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Fidelity Issue of Engineering Analysis and Computer Aided Calculations in Sign Models of Dynamic Systems

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Abstract

This article is devoted to solution of fidelity issue of engineering analysis and computer-aided calculations in ideal sign models of dynamic systems, presented in the form of various mathematical expressions, interrelating physical quantities which describe quantitatively the state of these systems. Developers of control systems of real dynamic systems are aware that upon analysis of nearly all technical objects it is necessary to account for variations of their parameters. This is attributed to the fact that the variables are almost always determined on the basis of experience or measurements, thus, they are of limited accuracy. In addition, these parameters are not ideally constant and vary in the course of operation. The proposed methods of control selection make it possible to develop control actions in dynamic systems, which are insensitive both to limited external impacts and to variation of parameters of these systems.

Keywords: model, engineering analysis, computer-aided calculations, stability, control, stabilization, ill-posed systems.

INTRODUCTION

Models as a research tool include various material and ideal objects. They are used while obtaining new information about object of research in the course of determination of fidelity of the obtained information. Let us denote a model as any system which is in certain relations with another system (denoted as original) so that the following conditions are valid: model and original are related by similarity, the form of which is obviously expressed, precisely defined; model in the course of scientific cognition is an analogue of considered object; study of model facilitates obtaining of information about original. Such model is simultaneously both an object of research and an experimental tool.

While analyzing control systems of real dynamic systems it should be taken into account that real unknown parameters of the considered object (hence, real values of parameters and coefficients of its mathematical model) are in a certain range $\pm\varepsilon$, and the following inequalities are valid:

$$a_i(1-\varepsilon_i) \le \overline{a_i} \le a_i(1+\varepsilon_i) \quad (1)$$

where $\overline{a_i}$ is the real unknown value of parameter; a_i is the value applied in calculations (nominal value); ε_i are the small numbers in comparison with unity (usually their values can be estimated). Quite often studies are restricted by solution of mathematical model of the considered object at nominal parameters. However, in order to provide reliability of the analysis it is necessary to estimate the degree to which the solution varies upon variations of nominal parameters equaling to $\pm\varepsilon_i a_i$.

Examples of ideal sign models, applied for description of wide spectrum of dynamic systems, including mathematical expression of dependence of physical quantities, are the models of the following scopes:

1. Stabilization of dynamic systems [1]-[4];
2. Various types of measure of dynamic systems and its definition [5]-[13];
3. Stability of dynamic systems [2], [14]-[19];
4. Optimization of dynamic systems [20]-[22];
5. Oscillations and waves in dynamic systems [23]-[29];
6. Optimization of management systems [30]-[38].

Mathematical models are known (for instance, in the form of differential equations), solutions of which vary by finite and even high values at arbitrarily small (and inevitable in practice) variations of parameters.

Definition: Ill-posed systems are the systems, solution of which vary by finite values at arbitrarily small variations of coefficients and parameters [39].

EXPERIMENTAL

The first step to provision of analysis fidelity is, before everything else, determination of ill-posed solutions. Reliable results (with rare exceptions) can be based only on correct solutions. Ill-posed solutions are unreliable, generally they do not have common sense.

In the course of researches at St. Petersburg University it was established that correctness of solutions can vary upon equivalent transformations of mathematical models (differential equation systems, in particular), which creates new approach to research of reliability of dynamic systems and their stability according to Lyapunov.

Indeed, according to general definitions equivalent (equal) transformations are those which do not vary solutions of transformed system. However, they should not obligatory maintain unchanged properties of the solutions, including such property as correctness.

This simple, though ignored for long time, circumstance lead to numerous and important consequences. Verification of solution correctness turned to be more complicated procedure than it was considered previously. Ambiguity of certain theorems and procedures was revealed, which previously were used in analyses.

1. **The Lyapunov's second method**. It is usually considered that if the Lyapunov function is constructed for a certain system of differential equations, then the zero solution of the system is stable. It has been revealed that existence of the Lyapunov function does not guarantee anything, since there are systems with the Lyapunov function which lose stability at arbitrarily small variations of parameters, inevitable in practice. A system, formally stable but losing its stability at arbitrarily small variations of parameters, is no better than unstable system and even more dangerous.
2. **Theorem on continuous dependence of solutions of differential equations on parameters**. It plays the central role in all practical theoretical applications. Nevertheless, it cannot be considered as valid since there are systems satisfying the hypotheses of the theorem (satisfying of the Lipschitz conditions for the right members of equations) and, nevertheless, having no continuous dependence of solutions on coefficients and parameters.
3. **Systems of differential equations with constant coefficients.** It is usually considered (and widely applied in calculations) that if a system of differential equations with constant coefficients has all characteristic polynomial roots in left half plane, far from imaginary axis, then the system is stable and maintain stability at minor variations of its parameters. In fact, this is not valid (not always valid, to be more exact).

