Double Layered Complete Fuzzy Graph (DLCFG)

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Abstract
In this paper, a new fuzzy graph named double layered complete fuzzy graph is proposed. The double layered complete fuzzy graph gives a 3-D structure. We have discussed about order, size, degree and $\mu$-complement of fuzzy graph.

Keywords: Order, Size, Vertex Degree, $\mu$-Compliment, Strong Fuzzy Graph, Double Layered Complete Fuzzy Graph.

1. INTRODUCTION
Azriel Rosenfeld introduced fuzzy graph in 1975 [5]. Though introduced recently, it has been growing fast and has numerous applications in various fields. During the same time Yeh and Bang have also introduced various concepts in connectedness with fuzzy graphs [7]. Mordeson and Peng introduced the concept of operations on fuzzy graphs, Sunitha and Vijayakumar discussed about the operations of union, join, Cartesian product and composition on two fuzzy graphs [4]. The degree of a vertex in some fuzzy graphs was discussed by Nagoorgani and Radha [6]. Nagoorgani and Malarvizhi have defined different types of fuzzy graphs and discussed its relationships with isomerism in fuzzy graphs [3].
In this paper we define double layered complete fuzzy graph (DLCFG) or 3-D Fuzzy graph which gives a 3-D structure in fuzzy graph theory and some of its properties were discussed. Section two contains the basic definitions in fuzzy graphs, in section three we introduce a new fuzzy graph called a double layered complete fuzzy graph, section four presents the theoretical concepts of DLCFG and finally we give conclusion on (DLCFG).

2. PRELIMINARIES

2.1 Definition

A fuzzy graph $G$ is a pair of functions $G: (\sigma, \mu)$ where a fuzzy subset of a non-empty set $V$ and $\mu$ is a symmetric fuzzy relation on $\sigma$. The underlying crisp graph of $G: (\sigma, \mu)$ is denoted by $G^*: (\sigma^*, \mu^*)$ [5].

2.2 Definition

Let $G: (\sigma, \mu)$ be a fuzzy graph, the order of $G$ is defined as $O (G) = \sum_{u \in V} \sigma(u)$ [8].

2.3 Definition

Let $G: (\sigma, \mu)$ be a fuzzy graph, the size of $G$ is defined as $S (G) = \sum_{u,v \in V} \mu(u, v)$ [8].

2.4 Definition

Let $G: (\sigma, \mu)$ be a fuzzy graph, the degree of a vertex $u$ in $G$ is defined as

$$d_G(u) = \sum_{v \in V} \mu(u, v)$$

and is denoted as $d_G(u)$ [10].

2.5 Definition

A fuzzy graph $G: (\sigma, \mu)$ is said to be strong fuzzy graph if $\mu(u, v) = \sigma(u) \land \sigma(v)$ for all $(u, v)$ in $\mu^*$ [9].

2.6 Definition

Let $G$ be a fuzzy graph, the $\mu$–compliment of $G$ is denoted as

$$G^\mu: (\sigma^\mu, \mu^\mu)$$

where $\sigma^* \cup \mu^*$ and $\mu^\mu(u, v) = \begin{cases} (\sigma(u) \land \sigma(v) - \mu(u, v) & \text{if } \mu(u, v) > 0 \\ 0 & \text{if } \mu(u, v) = 0 \end{cases}$ [4].
3. DOUBLE LAYERED COMPLETE FUZZY GRAPH (DLCFG)

3.1 Definition

Let $\sigma_{DL}: V \rightarrow [0, 1]$ be a subset of $V$ and $\mu_{DL}: V \times V \rightarrow [0, 1]$ be a symmetric fuzzy relation on $\sigma_{DL}$. Any two vertex of the double layered complete fuzzy graph is adjacent. The vertex set of complete double layered fuzzy graph be $\sigma \cup \mu$ and it’s denoted by $K_{\sigma \cup \mu}$.

Or

Let $\sigma_{DL}: V \rightarrow [0, 1]$ be a fuzzy subset of $V$ then the complete double layered fuzzy graph on $\sigma_{DL}$ is defined on $K_{\sigma \cup \mu}=(\sigma_{DL}, \mu_{DL})$. Any two vertices of the DLCFG is adjacent.

Example: 3.1.1. Consider the complete fuzzy graph with vertex 3 ($K_3$)

![Graph diagram](image.png)

Figure 1. A complete fuzzy graph ($K_3$)
Figure 2. DLCFG of K₃

Figure 3. Image of DL (K₃)
**Example 3.1.2**: Consider the complete fuzzy graph with vertex 4 ($K_4$).

