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Quasi-controlling of chaotic discrete dynamical systems

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Abstract

The aim of this study is to synchronize two phenomena described by two discrete chaotic systems although the dimensions are different in the sens that the first runing in the space and the other in the plane and vice versa. Because the synchronisation is incomplete we will launch a bi-control name or quasi-synchronization. The legitimacy of our choice of controller vectors is demonstrated by the trend towards zero errors using Lyapunov stability theory.

AMS subject classification: Primary 05C38, 15A15; Secondary 05A15, 15A18.
Keywords: Discrete dynamical system, synchronization, quasi-controlling, chaotic, lyapunov function.

1. Introduction

Often we find ourselves obliged to ensure the arrival of two phenomena which are described by two different discrete chaotic systems at the same time, and the word “time” here means the rank or iteration, because the systems are discrete. Furthermore, the two representative discrete dynamical systems are of different dimensions, which brings us to control a part of the phenomena, and this takes place in several research areas and life.

From a certain rank (time), we needed to couple (synchronize) the two systems that are not with the same number of components, for example one with two components and the other is with three components,and this has existence, such as a communication represented by a system of three dimensions, and a signal represented by a system of two dimentions which appeared at the same time, information-signal.

Various mathematical models of biological processe, physical processe, nonlinear optics, fluid dynamics, communication network, and chemical processe were defined using discrete-time dynamical systems. [9, 13, 15] However, more attentions were paid to the chaos synchronization in discrete-time dynamical systems [6–8, 12, 14, 16]. Various methods have been devloped for chaos synchronization in discrete-time dynamical systems such as active control, backstepping design and sliding mode control, etc. [6–8, 12, 14, 16].

The majority of the works of synchronization between two systems of different dimensions based on a function φ that increases or reduces the size of the synchronized system as appropriate, however, this method requires some analytical conditions on φ, which has been a major drawback to the use.

A simple vector controller to avoid difficulty is chosen, and based on Lyapunov stability theory, a new control design is proposed to guarantee synchronization: firstly, between the 2D Lozi map and the 3D generalized Hénon-like map, secondly, between the 3D generalized Hénon map and 2D Hénon map.

The last part of this paper is arranged as follows: simulation and finally, conclusion is given in Section 4.

2. Quasi-Synchronization betwen 3D Master System and 2D Slave System

Firstly, we try to synchronize the master system the 3D generalized Hénon map [2, 5], described in $\mathbb{R}^3$ by:

$$\begin{cases} x_1(k+1) = -0.1x_3(k) - x_2^2(k) + 1.76 \\ x_2(k+1) = x_1(k) \\ x_3(k+1) = x_2(k) \end{cases} \tag{1}$$

with the slave system wich is the Hénon map [2, 5], described in $\mathbb{R}^2$ by:

$$\begin{cases} y_1(k+1) = y_2(k) - ay_1^2(k) + 1 + u_1, \\ y_2(k+1) = by_1(k) + u_2, \end{cases} \tag{2}$$

where $(a, b) = (1.4, 0.3)$ and $(u_{1,}u_2)$ is the vector controller.

As we remark, the first system play in the space with three components, and the second play in the plane. We define the quasi-synchronization errors by

$$\begin{cases} e_1(k) = y_1(k) - x_1(k) \\ e_2(k) = y_2(k) - x_2(k) \end{cases} \tag{3}$$

The synchronization errors between master system (1) and slave system (2), can be derived as:

$$\begin{cases} e_1(k+1) = y_2(k) - a y_1^2(k) + 0.1 x_3(k) + x_2^2(k) - 0.76 + u_1 \\ e_2(k+1) = b y_1(k) - x_1(k) + u_2 \end{cases} \tag{4}$$

To attain synchronization between systems (1) and (2), we can choose the vector controller U as follow:

$$\begin{cases} u_1 = -\frac{1}{2} y_2(k) - \frac{1}{2} x_2(k) + a y_1^2(k) - 0.1 x_3(k) - x_2^2(k) + 0.76 \\ u_2 = (1-b) x_1(k) \end{cases} \tag{5}$$

