

Odd Region and Even Region of 3-Regular Planar Graph with its Application

Atowar ul Islam¹, Jayanta Kr. Choudhury², Anupam Dutta³ & Bichitra Kalita⁴

¹ *Department of Computer Science & IT, Cotton College
Guwahati, Assam-781001, India.*

² *Department of Mathematics, Swadeshi College of Commerce,
Guwahati, Assam-781007, India.*

³ *Department of Mathematics, APEX Professional University,
Pasighat, Arunachal Pradesh, India.*

⁴ *Department of Computer Application(MCA), Assam Engineering College,
Guwahati, Assam-781013, India.*

Abstract

In this article, construction for the structures of three regular planar graph from the graph $G(2m + 2, 3m + 3)$ for $m \geq 2$ are discussed and its numbers of even and odd regions are studied. A symmetric structure is discussed using structure-1. Also, an algorithm is developed. Finally an application is given in region base segmentation and also mentions some odd structure and even structure which is resembles with different structure.

Keywords: three regular planar graphs, even region, odd region, degree, algorithm.

INTRODUCTION

Various researchers have been working from different point of view. Boykov Yuri Y et al [1] discussed a new technique for general purpose interactive segmentation of N-

dimensional images. The user marks certain pixels as “object” or “background” to provide hard constraints for segmentation. Additional soft constraints incorporate both boundary and region information. Graph cuts are used to find the globally optimal segmentation of the N-dimensional image. The obtained solution gives the best balance of boundary and region properties among all segmentations satisfying the constraints. The topology of their segmentation is unrestricted and both “object” and “background” segments may consist of several isolated parts. Some experimental results are presented in the context of photo/video editing and medical image segmentation. They also demonstrate an interesting Gestalt example. Fast implementation of their segmentation method is possible via a new max-flow algorithm. Shirinivas S.G. et al [2] discuss the field of mathematics plays vital role in various fields. One of the important areas in mathematics is graph theory which is used in structural models. This structural arrangements of various objects or technologies lead to new inventions and modifications in the existing environment for enhancement in those fields. The field graph theory started its journey from the problem of Koinsberg bridge in 1735. This paper gives an overview of the applications of graph theory in heterogeneous fields to some extent but mainly focuses on the computer science applications that uses graph theoretical concepts. Dutta Anupam et al. [3] has discussed about regular planar sub graph of complete graph and their various application. First of all they have found some regular sub-graph of the complete graph which is obviously a planar. The minimum vertex cover of a class of regular planar sub-graph $H(2m+2, 3m+3)$, $K(2m+2, 4m+4)$ for $m \geq 2$ and $J(2m+2, 5m+5)$ for $m \geq 5$ obtained from the complete graph K_{2m+2} had already been discussed by kalita et.al [4]. An algorithm has been developed to find the minimum vertex cover of these types of regular planar sub-graph. Finally the application of minimum vertex cover has been found to reduce the power consumption of sensor network. Arya Sapna et al [5] proposed an efficient graph coloring algorithm for less number of coloring problems. The algorithm used in all types of graph. Using the algorithm they are dividing the vertices into two clusters. The two clusters are non-visited type of clusters which includes the non colored nodes and visited type of clusters which includes the colored nodes and finds maximum number of color that has been filled into visited nodes. Along with well known ECG algorithm, the proposed algorithm is implemented on random graphs and most of the cases the proposed algorithm provides better result and uses less number of colors as compare to ECG algorithm. Auvert Geoffroy et al [6] discussed that Lewis developed a 2D-representation of molecules, charged or uncharged, which known as structural formula, and stated the criteria to draw it. At the time, the huge majority of identified molecules followed the octet-rule, one of Lewis’s criteria. The same method was however quickly applied to represent compounds that do not follow the octet-rule.. In this article, with the objective to encompass all single-bonded ions in one group the even-odd rule is extended, these are Lewis’s ions, hypo- and hypervalent ions. The base of the even-odd representation is compatible with Lewis’s diagram. In addition, each atom is subscripted with an even number which is calculated by adding the valence number, the number of covalent bonds of the element, and its electrical charge. This article explains how to calculate the latter number and how charge and

