Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 5 (2016), pp. 4299-4315

http://www.ripublication.com/gjpam.htm

Application of Fuzzy $\tilde{\bar{\bar{X}}} - \tilde{S}$ Charts for Solder Paste Thickness

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Abstract

Control chart, one of the seven problem solving tools in statistical process control (SPC) is very popular technique in improving productivity, preventing defects and avoid purposeless process adjustment. Traditional control charts of $\bar{X} - S$ were introduced by Shewhart that used to detect whether assignable causes are exist. In SPC, an early assumption for traditional control charts where the sample observations must be independent and process observation must follow a normal distribution is needed. However, real world data or problems are too complicated to handle and the difficulty involves with the level of uncertainty. Uncertainty is a part of many situations in real life. Uncertainty can come from human, measurement devices or environmental conditions. The problem in using control chart will arise when uncertain data and not precise data present. Since the fuzzy set theory's concept can dealt with uncertainty, the control limits of $\bar{X} - S$ control chart are transformed into fuzzy control limits. In this paper, the fuzzy $\tilde{\bar{\bar{X}}} - \tilde{S}$ control charts using α-level fuzzy midrange transformation are applied to solder paste thickness data. The control charts performance can be measured by average run length (ARL). It is found that fuzzy $\tilde{\bar{\bar{X}}} - \tilde{S}$ charts are better than traditional $\bar{X} - S$ control charts in monitoring the product quality with lower value of average run length.

Keywords: Fuzzy $\tilde{\bar{\bar{X}}} - \tilde{S}$ control chart, $\bar{X} - S$ control chart, α-cuts, average run length (ARL).

INTRODUCTION

Control chart was first discovered by Walter A. Shewhart in 1920s. It is a very popular problem solving tool used as it have been proved in improving productivity, preventing defects and avoid purposeless process adjustment. Besides, control chart can also provide diagnostic information to allow the modification in the process that can improves its performance and information of process capability. Control chart is also a kind of quality characteristic's graphic presentation that was calculated from a sample against the sample number of time that used to study how process is changing from time to time and data are plotted in time sequence. It has a centre line and two horizontal lines which are upper and lower control limits to detect whether process is stable or not [1].

Moreover, control charts can be used when controlling continuous processes by correcting any issues that occurred. It also can be considered to ensure the process is stable. This means that the process shows sign of in statistical control state. Apart from that, control chart also useful in analyzing patterns of process variation and determine if there are needs to deter specific issues or to make transformation to the process in providing better quality.

The fuzzy theory was first proposed in 1965 by Zadeh. It is a mathematical tool that deals with uncertainty which comes from shortage of information, incompleteness, vagueness and inaccurate of measurements. The fuzzy set theory utilizes fuzzy set membership's concept. It describes the ambiguity of an event. It counts the level to which a situation is happening or not. This can be said that, either a situation happened is "random" whereas to what level it happened is fuzzy [2]. It used concept of degree of truth which ones cannot choose and say clearly whether an event occurred or not such as how much variable is in a set.

Fuzzy data is a natural kind of data, such as inaccurate data or uncertainty data that is not due to randomness. This type of data is common in natural language, social science, psychometrics, environments and others [3]. Triangular fuzzy numbers is used to represent fuzzy data and to model fuzziness of data. Fuzzy theory uses the whole interval between 0 and 1 to describe human reasoning. Real world problems or circumstances are too complicated to handle and the complication involves with the level of uncertainty. Uncertainty is a part of many situations in real life. Fuzzy logic can be found in various fields like engineering's control systems, power engineering, robotics, electronics and so on [4].

There are quite considerable studies were carried out to merge statistical methods with fuzzy sets theory as fuzzy logic has very broad application areas. Statistical process control is one of the area that developed solutions with fuzzy theory such as fuzzy control charts. The advantage of using fuzzy control chart is that it does not need to fulfill assumption of normality, different with traditional control chart which need typical assumption to build a control chart where the data or measurements are distributed as normal distribution. Fuzzy control charts is also useful in monitoring process when vagueness and uncertainty arise since the concept of fuzzy theory can deal with uncertain data. The aim of this study is to apply the traditional $\bar{X} - S$ control charts and fuzzy $\tilde{\bar{\bar{X}}} - \tilde{S}$ control charts using α-cuts to solder paste thickness data. The control charts performance can be measured by average run length (ARL) to compare the effectiveness between traditional $\bar{X} - S$ charts and fuzzy $\tilde{\bar{\bar{X}}} - \tilde{S}$ charts.

RESEARCH METHODOLOGY

In this research, the data were obtained from one of manufacturing industry in Malaysia. It is on solder paste thickness which the solder paste is used to joint electrical components on printed circuit board (PCB). The data measurement is measured by using high precision equipment and slow process. There were 50 sets of sample for each containing 5 observations on the quality characteristic chosen which is the solder paste thickness. In reality, this 50 × 5 = 250 is enough because the truth behind the sampling and cost of inspection related with variables measurements are normally quite large.

Traditional Control Charts

When deal with variable quality characteristics, it is essential to keep an eye on value of mean and quality characteristics' variability. Controlling mean of a process is often done using control charts for process average which is known as $\bar{X}$ control chart whereas the process variability can be monitored with a control chart for standard deviation (S chart) [1]. The assumption for constructing control chart is that data should be distributed as normal distribution. Montgomery has outlined the methodology of constructing $\bar{X}$-S control charts [1].