![Figure 4. A complete fuzzy graph ($K_4$)](image)

![Figure 5. DLCFG of $K_4$](image)
The figure 6(a) and 6(b) which is given below shows in different projections of Figure 5.

**Figure 6(a).** Image of DL($K_4$)

**Figure 6(b).** Image of DL($K_4$)
**Example 3.1.3:** Consider the complete fuzzy graph with vertex 5 ($K_5$).

![Figure 7](image1.png) **Figure 7.** A complete fuzzy graph ($K_5$)

![Figure 8](image2.png) **Figure 8.** DLCFG of $K_5$
Similarly we can convert complete fuzzy graph into double layered complete fuzzy graph.

4. THEORITICAL CONCEPTS

4.1 THEOREM

The order of double layered complete fuzzy graph $K_{\sigma U \mu}$ is equal to the sum of the order and size of the complete graph

Proof:

Let $\sigma U \mu$ be a node set of complete double layered fuzzy graph and the fuzzy subset $\sigma_{DL}$ on $\sigma^* U \mu^*$ is defined as,

$$\sigma_{DL} = \begin{cases} \sigma(u) & \text{if } u \in \sigma^* \\ \mu(uv) & \text{if } uv \in \mu^* \end{cases}$$

By the definition, order of the double layered fuzzy graph is,

$$O(DL(G)) = \sum_{u \in \sigma \cup \mu} \sigma_{DL}(u)$$

$$= \sum_{u \in \sigma} \sigma_{DL}(u) + \sum_{u \in \mu} \sigma_{DL}(u)$$

$$= \sum_{u \in \sigma} \sigma(u) + \sum_{u \in \mu} \mu(u)$$

$$O(DL(G)) = \text{Order}(G) + \text{size}(G)$$

4.2 Theorem

Every double layered complete fuzzy graph is a strong fuzzy graph

Proof:

As the node set of $DL(G)$ is $\sigma^* U \mu^*$ and the fuzzy subset $\sigma_{DL}$ on $\sigma^* U \mu^*$ is defined as,

$$\sigma_{DL} = \begin{cases} \sigma(u) & \text{if } u \in \sigma^* \\ \mu(uv) & \text{if } uv \in \mu^* \end{cases}$$

By the definition of double layered complete fuzzy graph

$$\mu(u, v) = \sigma(u) \land \sigma(v) \quad \text{--------------------------(1)}$$
And also by the definition of strong fuzzy graph

\[ \mu(u, v) = \min(\sigma(u), \sigma(v)) \quad (2) \]

From equation (1) & (2); we get

Every double layered complete fuzzy graph is a strong fuzzy graph

**Example 4.2.1**

We choose DL (G) of K3 graph,

\( v_1 = 0.4; v_2 = 0.6; v_3 = 0.8 \) and \( e_1 = 0.4; e_2 = 0.6; e_3 = 0.4 \)

![Figure 9. DLCFG of K3](image-url)
(i) \( \mu(v_1, v_2) = \sigma(v_1) \land \sigma(v_2) \)
\[= 0.4 \land 0.6 \]
\[= 0.4 \]

(ii) \( \mu(e_1, e_2) = \sigma(e_1) \land \sigma(e_2) \)
\[= 0.4 \land 0.6 \]
\[= 0.4 \]

(iii) \( \mu(v_1, e_1) = \sigma(v_1) \land \sigma(e_1) \)
\[= 0.4 \land 0.4 \]
\[= 0.4 \]

Every double layered fuzzy graph is a strong fuzzy graph

### 4.3 Theorem

If \( G \) is a strong fuzzy graph then the \( \mu \)-complement of \( DL(G) \) is isolated vertices

**Proof**

Let \( G \) be a strong fuzzy graph by the previous theorem,

Every double layered complete graph is a strong fuzzy graph

\[ \mu(u, v) = \sigma(u) \land \sigma(v) \rightarrow ① \]

And by the definition of \( \mu \)-complement,

\[ \mu^\mu(u, v) = \sigma(u) \land \sigma(v) - \mu(u, v) \]
\[= \mu(u, v) - \mu(u, v) \]
\[= 0 \]

\[ \mu^\mu(u, v) = 0 \text{ for all } u, v \text{ in } \sigma^*U\mu^* \]
\[d_{DL}(u) = 0 \text{ for all } u \text{ in } \sigma^*U\mu^* \]

