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Dominator and Total Dominator Coloring on Square Chessboard

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Abstract

The proper coloring of a graph G is said to be a dominator coloring if each vertex of the graph dominates every vertex of some color class. The minimum number of color classes required to satisfy the condition of dominator coloring is said to be dominator chromatic number which is denoted by $\chi_d(G)$. Total dominator coloring is defined to be a proper coloring of G with a property that every vertex of G dominates all the vertices of at least one color class (other than the class itself). The minimum number of color classes required to satisfy the condition of total dominator coloring is called total dominator chromatic number and is denoted by $\chi_{td}(G)$. In this paper, we would discuss the dominator and total dominator coloring parameters of bishops and rooks on square chessboard and give the values for dominator chromatic number and total dominator chromatic number for these chessboard graphs.

Keywords: Proper Coloring, Dominator Coloring, Total Dominator Coloring, and Chessboard Graphs

INTRODUCTION

In Graph theory, coloring and domination are the two broad areas of study. A detailed study on domination was given in the book by Haynes et al. [5]. A proper coloring of a graph G is defined as assignment of colors to each vertex of G such that no two adjacent vertices have same color. dominator coloring of graph G is proper coloring of vertices of G such that every vertex dominates all the vertices of some color class. The minimum cardinality of a dominator coloring set is called dominator chromatic number $\chi_d(G)$.Total dominator coloring is said to be a proper coloring of G with a property that every vertex of G dominates every vertex of at least one color class. The total dominator chromatic number is the minimum number of colors required to satisfy the condition of total dominator coloring which is denoted by $\chi_{td}(G)$. The number of squares in the entire chessboard of order $n \times n$ can be considered as vertices, while the edges are formed between two vertices or squares if they are adjacent. In Bishops graph B_n two squares are adjacent if one square can cover the other in the same diagonal, whereas, in Rooks graph R_n two squares are adjacent if they are in same row or column.

Dominator coloring was introduced in [2] by Ralucca Gera and this concept was studied on some classes of graphs. Gera studied the problem further in [3, 4], she proved its NP-completeness and showed that every graph G satisfies $\max\{\gamma(G), \chi(G)\} \leq \chi_d(G) \leq \gamma(G) + \chi(G)$. Arumugam et al. in [1] showed that the dominator coloring problem is NP-hard on bipartite, planar and split graphs. A. P. Kazemi in [7] introduced the concept of total dominator coloring and showed that total dominator chromatic number is NP-complete. Bounds of the dominator chromatic number of a graph in terms of chromatic number and the domination number, and also total dominator chromatic number on some classes of graphs were considered in [7].

Domination on chessboard graph was considered to be the origin of study of domination problems in [5]. We also have other domination parameters studied on different chessboard graphs. While, the coloring on square chessboards was introduced for the first time in 1966 by Iyer et al. as mentioned in [6] where the chromatic number of all the chess pieces were discussed. Further studies were made on Grundy coloring chessboard graphs by Casey parks et al. in [8]. In this paper we would extend the concept of dominator and total dominator coloring on chessboard graphs and give the dominator chromatic number and total dominator chromatic number for Bishops graph (B_n) and Rooks graph (R_n) on a square chessboard. In section 1 we discuss the concept of dominator coloring on bishops and rooks, in section 2 we discuss about total dominator coloring on bishops and rooks on square chessboard.

DOMINATOR COLORING

Theorem 1: For a Bishops graph on a square chessboard dominator coloring is

$$\chi_d(B_n) = 2n - 1.$$

Proof: To show the dominator coloring on bishops graph we have to prove that each vertex in the chessboard is dominated by some color class (includes the class itself). As we know that bishops movement on chessboard graphs is restricted to the diagonals, it implies that a vertex is adjacent only with the vertices that lie on the diagonal passing through it. Also it implies that two vertices cannot be placed on the same diagonal. Therefore, first assign *'n'* distinct colors from 1 to n to the center row. This would make all the cells other than the row itself dominated by some color class. Thus assign $'n-1'$ more colors one to each row distinct from the $'n'$ colors because assigning one of the colors from the used $'n'$ colors would give us some cells left without being dominated by any color class. Since each vertex in the row colored with $'n'$ colors at the beginning are distinct each vertex dominates its own color class. This results in every vertex of the board being dominated by some color class. Hence total number of colors required to satisfy the condition to become a dominator coloring would be $2n-1$. Here we assign the $'n'$ distinct colors to a row or a column but not to the diagonal because placing them in a column would leave $'n-1'$ cells in $'n'$ columns thus resulting in assigning $'n'$ more distinct colors. It implies that 2n colors are required which is not minimum. Thus, dominator chromatic number for Bishops on square chessboard is $2n-1$. Fig.1 shows the bishops dominator chromatic number on 5×5 board.

Theorem 2: For a Rooks graph on a square chessboard dominator coloring $\chi_d(R_n) = 2n-1$.

Proof: We prove this in the same way as proved for the Bishops in Theorem 1. First take *'n'* distinct colors from 1 to n and assign them to the main diagonal. As we know that rooks movement on chessboard graphs is restricted to the horizontal and vertical moves it implies that each vertex is adjacent only to the vertices lying on the same row and column. Therefore, only two vertices of a particular cell i.e. one being the row and the other being the column could be colored with same color, because assigning the same color to more than two vertices would leave us with an improper coloring. This would make all the cells other than the diagonal itself dominated by some color class. Thus assign $'n-1'$ more colors one to each diagonal distinct from the $'n'$ colors. Since each vertex in the diagonal colored with $'n'$ colors at the beginning are distinct each vertex dominates its own color class. Hence total number of colors required to satisfy the condition to become a dominator coloring would be $2n-1$ which is minimum. Thus, dominator chromatic number of Rooks (R_n) on square chessboard is $2n-1$. Fig.2 shows the rooks dominator chromatic number on 4×4 chessboard.