A quality data are supposed to be distributed as normal distribution with mean μ and standard deviation σ, where both μ and σ are known. If $x_1, x_2, \dots, x_q$ is a sample size q, then the average of this sample is:

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_q}{q} \tag{1}$$

Supposed that there are p samples, each sample contain q observations on the quality characteristics. Let $\bar{x}_1 + \bar{x}_2 + \dots + \bar{x}_q$ be the average of each sample. Then, the process average is given by:

$$\bar{\bar{X}} = \frac{\bar{x}_1 + \bar{x}_2 + \dots + \bar{x}_q}{q} \tag{2}$$

Parameters μ and σ are normally unknown in real life, $\bar{\bar{X}}$ is used as an estimator of μ and $\frac{\bar{R}}{d_2}$ as an estimator of σ [5]. The traditional $\bar{X}$ control chart based on standard deviation can be defined as following equations.

$$UCL = \bar{\bar{X}} + A_3\bar{S} \quad (3)$$

$$CL = \bar{\bar{X}} \quad (4)$$

$$LCL = \bar{\bar{X}} - A_3\bar{S} \quad (5)$$

where A_3 is the coefficient's of control chart and $\bar{S}$ can be defined from equation (6) and (7) below:

$$S_l = \sqrt{\frac{\sum_{k=1}^{q}\left(X_{kl} - \bar{X}_l\right)^2}{q-1}}, \quad (6)$$

$$\bar{S} = \frac{\sum_{l=1}^{p} S_l}{p}, \quad (7)$$

where S_l is the standard deviation of a sample *l* and $\bar{S}$ is the average of S_l.
The traditional *S* control chart can be calculated by the following equation:

$$UCL = B_4\bar{S} \quad (8)$$

$$CL = \bar{S} \quad (9)$$

$$LCL = B_3\bar{S} \quad (10)$$

where constants of B_4 and B_3 are the coefficient of control chart.

Fuzzy Control Charts For $\tilde{\bar{\bar{X}}}$ - $\tilde{S}$

Senturk and Erginel have outlined the methodology for constructing fuzzy control chart for $\tilde{\bar{\bar{X}}}$-$\tilde{S}$ [6]. Triangular fuzzy number (*d*,*e*,*f*) as in Figure 1 are assigned to each sample or subgroup (*p*) of the fuzzy control chart. The triangular fuzzy numbers are assigned by (X_d, X_e, X_f) for each fuzzy observation. Each observation was fuzzified as triangular fuzzy number with consideration of operators and gauge variability as in Table 1 below.

Table 1: Fuzzification of observations

X_d (Left)	X_e	X_f (Right)
X_e-(0 to 1.2)% × X_e	X_e	X_e+(0 to 1.2)% × X_e

Random numbers are generated between 0 to 1.2 for both left and right triangular fuzzy number for each observation x_i. Left triangular fuzzy number will take a range about X_e-(0 to 1.2)% × X_e while right triangular fuzzy number will take a range about X_e+(0 to 1.2)% × X_e. Figure 1 shows the graph for sample's transformation from crisp set to fuzzy set.

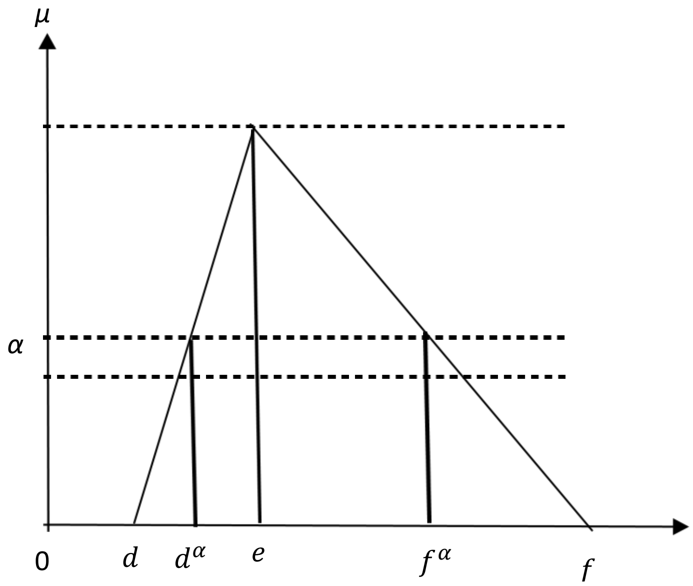


Figure 1: Graph for sample's transformation using triangular fuzzy numbers

The center line of the chart, $\widetilde{CL}$ is the arithmetic mean of fuzzy samples, and is shown as $(\bar{\bar{X}}_d, \bar{\bar{X}}_e, \bar{\bar{X}}_f)$ where $\bar{\bar{X}}_d, \bar{\bar{X}}_e, \bar{\bar{X}}_f$ are called general means as defined below:

$$\bar{X}_{jl} = \frac{\sum_{k=1}^{q} X_{jl}}{q}; \qquad j = d, e, f;\ \ k = 1,2, \dots, q;\ \ l = 1,2, \dots, p. \tag{11}$$

$$\bar{\bar{X}}_{j} = \frac{\sum_{l=1}^{p} \bar{X}_{jl}}{p}; \qquad j = d, e, f;\ l = 1,2, \dots, p \tag{12}$$

$$C\tilde{L} = \left(\bar{\bar{X}}_d, \bar{\bar{X}}_e, \bar{\bar{X}}_f\right) = \left(\frac{\sum_{l=1}^{p} \bar{X}_{dl}}{p}, \frac{\sum_{l=1}^{p} \bar{X}_{el}}{p}, \frac{\sum_{l=1}^{p} \bar{X}_{fl}}{p}\right). \tag{13}$$

where:
q = fuzzy sample size
n = number of fuzzy samples
$C\tilde{L}$ = centre line for fuzzy $\tilde{\bar{\bar{X}}}$ control chart.