electron-pairs can actually be accurately localized. Using ions known to be compatible with Lewis's rule of eight, the even-odd rule is compared with the former. The even-odd rule is then applied to ions known as hypo- or hypervalent. They also discussed the side effect of the presented rule. The side effect is due to charge and electron-pairs are unambiguously assigned to one of the atoms composing the single-charged ion. Ions that follow the octet rule and ions that do not, are thus reconciled in one group called "electron-paired ions" due to the absence of unpaired electrons. Tuvi-Arad Inbal [7] proposed that a website that helps students visualize and locate symmetry elements on three-dimensional molecular structures was developed. It includes textual explanations, an interactive example window and a Microsoft-Excel based symmetry toolkit that enables students to draw symmetry elements in three dimensions. Preliminary qualitative research aimed at exploring how students learned with this tool was performed. It was found that the three-dimensional graphical capabilities of the toolkit (1) helped students overcome difficulties in three-dimensional visualization, (2) enabled students to find symmetry elements of complex molecules generally not accessible from drawings and (3) contributed to a deeper understanding of molecular structure and chemical symmetry. Mainzer Klaus [8] proposed that molecules have more or less symmetric and complex structures which can be defined in the mathematical framework of topology, group theory, dynamical systems theory, and quantum mechanics. But symmetry and complexity are by no means only theoretical concepts of research. Modern computer aided visualizations show real forms of matter which nevertheless depend on the technical standards of observation, computation, and representation. Furthermore, symmetry and complexity are fundamental interdisciplinary concepts of research inspiring the natural sciences since the antiquity.

The paper is organized as follows – The section 1 includes the introduction which contains the works of other researcher. Section 2 includes the definition. Section 3 contains the construction of three structure and experimental results. Section 4 includes an algorithm. Section 5 includes Discussion. Section 6 includes the application. Section 7 includes the conclusion.

DEFINITION

Region: An area covered by a number of vertices with edges is known as a region. A region is two types –inner region and outer region. The inner region are two types – odd region and even region.

Odd Region: If a region is covered by odd number of vertices and odd number of edges than it is called odd region .Example

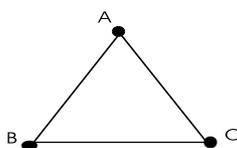


Figure 1: (bounded by odd edges)

Even Region: If a region covered by even number vertices and even number of edges, called even region. Example-

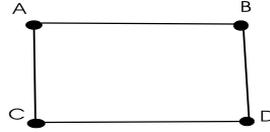


Figure 2: (bounded by even edges)

Construction of Three Regular Planar Graph: There are so many structure of three regular planar graph. In this paper we focus only three types of structure for discussing the odd region and even region of the graph.

OUR WORK:

For the graph $G(2m+2, 3m+3)$ where $m \geq 2$, we construct the structures-1, structure-2 and structure- 3 of three-regular planar graphs in the following ways-

Structure-1 :

Let G be a graph having $(2m+2)$ vertices and $3(m+1)$ edges for $m \geq 2$.

For $m=2$, G contains six vertices $\{v_1, v_2, v_3, v_4, v_5, v_6\}$ and nine edges. Let us join these six vertices by nine edges as follows:

$$\alpha(v_i) = \begin{cases} v_{i+1} & \text{for } 1 \leq i \leq 5, i \neq 3 \\ v_1 & \text{for } i = 3 \\ v_4 & \text{for } i = 6 \end{cases}$$

and $\alpha(v_j) = v_{j+3}$ for $1 \leq j \leq 3$

Then we have the edge set $\{v_1 v_2, v_2 v_3, v_4 v_5, v_5 v_6, v_3 v_1, v_6 v_4, v_1 v_4, v_2 v_5, v_3 v_6\}$ and we obtain a graph [Figure-3], which is planar and regular of degree three.

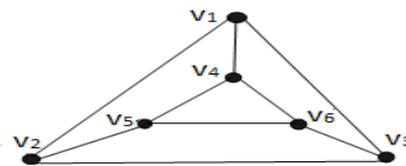


Figure 3: Three regular planar graph for $m=2$

From the above graph [Figure-3] we observed that it has two odd regions (each bounded by three edges) and three even regions (each bounded by four edges).

For $m=3$ vertex set is $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ i.e eight vertices and twelve edges. Let us join eight vertices with twelve edges by the following rule.

$$\alpha(v_i) = \begin{cases} v_{i+1} & \text{for } 1 \leq i \leq 7, i \neq 4 \\ v_1 & \text{for } i = 4 \\ v_5 & \text{for } i = 8 \end{cases}$$

and $\alpha(v_j) = v_{j+4}$ for $1 \leq j \leq 4$

Then we have the edge set $\{v_1v_2, v_2v_3, v_3v_4, v_5v_6, v_6v_7, v_7v_8, v_4v_1, v_8v_5, v_1v_5, v_2v_6, v_3v_7, v_4v_8\}$ and we obtain a graph [Figure-4], which is planar and regular of degree three.