6	6	6	6	6
7	7	7	7	7
1	2	3	4	5
8	8	8	8	8
9	9	9	9	9

Fig.1 Bishops Dominator Chromatic number on 5×5 chessboard

1	6	7	5
5	2	6	7
7	5	3	6
6	7	5	4

Fig.2 Rooks Dominator Chromatic number on 4×4 chessboard

TOTAL DOMINATOR COLORING

Theorem 3: For Bishops graph on a square chessboard total dominator coloring

$$\chi_{td}(B_n) = 3(n-1).$$

Proof: We prove this by first taking *'n'* distinct colours 1 to n and assigning them to $\left\lceil\frac{n+1}{2}\right\rceil^{th}$ row from lower left-hand corner. Therefore, according to the movement of bishops on chessboard as mentioned in Theorem.1 each vertex in the $\left\lceil\frac{n+1}{2}\right\rceil^{th}$ row can be dominated only if same color is assigned to the two diagonals lying on the same side of that particular cell. This results in every vertex of the board being dominated by at least one color class other than the row coloured with 1 to n distinct colors. Now to get the row coloured with 1 to n dominated by at least one color class we assign n+1 to 2n distinct colors to the rows adjacent to it. Thus coloring these three rows i.e. one being the $\left\lceil\frac{n+1}{2}\right\rceil^{th}$ row and two being its adjacent rows would require 2n colors. Now, as the remaining vertices in each row were already dominated by at least one color class, each row can be colored with a color distinct from the 2n colors. Otherwise, repeating the colors used for center row would leave some of the cells of the board without dominating any color class, while repeating the colors used for the adjacent rows leaves the center row without dominating any color class which contradicts the condition for dominator coloring. Continuing in this way we require (n-3) colors. Hence total number of colors required to satisfy the condition to become a dominator coloring would be $3(n-1)$ which is minimum. Thus, dominator chromatic number for Bishops on square chessboard is $3(n-1)$. Fig.3 shows bishops total dominator chromatic number on 6×6 chessboard.

13	**13**	**13**	**13**	**13**	**13**
7	**8**	**9**	**10**	**11**	**12**
1	**2**	**3**	**4**	**5**	**6**
7	**8**	**9**	**10**	**11**	**12**
14	**14**	**14**	**14**	**14**	**14**
15	**15**	**15**	**15**	**15**	**15**

Fig.3 Bishops Total Dominator Chromatic number on 6×6 chessboard

Theorem 4: For Rooks graph on a square chessboard total dominator coloring is

$$\chi_{td}(R_n) = \frac{n^2}{2} \text{ when } 'n' \text{ is even and } \frac{n^2+1}{2} \text{ when } 'n' \text{ is odd.}$$

Proof: We prove this in two cases.

(i) When *'n'* is even first divide the board into sub-boards of size 2×2. According to the movement of rooks on chessboard as mentioned in Theorem.2. This results in each color being assigned to exactly two vertices of the board. Now, assign two colors to each sub-board of size 2×2 which would dominate each cell in the sub-board by one of the two colors assigned in it. Thus, assigning same colors of one sub-board to others would lead us to a contradiction of dominator coloring. Since, there are n^2 cells and as mentioned each color is assigned to only two cells we require $\frac{n^2}{2}$ colors which is minimum. Thus the total dominator coloring of Rooks on square board is $\frac{n^2}{2}$. Fig.4 shows the rooks total dominator chromatic number on 4×4 chessboard.

(ii) When $'n'$ is odd first divide the board at the center row and column giving four quadrants. Now, divide the each quadrant of the board into sub-boards of size 2×2 and assign colors as mentioned above for the case when $'n'$ is even. Now, we have all the sub-boards being dominated by at least one color class. The left out center row and column has a cell in common which is assigned a distinct color. Now, the remaining $'2n-1'$cells in the center row and column can be assigned with $'n-1'$ distinct colors with each color being given to exactly two cells with one being the row and other in the column. Thus, assigning same color would contradict the concept of proper coloring and hence contradicts total dominator coloring. Therefore, n^2-1 cells are colored with $\frac{n^2-1}{2}$ colors and the remaining central vertex is colored with a color distinct from others which results in $\frac{n^2+1}{2}$ minimum colors. Thus, the total dominator coloring of rooks on square chessboard is $\frac{n^2+1}{2}$. Fig.5 shows the rooks total dominator chromatic number on 5×5 chessboard.

1	**2**	**3**	**4**
2	**1**	**4**	**3**
5	**6**	**7**	**8**
6	**5**	**8**	**7**

Fig.4 Rooks Total Dominator Chromatic number on 4 × 4 chessboard

5	**6**	**9**	**7**	**8**
6	**5**	**10**	**8**	**7**
9	**10**	**11**	**12**	**13**
1	**2**	**12**	**3**	**4**
2	**1**	**13**	**4**	**3**

Fig.5 Rooks Total Dominator Chromatic number on 5 × 5 chessboard

CONCLUSION

In this paper the exact values and placement for the Bishops and Rooks on the square chessboard using the dominator and total dominator chromatic number were presented. We are extending this work further on to various other chessboard graphs.

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