There are four techniques of fuzzy transformation which are fuzzy median, fuzzy mode, fuzzy average and α-level fuzzy midrange [7]. The α-level fuzzy midrange transformation technique is used in this study to construct control charts of $\tilde{\bar{\bar{X}}}$ - $\tilde{R}$ and $\tilde{\bar{\bar{X}}}$ - $\tilde{S}$ by converting fuzzy sets into scalar. The α-level fuzzy midrange is the midpoint of the ends of α-level cuts denoted by A^α where it is a non fuzzy set which consist of all elements that has membership greater than or equal to α [6]. If d^α and f^α are the endpoints of A^α, then,

$$f_{mr}^{\alpha} = \frac{1}{2}\left(d^{\alpha} + f^{\alpha}\right) \tag{14}$$

The α-level fuzzy midrange of sample j is defined as follow:

$$S_{mr,l}^{\alpha} = \frac{\left(d_l + f_l\right) + \alpha\left[\left(e_l + d_l\right) - \left(f_l + e_l\right)\right]}{2} \tag{15}$$

Fuzzy $\tilde{\bar{\bar{X}}}$ control charts based on standard deviation

The control limits of fuzzy $\tilde{\bar{\bar{X}}}$ charts based on standard deviation are calculated as equation (16), (17) and (18) where the upper and lower control limits is represented by triangular fuzzy number.

$$\begin{aligned} U\tilde{C}L = C\tilde{L} + A_2\bar{S} &= \left(\bar{\bar{X}}_d, \bar{\bar{X}}_e, \bar{\bar{X}}_f\right) + A_3\left(\bar{S}_d, \bar{S}_e, \bar{S}_f\right) \\ &= \left(\bar{\bar{X}}_d + A_3\bar{S}_d, \bar{\bar{X}}_e + A_3\bar{S}_e, \bar{\bar{X}}_f + A_3\bar{S}_f\right) \end{aligned} \tag{16}$$

$$C\tilde{L} = \left(\bar{\bar{X}}_d, \bar{\bar{X}}_e, \bar{\bar{X}}_f\right) \tag{17}$$

$$\begin{aligned} L\tilde{C}L = C\tilde{L} - A_3\bar{S} &= \left(\bar{\bar{X}}_d, \bar{\bar{X}}_e, \bar{\bar{X}}_f\right) - A_3\left(\bar{S}_d, \bar{S}_e, \bar{S}_f\right) \\ &= \left(\bar{\bar{X}}_d - A_3\bar{S}_f, \bar{\bar{X}}_e - A_3\bar{S}_e, \bar{\bar{X}}_f - A_3\bar{S}_d\right) \end{aligned} \tag{18}$$

The fuzzy $\tilde{S}_l$ is a standard deviation of sample l and can be determined as equation (19) below:

$$\tilde{S}_l = \sqrt{\frac{\sum_{k=1}^{q}\left[\left(X_d, X_e, X_f\right)_{kl} - \left(\bar{X}_d, \bar{X}_e, \bar{X}_f\right)_l\right]^2}{q-1}}, \tag{19}$$

The fuzzy average $\tilde{\bar{S}}$ represented by triangular fuzzy number is calculated using standard deviation as below:

$$\tilde{\bar{S}} = \left(\frac{\sum_{l=1}^{p} S_{dl}}{p}, \frac{\sum_{l=1}^{p} S_{el}}{p}, \frac{\sum_{l=1}^{p} S_{fl}}{p}\right) = \left(\bar{S}_d, \bar{S}_e, \bar{S}_f\right). \tag{20}$$

The α-cut fuzzy $\tilde{\bar{\bar{X}}}$ control chart limits based on standard deviation can be determined as the following equations:

$$U\tilde{C}L^{\alpha} = \left(\bar{\bar{X}}_d^{\alpha}, \bar{\bar{X}}_e, \bar{\bar{X}}_f^{\alpha}\right) + A_3\left(\bar{S}_d^{\alpha}, \bar{S}_e, \bar{S}_f^{\alpha}\right) = \left(\bar{\bar{X}}_d^{\alpha} + A_3\bar{S}_d^{\alpha}, \bar{\bar{X}}_e + A_3\bar{S}_e, \bar{\bar{X}}_f^{\alpha} + A_3\bar{S}_f^{\alpha}\right) \tag{21}$$

$$C\tilde{L}^{\alpha} = \left(\bar{\bar{X}}_d^{\alpha}, \bar{\bar{X}}_e, \bar{\bar{X}}_f^{\alpha}\right) \tag{22}$$

$$L\tilde{C}L^{\alpha} = \left(\bar{\bar{X}}_d^{\alpha}, \bar{\bar{X}}_e, \bar{\bar{X}}_f^{\alpha}\right) - A_3\left(\bar{S}_d^{\alpha}, \bar{S}_e, \bar{S}_f^{\alpha}\right) = \left(\bar{\bar{X}}_d^{\alpha} - A_3\bar{S}_f^{\alpha}, \bar{\bar{X}}_e - A_3\bar{S}_e, \bar{\bar{X}}_f^{\alpha} - A_3\bar{S}_d^{\alpha}\right) \tag{23}$$

where,

$$\bar{\bar{X}}_d^{\alpha} = \bar{\bar{X}}_d + \alpha\left(\bar{\bar{X}}_e - \bar{\bar{X}}_d\right) \tag{24}$$

$$\bar{\bar{X}}_f^{\alpha} = \bar{\bar{X}}_f + \alpha\left(\bar{\bar{X}}_f - \bar{\bar{X}}_e\right) \tag{25}$$

$$\bar{S}_d^{\alpha} = \bar{S}_d + \alpha\left(\bar{S}_e - \bar{S}_d\right), \tag{26}$$

$$\bar{S}_f^{\alpha} = \bar{S}_f + \alpha\left(\bar{S}_f - \bar{S}_e\right). \tag{27}$$