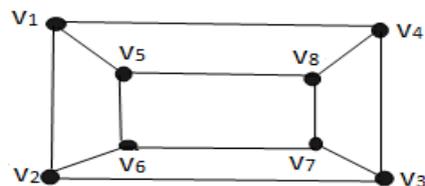


Figure 4: Three regular planar graph for m=3

We observed that the graph [Figure-4] having six even region, each bounded by four edges.

For m=4, vertex set is $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$ i.e ten vertices and fifteen edges. These fifteen edges can connect ten vertices as follows:

$$\alpha(v_i) = \begin{cases} v_{i+1} & \text{for } 1 \leq i \leq 9, i \neq 5 \\ v_1 & \text{for } i = 5 \\ v_6 & \text{for } i = 10 \end{cases}$$

and $\alpha(v_j) = v_{j+5}$ for $1 \leq j \leq 5$

Then the edges set is

$\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_6v_7, v_7v_8, v_8v_9, v_9v_{10}, v_5v_1, v_{10}v_6, v_1v_6, v_2v_7, v_3v_8, v_4v_9, v_5v_{10}\}$ and we obtain a graph [Figure-5] which is planar and regular of degree three.

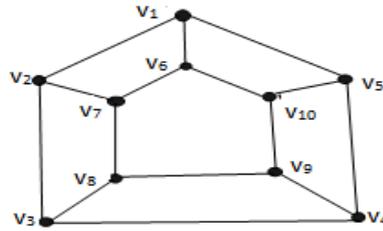


Figure 5: Three Regular planar graph for m=4

In this graph [Figure-5] there are two odd region (each bounded by five edges) and five even region (each bounded by four edges).

For m=5 the vertex set with twelve vertices is $\{ v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12} \}$ and number of edges is eighteen. We join the vertices by the following rule.

$$\alpha(v_i) = \begin{cases} v_{i+1} & \text{for } 1 \leq i \leq 11, i \neq 6 \\ v_1 & \text{for } i = 6 \\ v_7 & \text{for } i = 12 \end{cases}$$

and $\alpha(v_j) = v_{j+6}$ for $1 \leq j \leq 6$

Then the edge set becomes

$\{ v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_7v_8, v_8v_9, v_9v_{10}, v_{10}v_{11}, v_{11}v_{12}, v_6v_1, v_{12}v_7, v_1v_7, v_2v_8, v_3v_9, v_4v_{10}, v_5v_{11}, v_6v_{12} \}$ and we have the graph which is planar and regular of degree three in Figure-6.

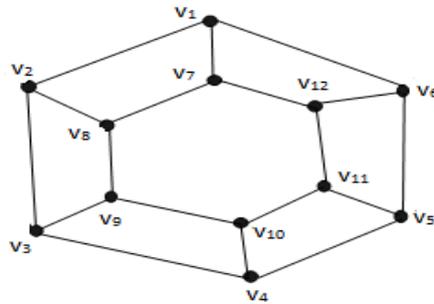


Figure 6: Three Regular planar graph for m=5

The above graph [Figure-6] contains eight even region two of which is bounded by six edges and remaining six region bounded by four edges.

Similarly we can construct regular planar graph of degree three having odd and even regions for $m=6,7,8,9$ ---and hence we can generalize the above cases by the following rule for constructing the graph.

For the graph G having $2m+2$ number of vertices and $3(m+1)$ edges for $m \geq 2$, we define

$\alpha: V_G \rightarrow V_G$ such that

$$\alpha(v_i) = \begin{cases} v_{i+1} & \text{for } 1 \leq i \leq 2m+1, i \neq m+1 \\ v_1 & \text{for } i = m+1 \\ v_{i-m} & \text{for } i = 2m+2 \end{cases}$$

and $\alpha(v_j) = v_{j+(m+1)}$ for $1 \leq j \leq m+1$

The experimental results of three regular planar graph $G(2m+2,3m+3)$ of structure-1 for different values of $m \geq 2$ are shown in Table-1.