Then, the sysnchronization errors between systems (1) and (2), simplifed as:

$$\begin{cases} e_1(k+1) = \frac{1}{2} e_2(k) \\ e_2(k+1) = b e_1(k) \end{cases} \tag{6}$$

We Consider the candidate Lyapunov function to study the stability of synchronization errors:

$$V(e(k)) = \sum_{i=1}^{3} e_i^2(k), \tag{7}$$

we obtain:

$$\begin{aligned} \Delta V(e(k)) &= \sum_{i=1}^{3} e_i^2(k+1) - \sum_{i=1}^{3} e_i^2(k) \\ &= \frac{1}{4} e_2^2(k) + b^2 e_1^2(k) - e_1^2(k) - e_2^2(k) \\ &= \left(b^2 - 1\right) e_1^2(k) - \frac{1}{4} e_2^2(k) < 0. \end{aligned}$$

then, by Lyapunov stability it is immediate that

$$\lim_{k\to\infty} e_i(k) = 0, \quad (i = 1,\ 2). \tag{8}$$

we conclude that the systems (1) and (2) are quasi-synchronized as showing in figure (1) and figure (2).

3. Quasi-Synchronization between 2D master system and 3D slave system

Secondly, we consider the master system wich is the Lozi map [1, 3–5], described in $\mathbb{R}^2$ by:

$$\begin{cases} x_1(k+1) = x_2(k) \\ x_2(k+1) = 1 + x_1(k) - a\,|x_2(k)| \end{cases} \tag{9}$$

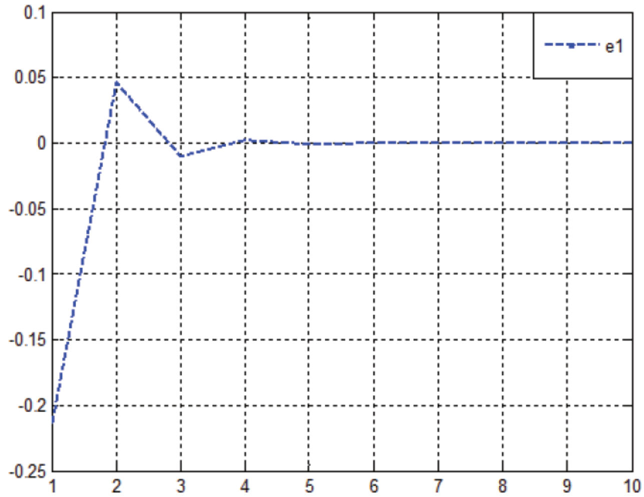

Figure 1: Synchronization error e1.

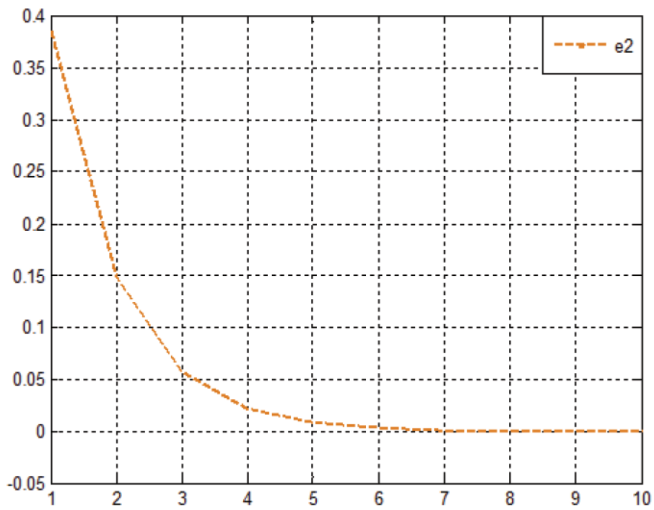

Figure 2: Synchronization error e2.

and the slave system wich is the 3D generalized Hénon-like map [2, 5], described in $\mathbb{R}^3$ by:

$$\begin{cases} y_1(k+1) = 1 + y_3(k) - \alpha y_2^2(k) + u \\ y_2(k+1) = 1 + \beta y_2(k) - \alpha y_1^2(k) + u_2 \\ y_3(k+1) = \beta y_1(k) + u_3 \end{cases} \tag{10}$$

where $(\alpha, \beta) = (1.4, 0.2)$ and $(u_{1,} u_2)$ is the vector controller.