The control limits and centre line for α-cut fuzzy $\tilde{\bar{\bar{X}}}$ control chart based on α-level fuzzy midrange are as follow:

$$U\tilde{C}L_{mr}^{\alpha} = C\tilde{L}_{mr}^{\alpha} + A_3\left(\frac{\bar{S}_d^{\alpha} + \bar{S}_f^{\alpha}}{2}\right), \tag{28}$$

$$C\tilde{L}_{mr}^{\alpha} = f_{mr}^{\alpha}\left(C\tilde{L}\right) = \left(\frac{CL_1^{\alpha} + CL_3^{\alpha}}{2}\right), \tag{29}$$

$$L\tilde{C}L_{mr}^{\alpha} = C\tilde{L}_{mr}^{\alpha} - A_3\left(\frac{\bar{S}_d^{\alpha} + \bar{S}_f^{\alpha}}{2}\right). \tag{30}$$

The definition of α-level fuzzy midrange of sample l for fuzzy $\tilde{\bar{\bar{X}}}$ control chart is as equation (31).

$$S_{mr,l}^{\alpha} = \frac{\left(\bar{X}_{dl} + \bar{X}_{fl}\right) + \alpha\left[\left(\bar{X}_{el} + \bar{X}_{dl}\right) - \left(\bar{X}_{fl} + \bar{X}_{el}\right)\right]}{2} \tag{31}$$

Then, the condition of process control for each sample can be defined as:

$$\text{Process control} = \begin{cases} \text{in control} & \text{for } L\tilde{C}L_{mr}^{\alpha} \le S_{mr,l}^{\alpha} \le U\tilde{C}L_{mr}^{\alpha} \\ \text{out of control} & \text{for otherwise} \end{cases}. \tag{32}$$

The α-level value was chosen according to the nature of production process which is defined as 0.65 [6].

Fuzzy $\tilde{S}$ control chart

Fuzzy $\tilde{S}$ control chart is defined about the same manner with traditional *S* control charts. However they are constructed using triangular fuzzy numbers approach as below:

$$U\tilde{C}L = B_4\bar{S} = B_4\left(\bar{S}_d, \bar{S}_e, \bar{S}_f\right) = \left(B_4\bar{S}_d, B_4\bar{S}_e, B_4\bar{S}_f\right) \quad (33)$$

$$C\tilde{L} = \bar{S} = \left(\bar{S}_d, \bar{S}_e, \bar{S}_f\right) \quad (34)$$

$$L\tilde{C}L = B_3\bar{S} = B_3\left(\bar{S}_d, \bar{S}_e, \bar{S}_f\right) = \left(B_3\bar{S}_d, B_3\bar{S}_e, B_3\bar{S}_f\right) \quad (35)$$

The control limits for α-Cut fuzzy $\tilde{S}$ control charts can be defined as below:

$$U\tilde{C}L^{\alpha} = B_4\bar{S}^{\alpha} = B_4\left(\bar{S}_d^{\alpha}, \bar{S}_e, \bar{S}_f^{\alpha}\right) = \left(B_4\bar{S}_d^{\alpha}, B_4\bar{S}_e, B_4\bar{S}_f^{\alpha}\right) \quad (36)$$

$$C\tilde{L}^{\alpha} = \bar{S}^{\alpha} = \left(\bar{S}_d^{\alpha}, \bar{S}_e, \bar{S}_f^{\alpha}\right) = \left(C\tilde{L}_1^{\alpha}, C\tilde{L}_2^{\alpha}, C\tilde{L}_3^{\alpha}\right) \quad (37)$$

$$L\tilde{C}L^{\alpha} = B_3\bar{S}^{\alpha} = B_3\left(\bar{S}_d^{\alpha}, \bar{S}_e, \bar{S}_f^{\alpha}\right) = \left(B_3\bar{S}_d^{\alpha}, B_3\bar{S}_e, B_3\bar{S}_f^{\alpha}\right) \quad (38)$$

Control limits of α-Level fuzzy midrange for α-Cut fuzzy $\tilde{S}$ control charts can be defined as the following equations:

$$U\tilde{C}L_{mr}^{\alpha} = B_4 f_{mr}^{\alpha}\left(C\tilde{L}\right), \quad (39)$$

$$C\tilde{L}_{mr}^{\alpha} = f_{mr}^{\alpha}\left(C\tilde{L}\right) = \left(\frac{\bar{S}_d^{\alpha} + \bar{S}_f^{\alpha}}{2}\right), \quad (40)$$

$$L\tilde{C}L_{mr}^{\alpha} = B_3 f_{mr}^{\alpha}\left(C\tilde{L}\right). \quad (41)$$

The definition of α-level fuzzy midrange of sample l for fuzzy $\tilde{S}$ control chart is as follow:

$$S_{mr,l}^{\alpha} = \frac{\left(S_{dl} + S_{fl}\right) + \alpha\left[\left(S_{el} + S_{dl}\right) - \left(S_{fl} + S_{el}\right)\right]}{2} \quad (42)$$