Table -1: (Odd and Even region of three regular planar graph for structure-1)

A	B	C	D	E	F	G	H
2	G(6,9)	5	2	3	3	4	Fig-3
3	G(8,12)	6	Nil	6	Nil	4	Fig-4
4	G(10,15)	7	2	5	5	4	Fig-5
5	G(12,18)	8	Nil	8	Nil	Out of 8, six are bounded by 4 edges and two are bounded by 6 edges	Fig-6
6	G(14,21)	9	2	7	7	4	
7	G(16,24)	10	Nil	10	Nil	Out of 10, eight are bounded by 4 edges and 2 are bounded by 8 edges.	

The letters A, B, C, D, E, F, G and H appears in the 8 columns as shown in table-1, where A= Value of m, B= G(2m+2,3m+3), C= Total Region, D= Number of Odd

Region, E= Number of Even Region, F= Odd Region Covered by no of Edges,
 G= Even Region Covered by no of edges, H= Structure of Graph

Structure-2 :

Let G be a graph having $(2m+2)$ vertices and $3(m+1)$ edges for $m \geq 2$.

For $m=2$, G contains six vertices $\{v_1, v_2, v_3, v_4, v_5, v_6\}$ and nine edges. Let us join these six vertices by nine edges as follows:

$$\partial(v_i) = \begin{cases} v_{i+1} & \text{for } 1 \leq i \leq 5 \\ v_1 & \text{for } i = 6 \end{cases}$$

$$\partial(v_k) = \begin{cases} v_1 & \text{for } k = 5 \\ v_2 & \text{for } k = 4 \end{cases}$$

$$\partial(v_{j+2}) = v_{7-j} \quad \text{for } 1 \leq j \leq 1$$

Then we have the edge set $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_1v_6, v_1v_5, v_2v_4, v_3v_6\}$ and we obtain a graph [Figure-7], which is planar and regular of degree three.

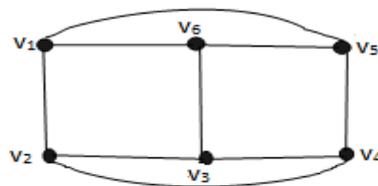


Figure 7: Three Regular planar graph for $m=2$

From the graph [Figure-7], we observed that it has two odd region (each bounded by three edges) and three even region (each bounded by four edges).

For $m=3$, vertex set is $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ i.e eight vertices and twelve edges. Let us join eight vertices with twelve edges by the following rule.

$$\partial(v_i) = \begin{cases} v_{i+1} & \text{for } 1 \leq i \leq 7 \\ v_1 & \text{for } i = 8 \end{cases}$$

$$\partial(v_k) = \begin{cases} v_1 & \text{for } k = 6 \\ v_2 & \text{for } k = 5 \end{cases}$$

$$\partial(v_{j+2}) = v_{9-j} \quad \text{for } 1 \leq j \leq 2$$

Then the edge set $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_7, v_7v_8, v_8v_1, v_1v_6, v_2v_5, v_3v_8, v_4v_7\}$ and we have the graph as shown in Figure-8, which is planar and regular of degree three.

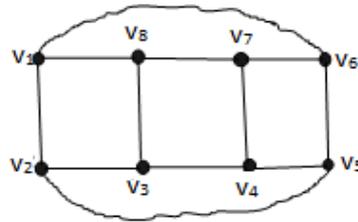


Figure 8: Three Regular planar graph for m=3

We observed that the graph in Figure-8 having six even region (each bounded by four edges) and there is no odd region.

For m=4, vertex set is $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$ i.e ten vertices and fifteen edges. The fifteen edges can connect ten vertices as follows:

$$\partial(v_i) = \begin{cases} v_{i+1} & \text{for } 1 \leq i \leq 9 \\ v_1 & \text{for } i = 10 \end{cases}$$

$$\partial(v_k) = \begin{cases} v_1 & \text{for } k = 7 \\ v_2 & \text{for } k = 6 \end{cases}$$

$$\partial(v_{j+2}) = v_{11-j} \quad \text{for } 1 \leq j \leq 3$$

Then the edges set is

$\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_7, v_7v_8, v_8v_9, v_9v_{10}, v_{10}v_1, v_1v_7, v_2v_6, v_3v_{10}, v_4v_9, v_5v_8\}$ and the graph is shown [Figure-9] which is planar and regular.