Every vertices of complement of \( DL(G) \) have isolated vertices.
Example 4.3.1

Figure 10. DLCFG of $K_3$

Figure 11. $\mu$-complement DLCFG of $K_3$
\[ \text{DLCFG} (K_n) = K_n + \text{DLCFG} (K_{n-1}) \]

**Example 4.3.1**

(i) \[ \text{DLCFG}(K_4) = K_4 + \text{DLCFG}(K_3) \]
\[ = K_4 + K_5 \]
\[ = K_{10} \]
\[ \text{DLCFG} (K_4) = \text{CFG} (K_{10}) \]

(ii) \[ \text{DLCFG}(K_5) = K_5 + \text{DLCFG}(K_4) \]
\[ = K_5 + K_{10} \]
\[ = K_{15} \]
\[ \text{DLCFG} (K_5) = \text{CFG} (K_{15}) \]

**Table 1:** Relation between complete fuzzy graph and Double layered complete fuzzy graph.

<table>
<thead>
<tr>
<th>COMPLETE FUZZY GRAPH</th>
<th>DOUBLE LAYERED COMPLETE FUZZY GRAPH</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_3 )</td>
<td>( \text{DLCFG}(K_3) = K_6 )</td>
</tr>
<tr>
<td>( K_4 )</td>
<td>( \text{DLCFG}(K_4) = K_{10} )</td>
</tr>
<tr>
<td>( K_5 )</td>
<td>( \text{DLCFG}(K_5) = K_{15} )</td>
</tr>
<tr>
<td>( K_6 )</td>
<td>( \text{DLCFG}(K_6) = K_{21} )</td>
</tr>
<tr>
<td>( K_7 )</td>
<td>( \text{DLCFG}(K_7) = K_{28} )</td>
</tr>
<tr>
<td>( K_8 )</td>
<td>( \text{DLCFG}(K_8) = K_{36} )</td>
</tr>
<tr>
<td>( K_9 )</td>
<td>( \text{DLCFG}(K_9) = K_{45} )</td>
</tr>
<tr>
<td>( K_{10} )</td>
<td>( \text{DLCFG}(K_{10}) = K_{55} )</td>
</tr>
<tr>
<td>( K_{11} )</td>
<td>( \text{DLCFG}(K_{11}) = K_{66} )</td>
</tr>
<tr>
<td>( K_{12} )</td>
<td>( \text{DLCFG}(K_{12}) = K_{78} )</td>
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</tbody>
</table>
**Double Layered Complete Fuzzy Graph (DLCFG)**

<table>
<thead>
<tr>
<th>$K_{13}$</th>
<th>DLCFG($K_{13}$) = $K_{91}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{14}$</td>
<td>DLCFG($K_{14}$) = $K_{105}$</td>
</tr>
<tr>
<td>$K_{15}$</td>
<td>DLCFG($K_{15}$) = $K_{120}$</td>
</tr>
<tr>
<td>$K_{16}$</td>
<td>DLCFG($K_{16}$) = $K_{136}$</td>
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<tr>
<td>$K_{17}$</td>
<td>DLCFG($K_{17}$) = $K_{154}$</td>
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<tr>
<td>$K_{18}$</td>
<td>DLCFG($K_{18}$) = $K_{173}$</td>
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<td>$K_{19}$</td>
<td>DLCFG($K_{19}$) = $K_{192}$</td>
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<tr>
<td>$K_{20}$</td>
<td>DLCFG($K_{20}$) = $K_{212}$</td>
</tr>
<tr>
<td>$K_{21}$</td>
<td>DLCFG($K_{21}$) = $K_{233}$</td>
</tr>
<tr>
<td>$K_{22}$</td>
<td>DLCFG($K_{22}$) = $K_{255}$</td>
</tr>
</tbody>
</table>

**Remark:**

The edge relation between complete fuzzy graph and Double layered complete fuzzy graph is,

$$\text{DLCFG} (K_n) = K_n + \text{DLCFG} (K_{n-1})$$

$$\text{Number of edges (}E_{DL}\text{)} = \frac{n_{DL}(n_{DL} - 1)}{2}$$

$n_{DL}$ represents number of vertices in DLCFG

**CONCLUSION**

In this paper we have find a double layered complete fuzzy graph and illustrated with some examples. Further structures can be developed by increasing number of cycles. These structural patterns with the cycles gives to prove further into different patterns in networking models.

**REFERENCES**