Then, the condition of process control for each sample can be defined as:

$$\text{Process control} = \begin{Bmatrix} \text{in control} & \text{for } L\tilde{C}L_{mr}^{\alpha} \le S_{mr,l}^{\alpha} \le U\tilde{C}L_{mr}^{\alpha} \\ \text{out of control} & \text{for otherwise} \end{Bmatrix} \quad (43)$$

Control Chart Performance

The most suitable or the best control charts are referring to the least value of average run length (ARL). ARL is the average number of points that must be plotted on the chart before a point indicates an out-of-control condition [6]. For Shewhart control charts, if process observations are uncorrelated, the ARL can be determined as follow:

$$ARL = \frac{1}{p} \tag{44}$$

where p is the probability that any point exceeds the control limits or for the in control ARL can be defined by:

$$ARL_0 = \frac{1}{\alpha} \tag{45}$$

and for out of control ARL can be defined by:

$$ARL_1 = \frac{1}{1-\beta} \tag{46}$$

For a control chart with control limits of UCL and LCL, the probability of type II error can be defined as

$$\beta = \Pr\left(LCL \le \omega \le UCL \middle| \mu = \mu_0 + k\sigma\right) \tag{47}$$

where k is the shift.

RESULTS AND DISCUSSION

The aim of this study is to measure the performance between traditional $\bar{X} - S$ charts and fuzzy $\tilde{\bar{\bar{X}}} - \tilde{S}$ charts using solder paste thickness data. The quality characteristic that was examined is the thickness of solder paste used to joint electrical components onto PCB. 50 samples with sample size of 5 were drawn from the production process. These samples were used to construct $\bar{X}$-S control charts.

Traditional $\bar{X} - S$ Control Charts

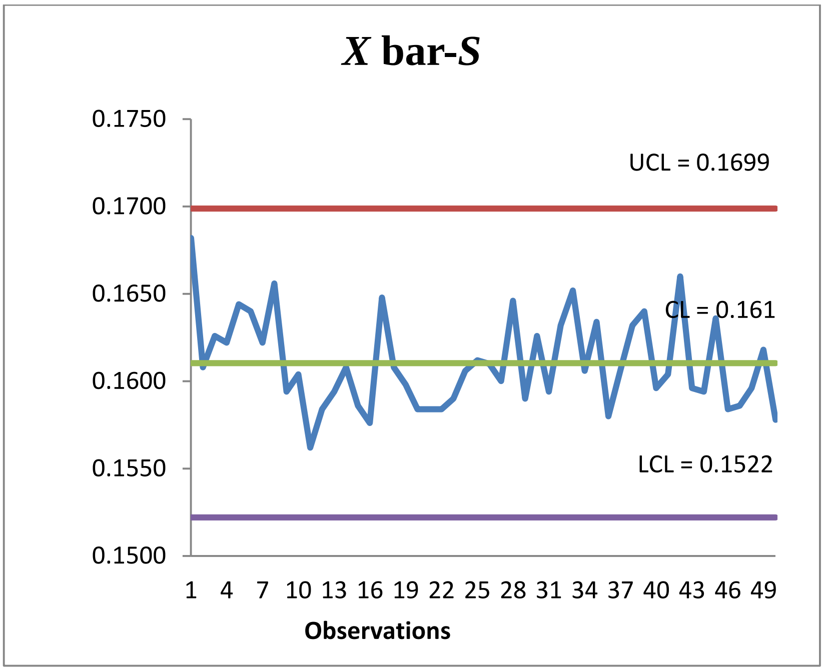
Setting up of $\bar{X}$ and S charts need similar approaches like for $\bar{X}$ and R charts. The centre line of S chart is $\bar{S} = 0.0062$. The value of A_3 = 01.427 for q = 5 from table of coefficient for control chart. The control limits are UCL = 0.1698 and LCL = 0.1522.

$$UCL = 0.1610 + (1.427)(0.0062) = 0.1698$$

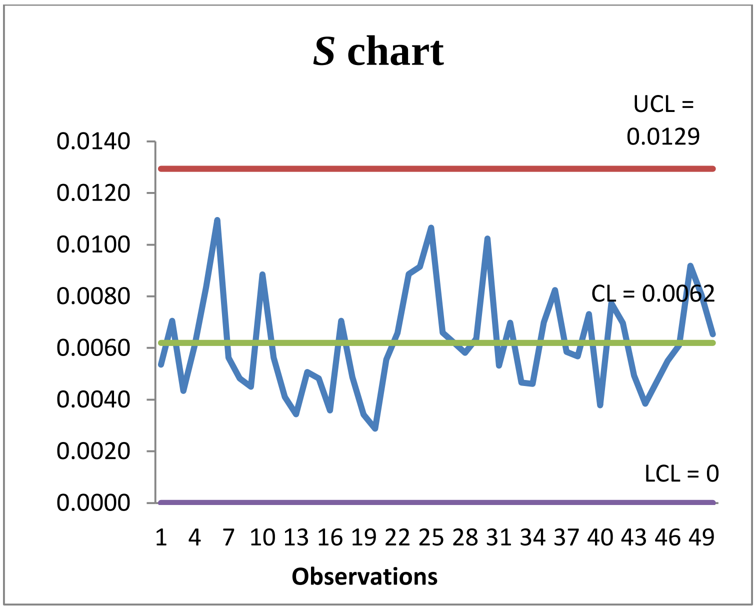
$$CL = \bar{\bar{X}} = 0.1610$$

$$LCL = 0.1610 - (1.427)(0.0062) = 0.1522$$

The $\bar{X}$ chart based on standard deviation can be seen in Figure 2(a). It shows no sign of out of control state when the 50 sample averages were plotted.