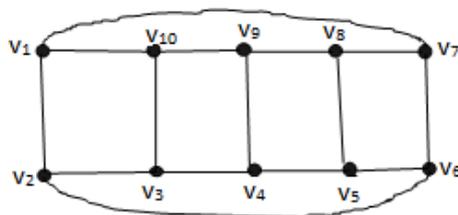


Figure 9: Three Regular planar graph for m=4

In this graph Figure-9 there are two odd regions (each bounded by five edges) and five even region (each bounded by four edges).

Similarly we can construct regular planar graph of degree three having odd and even regions for $m=5, 6, 7, 8, 9$ ---and hence we can generalize the above cases by the following rule.

For the graph G having $2m+2$ no of vertices and $3(m+1)$ edges for $m \geq 2$, we define

$\partial: V_G \rightarrow V_G$ such that

$$\begin{aligned} \partial(v_i) &= \begin{cases} v_{i+1} & \text{for } 1 \leq i \leq 2m+1 \\ v_1 & \text{for } i = 2m+2 \end{cases} \\ \partial(v_k) &= \begin{cases} v_1 & \text{for } k = m+3 \\ v_2 & \text{for } k = m+3-1 \end{cases} \\ \partial(v_{j+2}) &= v_{2m+3-j} \quad \text{for } 1 \leq j \leq (m-1) \end{aligned}$$

The experimental results of three regular planar graph $G(2m+2, 3m+3)$ of structure-2 for different values of $m \geq 2$ are shown in Table-2.

Table 2: (Odd and Even region of three regular planar graph for structure-2)

A	B	C	D	E	F	G	H
2	G(6,9)	5	2	3	3	4	Fig-7
3	G(8,12)	6	Nil	6	Nil	4	Fig-8
4	G(10,15)	7	2	5	5	4	Fig-9
5	G(12,18)	8	Nil	8	Nil	Out of 8, six are bounded by 4 edges and two are bounded by 6 edges	
6	G(14,21)	9	2	7	7	4	
7	G(16,24)	10	Nil	10	Nil	Out of 10, eight are bounded by 4 edges and 2 are bounded by 8 edges.	

The letters A , B, C, D, E, F, G and H appears in the 8 columns as shown in table-2, where A= Value of m , B= $G(2m+2, 3m+3)$, C= Total Region, D= Number of Odd Region, E= Number of Even Region, F= Odd Region Covered by no of Edges, G= Even Region Covered by no of edges, H= Structure of Graph

Structure-3 :

Let G be a graph having $(2m+2)$ vertices and $3(m+1)$ edges for $m \geq 2$.

For $m=2$, G contains six vertices $\{v_1, v_2, v_3, v_4, v_5, v_6\}$ and nine edges. Let us join these six vertices by nine edges as follows

$$\alpha(v_i) = \begin{cases} v_{i+1} & \text{for } 1 \leq i \leq 5 \\ v_1 & \text{for } i = 6 \end{cases}$$

$$\alpha(v_{j+1}) = v_{7-j} \quad \text{for } 1 \leq j \leq 2$$

$$\alpha(v_k) = v_1 \quad \text{for } k = 4$$

Then we have the edge set as $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_1v_6, v_1v_4, v_2v_6, v_3v_5\}$. Then we obtain a graph [Figure-10] which is planar and regular of degree three.

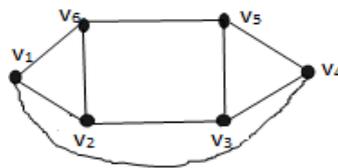


Figure 10: Three Regular planar graph for $m=2$

From the above graph in Figure-10, we observed that it has two odd region (each bounded by three edges) and three even region (each bounded by four edges).

For $m=3$, vertex set is $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ i.e eight vertices and twelve edges. Let us join eight vertices with twelve edges by the following rule.

$$\alpha(v_i) = \begin{cases} v_{i+1} & \text{for } 1 \leq i \leq 7 \\ v_1 & \text{for } i = 8 \end{cases}$$

$$\alpha(v_{j+1}) = v_{9-j} \quad \text{for } 1 \leq j \leq 3$$

$$\alpha(v_k) = v_1 \quad \text{for } k = 5$$

Then the edge set $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_7, v_7v_8, v_8v_1, v_1v_5, v_2v_8, v_3v_7, v_4v_6\}$ and we have the graph [Figure-11], which is planar and regular of degree three.

