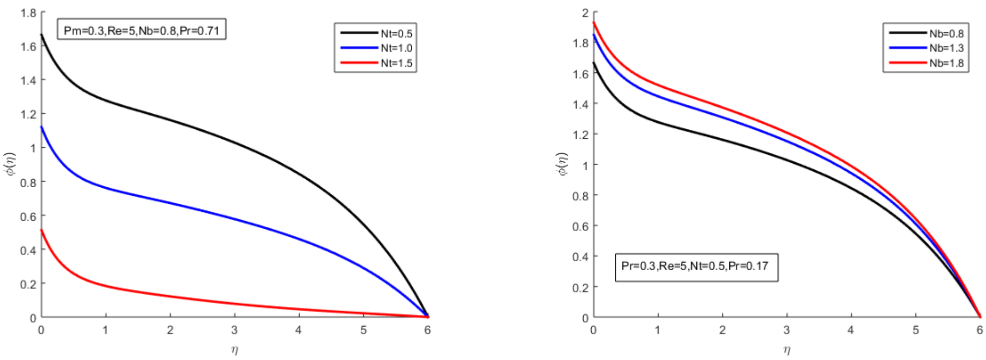

Figure 9: LEFT:FIGURE 21;Effect of Thermophoresis on concentration RIGHT:FIGURE 25;Effect of brownian motion on concentration

From figure 5, it is observed when suction parameter(S) is increased the concentration profile was increased. It was observed that an increase in magnetic Prandtl number reduced the concentration profile which is observed in figure 9.From figure 13, increased Lewis number(Le) decreases mass diffusivity hence a higher concentration in the fluid. It was also found that an increase in Hartman Number(M) increases concentration profile as seen in figure 17. From the study it was observed that an increase in thermophoresis parameter (Nt) reduced concentration profile as depicted in figure 21. It was also observed that an increase in Brownian motion parameter(Nb) increases concentration profile because there is an increase in mass diffusivity as seen in figure 25.

MAGNETIC PROFILES:

From the graphs presented different parameters were seen to have varying effects on the magnetic profile. In when suction parameter (S) was increased magnetic profile was reduced as shown in figure 3.Also an increase in Magnetic Prandtl number(Pm) the magnetic field was found to increase too, as depicted in figure 7.

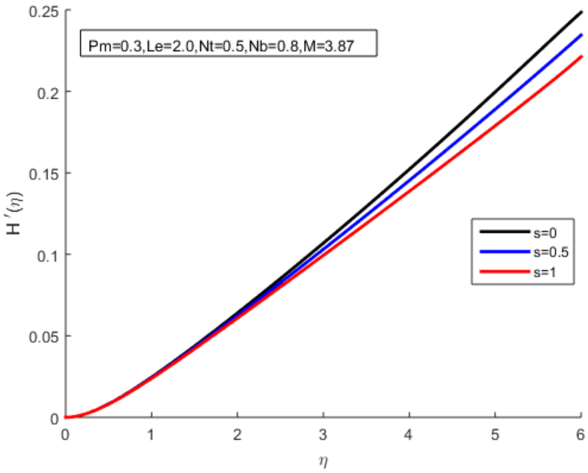

Figure 10: FIGURE 3:Effect of suction on Magnetic profile

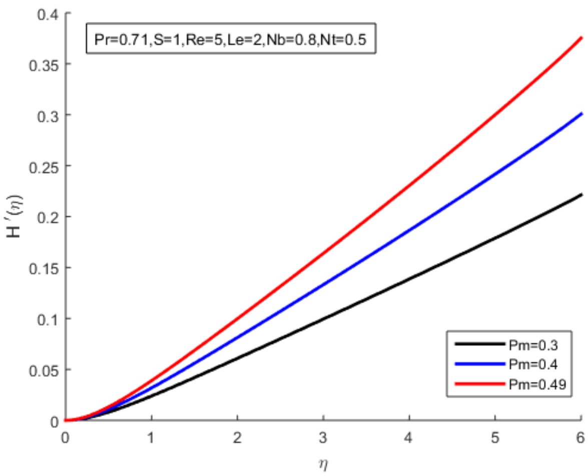

Figure 11: FIGURE 7:Effect of prandtl number on magnetic profile

5. CONCLUSION

In this study governing equations for an MHD flow of a Nano fluid over an exponentially shrinking sheet with heat and mass fluxes were formed. These equations were transformed from PDEs to ODEs using suitable similarity transforms. The resulting ODEs were reduced to first order and solved numerically using collocation method through MATLAB function bvp4c function. Graphs were drawn to show the effect of varying various parameters on velocity, temperature, concentration and magnetic profiles. The following conclusions were made;

(i) Induced magnetic field increases flow velocity for flow over an exponentially shrinking sheet. This is contrary effect of MHD on linear shrinking surface flow where it retards flow. This is in good agreement with the study [2]

(ii) Induced magnetic field reduces the amount of wall suction required to achieve a steady flow over an exponentially shrinking sheet with heat and mass fluxes.

(iii) Induced magnetic field increases mass flux. This was clearly shown by concentration graph (figure 7)

(iv) Induced magnetic field increased temperature flux. This is depicted by temperature graph (figure 16)

5.1. Recommendations

This study investigated the Numerical solution of MHD boundary layer steady flow of a Nano fluid over a porous exponentially shrinking surface with heat and mass fluxes. It is recommended that further research can be carried out on the same area on;

(i) Numerical solution of an unsteady MHD boundary layer flow of a Nano fluid over a porous exponentially shrinking surface with heat and mass flux

(ii) Effect of an inclined magnetic field on a Nano fluid flow over a porous exponentially shrinking surface with heat and mass fluxes

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