(a)



(b)

Figure 2: $\bar{X} - S$ charts

The centre line for S chart is 0.0062. The value of B_3 is 0 and B_4 is 2.089 for $q = 5$ from table of coefficient for control chart. Therefore, S chart control limits are:

$$UCL = (2.089)(0.0062) = 0.0130$$
$$CL = \bar{S} = 0.0062$$
$$LCL = (0)(0.0062) = 0$$

The S chart shown in Figure 2(b) above indicates no sign of out of control state. In this case, since both $\bar{X}$ chart and S chart are in control, it can be concluded that the process is in control and can be used to observe future production process.

Fuzzy $\tilde{\tilde{\bar{X}}} - \tilde{S}$ Control Charts

The 50 samples of solder paste thickness taken from production process were measured and recorded as triangular fuzzy number. The fuzzy measurement values and their fuzzy means and fuzzy standard deviations for each sample are given in Table 2 and Table 3. Fuzzy control limits were determined by using equation (11) to (43).

Table 2: Fuzzy observations (*d*, *e*, *f*)

sample		1	2	3	4	5
1	*Xd*	0.1658	0.1710	0.1727	0.1597	0.1652
	Xe	0.1670	0.1730	0.1740	0.1610	0.1660
	Xf	0.1677	0.1739	0.1746	0.1629	0.1665
2	*Xd*	0.1571	0.1708	0.1571	0.1606	0.1514
	Xe	0.1580	0.1720	0.1590	0.1620	0.1530
	Xf	0.1591	0.1735	0.1600	0.1632	0.1537
3	*Xd*	0.1614	0.1684	0.1606	0.1575	0.1592
	Xe	0.1620	0.1700	0.1620	0.1590	0.1600
	Xf	0.1628	0.1714	0.1632	0.1596	0.1614
4	*Xd*	0.1661	0.1587	0.1681	0.1542	0.1583
	Xe	0.1670	0.1600	0.1700	0.1550	0.1590
	Xf	0.1680	0.1611	0.1719	0.1564	0.1601
5	*Xd*	0.1611	0.1571	0.1778	0.1593	0.1598
	Xe	0.1630	0.1580	0.1790	0.1610	0.1610
	Xf	0.1642	0.1593	0.1799	0.1620	0.1621
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
50	*Xd*	0.1487	0.1527	0.1664	0.1603	0.1560
	Xe	0.1500	0.1540	0.1670	0.1610	0.1570
	Xf	0.1515	0.1552	0.1682	0.1615	0.1581

Table 3: Fuzzy mean, fuzzy standard deviation and their representative values

Sample	Fuzzy mean			$S^{0.65}_{mr-\tilde{\bar{X}},l}$	Fuzzy standard deviation			$S^{0.65}_{mr-S,l}$
1	0.1669	0.1682	0.1691	0.1681	0.0052	0.0054	0.0050	0.0053
2	0.1594	0.1608	0.1619	0.1608	0.0072	0.0070	0.0073	0.0071
3	0.1614	0.1626	0.1637	0.1626	0.0042	0.0043	0.0045	0.0043
4	0.1611	0.1622	0.1635	0.1622	0.0058	0.0061	0.0063	0.0061
5	0.1630	0.1644	0.1655	0.1644	0.0084	0.0084	0.0083	0.0083
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
50	0.1568	0.1578	0.1589	0.1578	0.0069	0.0065	0.0064	0.0066

Setting up and construction of fuzzy $\tilde{\bar{X}}$-$\tilde{S}$ charts need similar measures as for $\tilde{\bar{X}}$-$\tilde{R}$ charts. Using the data in Table 2, the fuzzy average $\tilde{\bar{S}}$ represented by triangular fuzzy number was calculated as below:

$$\tilde{\bar{S}} = \left(\frac{0.3145}{50}, \frac{0.3161}{50}, \frac{0.3182}{50} \right) = (0.0063, 0.0063, 0.0064)$$

The value of A_3 = 1.427 for q = 5 from table of coefficient. The control limits for fuzzy $\tilde{\bar{X}}$ chart based on standard deviation were calculated as follow:

$$U\tilde{C}L = (0.1599, 0.1610, 0.1621) + 1.427(0.0063, 0.0063, 0.0064)$$
$$= (0.1689, 0.1700, 0.1712)$$

$$C\tilde{L} = (0.1599, 0.1610, 0.1621)$$

$$L\tilde{C}L = (0.1599, 0.1610, 0.1621) - 1.427(0.0063, 0.0063, 0.0064)$$

$$= (0.1508, 0.1520, 0.1531)$$

The control limits for α-cut fuzzy $\tilde{\bar{X}}$ control chart based on standard deviation at α-level of 0.65 were determined as below:

$$U\tilde{C}L^{0.65} = (0.1606, 0.1610, 0.1614) + 1.427(0.0063, 0.0063, 0.0063)$$

$$= (0.1696, 0.1700, 0.1704)$$

$$C\tilde{L}^{0.65} = (0.1606, 0.1610, 0.1614)$$

$$L\tilde{C}L^{0.65} = (0.1606, 0.1610, 0.1614) - 1.427(0.0063, 0.0063, 0.0063)$$
$$= (0.1516, 0.1520, 0.1524)$$