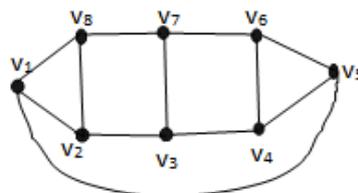


Figure 11: Three regular planar graph for $m=3$

We observed that the graph in Figure-11 having two even region , each bounded by four edges and three odd region and out of three region two is bounded by three edges and one is bounded by five edges .

For $m=4$, vertex set is $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$ i.e ten vertices and fifteen edges. The fifteen edges can connect ten vertices as follows:

$$\begin{aligned} \alpha(v_i) &= \begin{cases} v_{i+1} & \text{for } 1 \leq i \leq 9 \\ v_1 & \text{for } i = 10 \end{cases} \\ \alpha(v_{j+1}) &= v_{11-j} \quad \text{for } 1 \leq j \leq 4 \\ \alpha(v_k) &= v_1 \quad \text{for } k = 6 \end{aligned}$$

Then the edges set is $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_7, v_7v_8, v_8v_9, v_9v_{10}, v_{10}v_1, v_1v_7, v_2v_{10}, v_3v_9, v_4v_8, v_5v_7\}$ and we have the graph [Figure-12], which is planar and regular of degree three.

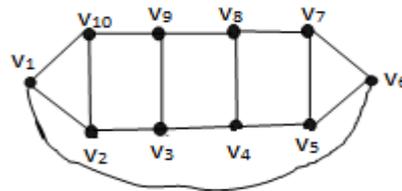


Figure 12: Three Regular planar graph for $m=4$

In this graph [Figure-12] there are two odd region , each bounded by three edges and five even region and out of five, three are bounded by four edges and remaining two are bounded by six edges.

Similarly we can construct regular planar graph of degree three having odd and even regions for $m=5,6,7,8,9$ ---and hence we can generalize the above cases by the following rule.

For the graph G having $2m+2$ no of vertices and $3(m+1)$ edges for $m \geq 2$, we define

$\alpha: V_G \rightarrow V_G$ such that

$$\begin{aligned} \alpha(v_i) &= \begin{cases} v_{i+1} & \text{for } 1 \leq i \leq 2m + 1 \\ v_1 & \text{for } i = 2m + 1 \end{cases} \\ \alpha(v_{j+1}) &= v_{2m+3-j} \quad \text{for } 1 \leq j \leq m \\ \alpha(v_k) &= v_1 \quad \text{for } k = m + 4 \end{aligned}$$

The experimental results of three regular planar graph $G(2m+2, 3m+3)$ of structure-3 for different values of $m \geq 2$ are shown in Table-3.

Table 3: (Odd and Even region of 3regular planar graph for structure-3)

A	B	C	D	E	F	G	H
2	G(6,9)	5	2	3	3	4	Fig-10
3	G(8,12)	6	4	2	Out of 4, two are covered by 3 edges and two are covered by 5 edges	4	Fig-11
4	G(10,15)	7	2	5	3	Out of 5, two are covered by 6 edges and 3 are covered by 4 edges.	Fig-12
5	G(12,18)	8	4	4	Out of 4, two are covered by 7 edges and 2 are covered by 3 edges.	4	
6	G(14,21)	9	2	7	3	Out of 7,two are covered by 8 edges and 5 are covered by 4dges.	
7	G(16,24)	10	4	6	Out of 4 two are covered by 9 edges and other two are covered by 3 edges	4	

The letters A , B, C, D, E, F, G and H appears in the 8 columns as shown in table-3, where A= Value of m, B= G(2m+2,3m+3), C= Total Region, D= Number of Odd Region, E= Number of Even Region, F= Odd Region Covered by no of Edges, G= Even Region Covered by no of edges, H= Structure of Graph

ALGORITHM:

Step-1: Start

Step-2: Take input graph G(2m+2,3m+3) which is regular and planar.

Step-3: For $m \geq 2$ we find different types of structure of graph which as

- a) Structure-1
- b) Structure -2
- c) Structure-3

Step-4: For different values of m in the same structure the graph will be different and R_g is also be different.(where R_g means regions of the graph)

Step-5: In structure-1,2,3, we find E_R and O_R for increasing the value of m where E_R and O_R means even regions and odd regions respectively and both $E_R, O_R \in R_g$.

Step-6: Stop.