The synchronization errors are defined by

$$\begin{cases} e_1(k) = y_1(k) - x_1(k) \\ e_2(k) = y_2(k) - x_2(k) \\ e_3(k) = y_3(k) - x_1(k) \end{cases} \tag{11}$$

As we remark, the third error is arbitrary, then the synchronization errors between master system (1) and slave system (2), can be derived as:

$$\begin{cases} e_1(k+1) = 1 + y_3(k) - \alpha y_2^2(k) - x_2(k) + u_1 \\ e_2(k+1) = \beta y_2(k) - \alpha y_1^2(k) - x_1(k) + a\,|x_2(k)| + u_2 \\ e_3(k+1) = \beta y_1(k) - x_2(k) + u_3 \end{cases} \tag{12}$$

To ensure synchronization between systems (1) and (2), we can choose the vector controller U as follow:

$$\begin{cases} u_1 = -\frac{1}{2} y_3(k) + x_2(k) - \frac{1}{2} x_3(k) + \alpha y_2^2(k) - 1 \\ u_2 = -\beta x_2(k) + \alpha y_1^2(k) + x_1(k) - a\,|x_2(k)| \\ u_3 = -\beta x_1(k) + x_2(k) \end{cases} \tag{13}$$

The sysnchronization errors between systems (1) and (2), ca be written as:

$$\begin{cases} e_1(k+1) = \frac{1}{2} e_3(k) \\ e_2(k+1) = \beta e_2(k) \\ e_3(k+1) = \beta e_1(k) \end{cases} \tag{14}$$

To study the stability of synchronization errors, we Consider the candidate Lyapunov function:

$$V(e(k)) = \sum_{i=1}^{3} e_i^2(k), \tag{15}$$

We obtain:

$$\begin{aligned} \Delta V(e(k)) &= \sum_{i=1}^{3} e_i^2(k+1) - \sum_{i=1}^{3} e_i^2(k) \\ &= \frac{1}{4} e_3^2(k) + \beta^2 e_2^2(k) + \beta^2 e_1^2(k) - e_1^2(k) - e_2^2(k) - e_3^2(k) \\ &= \left(\beta^2 - 1\right) e_1^2(k) + \left(\beta^2 - 1\right) e_1^2(k) - \frac{3}{4} e_3^2(k) < 0 \end{aligned}$$

Thus, by Lyapunov stability it is immediate that errors tend to zero at infinity

$$\lim_{k \to \infty} e_i(k) = 0, \quad (i = 1,\ 2, 3). \tag{16}$$

we conclude that the systems (1) and (2) are quasi-synchronized as showing in figure (3).

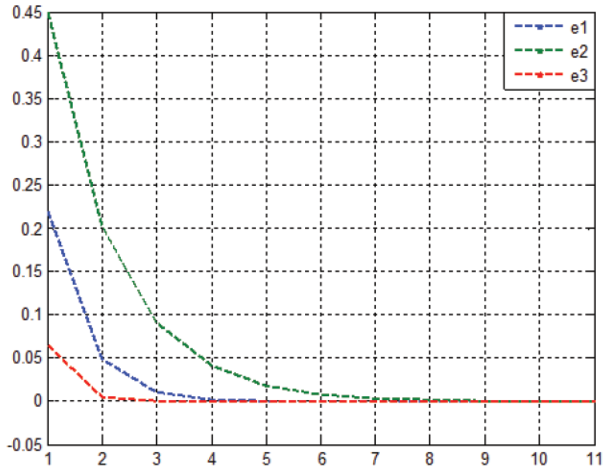

Figure 3: Synchronization errors: e1, e2, e3.

4. Conclusion

In this paper, we analysed the problem of synchronization for chaotic dynamical systems in discrete-time with different dimensions. A new control method was proposed and numerical simulations were given to show the effectiveness of the proposed scheme.We couple two phenomena, which are represented by two dynamical systems having different dimensions, and more with different topological properties, and different bifurcation structure, we show that the errors are tend to zero, and finally the graphs shows the convergence of errors to zero. Because the dimensions are differet, we propose the name of quasi-cotrolling on this type of synchronization.

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