where,

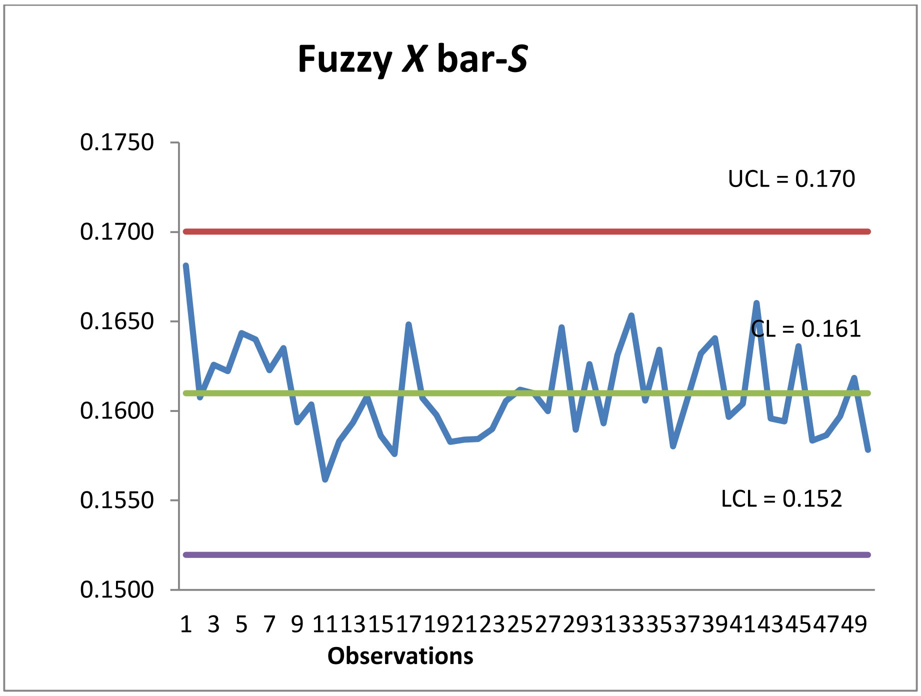
$$\bar{\bar{X}}_d^{0.65} = 0.1599 + 0.65(0.1610 - 0.1599) = 0.1606$$
$$\bar{\bar{X}}_f^{0.65} = 0.1621 - 0.65(0.1621 - 0.1610) = 0.1614$$
$$\bar{S}_d^{0.65} = 0.0063 + 0.65(0.0063 - 0.0063) = 0.0063$$
$$\bar{S}_f^{0.65} = 0.0064 + 0.65(0.0064 - 0.0063) = 0.0063$$

The control limits and centre line for α-cut fuzzy $\tilde{\bar{\bar{X}}}$ control chart based on standard deviation using α-level fuzzy midrange were obtained as follow:

$$U\tilde{C}L_{mr}^{0.65} = 0.1610 + 1.427\left(\frac{0.0063 + 0.0063}{2}\right) = 0.1700$$
$$C\tilde{L}_{mr}^{0.65} = f_{mr}^{0.65}\left(C\tilde{L}\right) = \left(\frac{0.1606 + 0.1614}{2}\right) = 0.1610$$
$$L\tilde{C}L_{mr}^{0.65} = 0.1610 - 1.427\left(\frac{0.0063 + 0.0063}{2}\right) = 0.1520$$

The α-level fuzzy midrange of 50 samples, $S_{mr,l}^{0.65}$, for fuzzy $\tilde{\bar{\bar{X}}}$ control chart based on standard deviation has been calculated and is shown in Table 3. These samples were then plotted on the fuzzy $\tilde{\bar{\bar{X}}}$ control chart based on standard deviation.

Figure 3(a) shows the fuzzy $\tilde{\bar{\bar{X}}}$ chart based on standard deviation. Using fuzzy triangular number, the control limits of the fuzzy $\tilde{\bar{\bar{X}}}$ charts after applying α-level fuzzy midrange transformation are UCL = 0.1700 and LCL = 0.1520. From the fuzzy $\tilde{\bar{\bar{X}}}$ chart, there is no sign of out of control condition is observed when 50 α-level fuzzy midrange sample averages were plotted on the chart. Hence, this control chart can also be used to observe production process.



(a)

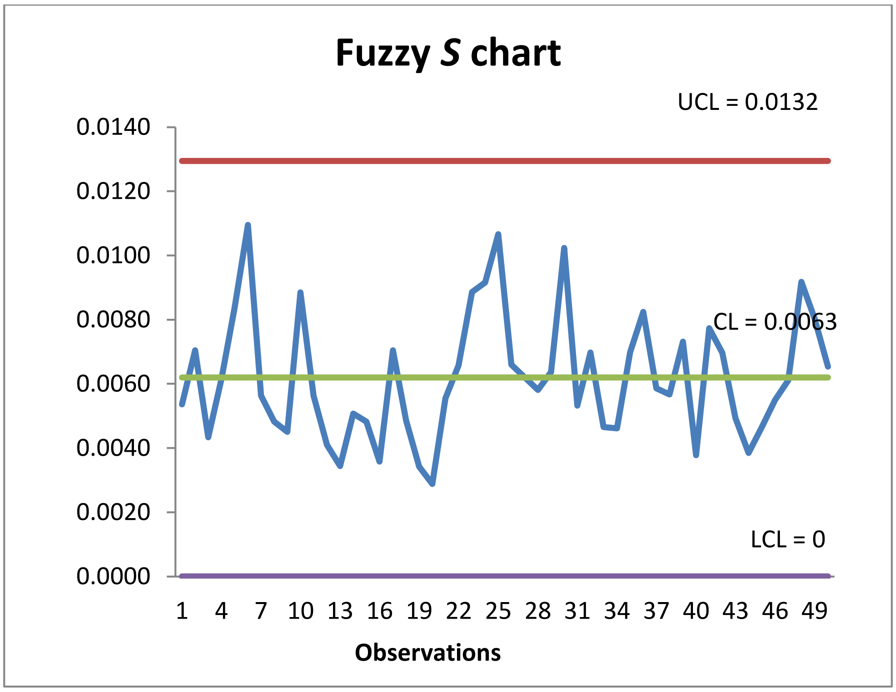


Figure 3: Fuzzy $\tilde{\bar{\bar{X}}} - \tilde{S}$ charts

The value of B_4 = 2.089 and B_3 = 0 for q = 5 were obtained from coefficients table of control chart. The fuzzy $\tilde{S}$ chart control limits are:

$$U\tilde{C}L = 2.089(0.0063, 0.0063, 0.0064) = (0.0132, 0.0132, 0.0134)$$

$$C\tilde{L} = (0.0063, 0.0063, 0.0064)$$

$$L\tilde{C}L = 0(0.0063, 0.0063, 0.0064) = (0, 0, 0)$$

Using the α-cut at α-level of 0.65, the control limits for α-Cut fuzzy $\tilde{S}$ control charts were defined as follow:

$$U\tilde{C}L^{0.65} = 2.089(0.0063, 0.0063, 0.063) = (0.0132, 0.0132, 0.0132)$$

$$C\tilde{L}^{0.65} = (0.0063, 0.0063, 0.063)$$

$$L\tilde{C}L^{0.65} = 0(0.0063, 0.0063, 0.063) = (0, 0, 0)$$

Control limits of α-level fuzzy midrange for α-Cut fuzzy $\tilde{S}$ charts were defined as the following:

$$U\tilde{C}L_{mr}^{0.65} = 2.089(0.0063) = 0.0132$$

$$C\tilde{L}_{mr}^{0.65} = f_{mr}^{0.65}\left(C\tilde{L}\right) = \left(\frac{0.0063 + 0.0063}{2}\right) = 0.0063$$

$$L\tilde{C}L_{mr}^{0.65} = 0(0.0063) = 0$$

The α-level fuzzy midrange of 50 samples, $S_{mr,l}^{0.65}$, for fuzzy $\tilde{S}$ chart has been calculated and is shown as in Table 3. These samples were then plotted on the fuzzy $\tilde{S}$ chart. Figure 3(b) shows the fuzzy $\tilde{S}$ chart. Using the fuzzy triangular number, control limits of the fuzzy $\tilde{S}$ chart after applying α-level fuzzy midrange transformation are UCL = 0.0132 and LCL = 0. The centre line for fuzzy $\tilde{S}$ chart is 0.0063. The fuzzy $\tilde{S}$ chart shows no sign of out of control state when the 50 α-level fuzzy midrange sample averages were plotted on the chart. Therefore, it can be concluded that the process is in control and can be used to monitor future production process since both fuzzy $\tilde{\tilde{\bar{X}}}$ chart and fuzzy $\tilde{S}$ chart are in control state.

Control Chart Performance

The aim of this study is to compare the performance between traditional $\bar{X} - S$ and fuzzy $\tilde{\tilde{\bar{X}}} - \tilde{S}$ charts by applying the ARL. Different value of shift, k were set at k = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5. After determining the UCL and LCL for required control chart, the ARL value with different value of shift, k was determined and the outcomes are provided as in Table 4. It shows that the value of ARL is decrease for both types of control chart. From Table 4, it can be seen that fuzzy control chart have lower value of ARL compared to traditional control charts. It shows that when fuzzy theory has been implemented to the control charts, the performance of control chart is better.

Table 4: ARL Value for Alpha = 0.65

Shift (*k*)	Fuzzy X bar-S	X bar-S
0	370.3704	370.3704
0.1	303.0303	303.0303
0.2	178.5714	185.1852
0.3	101.0101	104.1667
0.4	57.1429	58.4795
0.5	33.6700	34.6021
0.6	20.7039	21.2314
0.7	13.2979	13.6054
0.8	8.9127	9.0909
0.9	6.2189	6.3291
1	4.5167	4.5893
1.1	3.4106	3.4578
1.2	2.6717	2.7034
1.3	2.1673	2.1891
1.4	1.8165	1.8318
1.5	1.5699	1.5808

CONCLUSION

Statistical Process Control is one of the fields where fuzzy theory approach can be implemented using fuzzy control charts. The advantage when using fuzzy control chart is, it does not need to fulfill assumption of normality unlike traditional control chart. Fuzzy control charts is useful in monitoring process when vagueness and uncertainty arise. In this paper, applications on traditional and fuzzy control chart for solder paste thickness of integrated circuit were developed. Observations of each sample were recorded as triangular fuzzy number to built fuzzy control charts. This study also aimed to make comparison in terms of performance between fuzzy and traditional control charts using solder paste thickness for integrated circuit data. All the control charts in this study showed that there were no sign of out of control state when the 50 samples were plotted on the chart. Therefore, it can be concluded that the production process is stable and can be used to monitor future process. In order to compare the performance between fuzzy control charts and traditional control charts, ARL criterion is used. It was shown that fuzzy control charts have smaller value of ARL and provide more flexibility for controlling production process. Hence, fuzzy

$\tilde{\bar{\bar{X}}} - \tilde{S}$ charts are better than traditional $\bar{X} - S$ control charts in monitoring the product quality in solder paste thickness.

ACKNOWLEDGEMENTS

This research work is supported by FRGS (Fundamental Research Grant Scheme) grant (Vot 1498), Ministry of Higher Education, Malaysia.

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