DISCUSSION

From the construction of three regular planar graph by structure -1 and structure-2 from the graph $G(2m+2,3m+3)$ for $m \geq 2$, it has been found that there is no odd region if m is odd. There are $(m+3)$ even regions where 2 are bounded by $(m+1)$ edges, each and the remaining $(m+1)$ even regions are bounded by 4 edges each. But the existence of both even and odd regions are established when m is even. It has been found that 2(always) odd regions bounded by $m+1$ edges each and $(m+1)$ even regions bounded by 4 edges each. Further in the construction of three regular planar graph by structure-3, the experiment results shows the existence of both even and odd regions when m is odd. There are 4 (always) odd regions, where 2 are bounded by 3 edges each and 2 are bounded by $(m+2)$ edges each. Again number of even regions is $(m-1)$ each of which is bounded by 4 edges. Moreover if m is even than also it has been found out that both types of regions exists. In this case, there are 2 (always) odd regions, each bounded by 3 edges and $(m-1)$ even regions each bounded by 4 edges and the remaining 2 even regions are bounded by $(m+1)$ edges. It is noteworthy to mention that in our investigation, the 3-Regular planar graph by structure-1 satisfy the chemical structure of D_{nh} -group according to plane of symmetry. This is really a new light in this direction. The term symmetry implies a structure in which the parts are in harmony with each other, as well as to whole structure that is the structure is proportional as well as balanced. Symmetry elements are the geometrical elements like line, plane with respect to which one or more symmetric operations are carried out. The symmetry of a molecule can be described by 5 types of symmetry elements. The symmetry elements are symmetry axis, plane of symmetry, centre of symmetry or inversion center, rotation axis and identity.

Symmetry Operations: A molecule or object is said to possess a particular symmetry operation if the molecule retains the identical structure after the operation. Each operation is performed relative to a point, line, or plane - called a symmetry element. There are 5 kinds of operations - Identity, n-Fold axis Rotation, Reflection, Inversion, Improper n-Fold axis of symmetry, C_n is a rotation through $\frac{360^\circ}{n}$. A H_2O has one twofold axis C_2 . An NH_3 molecules has one threefold axis C_3 with which is associated two symmetry operations, being 120° rotation in clockwise sense and the other 120° rotation in anticlockwise sense. There is only one twofold rotation associate with a C_2 axis because clockwise and anticlockwise 180° rotations are identical. A pentagonal has a C_5 axis, with two rotations through 72° associated with clockwise and

anticlockwise rotations. Some structures of symmetry operation are given in figure - 15

In the Structure-1, Figure-3 has the different symmetry element which is explained bellow.

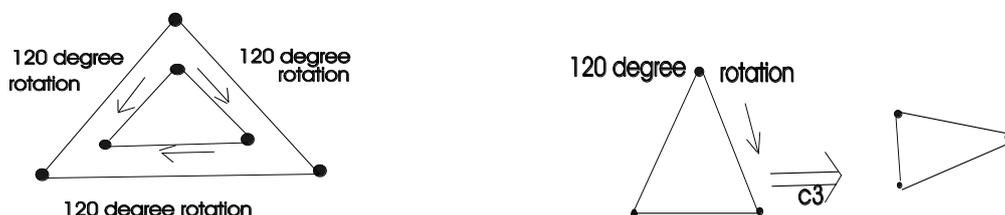


Figure-13: Rotation of Figure -3

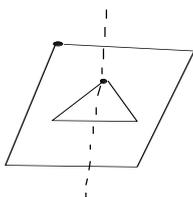
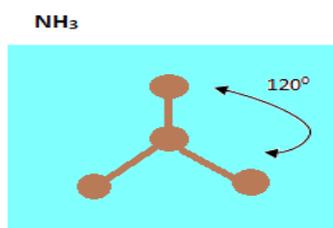
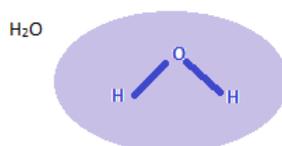


Figure 14: $3C_2 \perp C_3 \Rightarrow D_{3h}$

Some structures of the group of H_2O , NH_3 , rotational structure are shown as follows:



The symmetry element (SE) of figure 3 i.e $SE \Rightarrow E, C_3, 3C_2, \sigma_h, 3\sigma_v$. Here E means identity and this operation does not do anything to the molecule. In Figure-13 C_3 means 120° rotation. After 120° rotation the molecule retains its identical structure.

The plane is perpendicular to the principal rotation axis so it is a horizontal plane which is defined by σ_h reflection. So $3C_2 \perp C_3 \Rightarrow D_{3h}$. Accordingly the figure-4, figure-5 and figure-6 satisfy the condition of D_{4h} , D_{5h} and D_{6h} respectively, so that the 3-regular planar graph of structure-1 satisfy the chemical structure of D_{nh} -group. Moreover a column of algorithms for verification of our reported result.

APPLICATION

Region based segmentation used in various image processing. If an image contain different picture than we can be identified and colors the different picture as region wise.

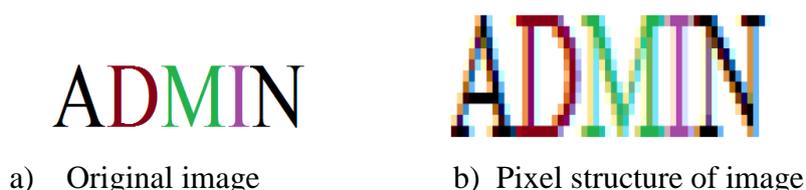


Figure 15

In the Figure 15 (a) contains original image of 'ADMIN' and 15(b) contains the pixel structure of 'ADMIN'. If we consider pixels to be a vertex of a graph then based on its color property we can find vertices that are connected. These connected vertices form a region and in this case the connected region is a set of pixels. These set of connected pixels form an isolated character image. Character segmentation from a word can be done using this region based segmentation technique. In the figure 15(b) connected pixels are colored with same color which is shown in Figure 15 (a).

In the Regular planar graph $G(2m + 2, 3m + 3)$ we find odd and even regions. We can found odd regions in odd structure and even region in even structure. Different figures of structure-1 resemble with roof of house, corridor etc and the structure-2 resemble with railway track and the structure-3 resemble with the structure of river boat.

CONCLUSION

The hypothesis of odd regions and even regions of some regular planar graphs give the rotation of symmetry operation about the principal axis. This will make a relationship between chemistry and mathematics, thereafter an application about the region coloring is given here. Consider that the words are the sums of some letters and generally a letter is covered by some odd and even regions as shown in figure 15(b), if we select every regions of a letter and using coloring properties then pixel color of any letter will give us original image without giving of high resolution of pixel structure of image. Anyone can use structure-1, structure-2 and structure -3 to find new application is a future study.

REFERENCES

- [1] Yuri Y. Boykov Marie-Pierre Jolly “Interactive Graph Cuts for Optimal Boundary & Region Segmentation of Objects in N-D Images” Proceedings of “International Conference on Computer Vision”, Vancouver, Canada, July 2001.
- [2] S.G.Shirinivas, S.Vetrivel, Dr. N.M.Elango “Application of graph theory in computer science an overview. International Journal of Engineering Science and Technology Vol. 2(9), 2010, 4610-4621.
- [3] Anupam Dutta, Bichitra Kalita and Hementa K.Baruah, “Regular Planar Sub-Graph of Complete Graph and their Application” International Journal of Applied Engineering Research ISSN 0973-4562 Volume 5 Number 3(2010) pp.377-386 © research India Publication.
- [4] Atowar ul Islam, Bichitra Kalita and Anupam Dutta ,“ Minimum Vertex Cover of Different Regular Planar Graph and Its Application” International Journal of Mathematical Archive-5(10), 2014, 175-184 Available online through www.ijma.info ISSN 2229 – 5046.
- [5] Sapna Arya, Manish Dixit, “A Clustered Based Novel Approach of Graph Coloring”, IOSR Journal of Computer Engineering (IOSR-JCE) e-ISSN: 2278-0661, p-ISSN: 2278-8727, Volume 16, Issue 6, Ver. II (Nov – Dec. 2014), PP 14-17
- [6] Geoffroy Auvert , “Chemical Structural Formulas of Single-Bonded Ions Using the “Even-Odd” Rule Encompassing Lewis’s Octet Rule: Application to Position of Single-Charge and Electron-Pairs in Hypo- and Hyper-Valent Ions with Main Group Elements”, Open Journal of Physical Chemistry, 2014, 4, 67-72 Published Online May 2014.
- [7] Inbal Tuvi-Arad^{*} and Paul Gorsky, “New visualization tools for learning molecular symmetry: a preliminary evaluation”, Chemistry Education Research and Practice, 2007, 8 (1), 61-72
- [8] Klaus Mainzer,” Symmetry and Complexity, Fundamental Concepts of Research in Chemistry, HYLE – An International Journal for the Philosophy of Chemistry, Vol. 3 (1997), 29-49.