RESULTS AND DISCUSSION

Let us consider such system of differential equations as an example:

$$D = \frac{d}{dt}$$

$$(D^3 + 4D^2 + 5D + 2)x_1 - (D^2 + 2D + 1)x_2 = 0 \qquad (2)$$

$$(D^2 + 4D + 4)x_1 - (D + 1)x_2 = 0 \qquad (3)$$

The characteristic polynomial of this system is as follows:

$$\Delta_{1=} \begin{vmatrix} \lambda^3 + 4\lambda^2 + 5\lambda + 2 & -\lambda^2 - 2 - 1\lambda \\ \lambda^2 + 4\lambda + 4 & -\lambda - 1 \end{vmatrix} =$$

$$= \lambda^3 + 4\lambda^2 + 5\lambda + 2 = (\lambda + 1)^2(\lambda + 2) \qquad (4)$$

The roots of characteristic polynomial (4) are $\lambda_1 = \lambda_2 = -1$, $\lambda_3 = -2$. The general solutions of Eqs. (2)-(3) with regard to variable $x_1(t)$ is as follows:

$$x_1 t = C_1 e^{-2t} + (C_2 t + C_3)e^{-t}$$

According to the Hurwitz criterion it is possible to state stability of this system at minor variations of the coefficients, since the roots of characteristic polynomial are in the left half plane. However, if the coefficient of D^2x_2 in Eq. (2) is varied by arbitrarily small value $\varepsilon < 0$, then it equals to $1-\varepsilon$, and the system is as follows:

$$(D^3+4D^2+5D+2)x_1-\big((1-\varepsilon)D^2+2D+1\big)x_2=0 \qquad (5)$$
$$(D^2+4D+4)x_1-(D+1)x_2=0,$$

and its characteristic polynomial equals to:

$$\Delta_2=\begin{vmatrix}\lambda^3+4\lambda^2+5\lambda+2 & -((1-\varepsilon)\lambda^2+2+1\lambda)\\ \lambda^2+4\lambda+4 & -\lambda-1\end{vmatrix}=$$
$$=-\varepsilon\lambda^4+(1-4\varepsilon)\lambda^3+(4-4\varepsilon)\lambda^2+5\lambda+2 \qquad (6)$$

The general solution for $x_1(t)$ is as follows:

$$x_1(t)=C_1e^{-2t}+(C_2t+C_3)e^{-t}+C_4e^{\frac{t}{\varepsilon}}$$

The function of this characteristic polynomial (6) is illustrated in Fig. 1.

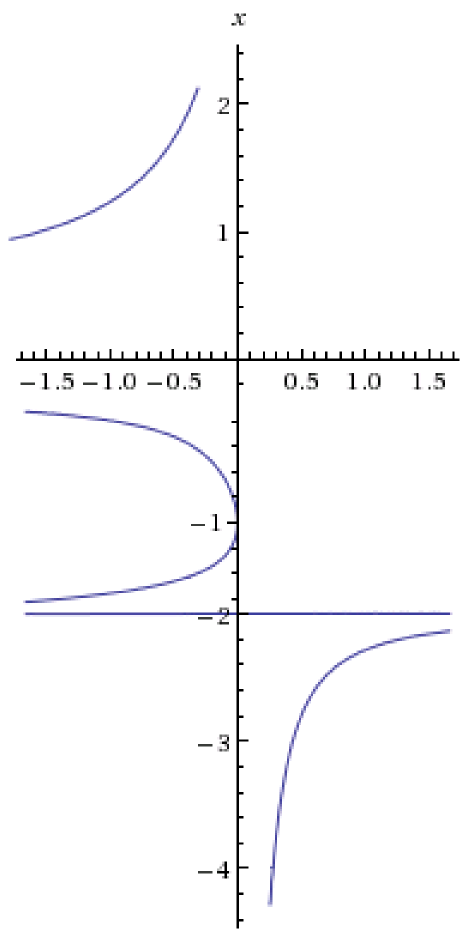

Figure 1. Graph of the characteristic polynomial (6)

As can be seen, upon deviation of the coefficient by arbitrarily small negative value we obtain the root of characteristic polynomial (6) of the system (5)-(3) in right half plane and the system stability is violated, and at the same deviation by positive value the stability remains the same.

It follows from this analysis that upon arbitrarily small variation of the coefficient in Eqs. (2)-(3) the stability is lost, and the solution pattern of $x_1(t)$ varies radically. It acquires rapidly increasing term of $C_4 e^{\frac{t}{\varepsilon}}$ type and there appears continuous dependence of solution on the varied by us coefficient (this is not so for Eqs. (2)-(3)). After normalizing Eqs. (2)–(3) using equivalent transformations (in classical sense) we obtain the following

$$\begin{aligned} \dot{x}_1 &= -2x_1 - 2x_2 - x_3 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -x_2 - 2x_3 \end{aligned} \tag{7}$$

Equation (7) has the following characteristic polynomial Δ_3:

$$\Delta_3 = \begin{vmatrix} \lambda+2 & 1 & 1 \\ 0 & \lambda & -1 \\ 0 & 1 & \lambda+2 \end{vmatrix} = \lambda^3 + 4\lambda^2 + 5\lambda + 2 = (\lambda+1)^2(\lambda+2) \tag{8}$$

This polynomial (8) identically equal to Δ_1 (4). Solution $x_1(t)$ of Eqs. (2)–(3) has the same form as Eq. (7). From this we can conclude that Eq. (7) is equivalent to Eqs. (2)–(3), however, as stipulated earlier, we obtained different properties of these solutions. The solutions of Eq. (7) continuously depend on all their coefficients and are stable at all variations (this can be easily confirmed). The solutions of Eqs. (2)–(3) do not possess such property, as demonstrated above.

It should be mentioned that for Eqs. (2)–(3) we can construct the Lyapunov function, since this system has constant coefficients and characteristic polynomial with the roots in left half plane. As demonstrated, this also will not guarantee preservation of stability upon variation of parameters.

This problem was discussed elsewhere [2], [3], [20], [21], [40]-[42].

Now let us consider variations of coefficients of other terms of Eqs. (2)–(3):

1) Then, at $2Dx_2$ Eqs. (2)–(3) are as follows:

$$(D^3 + 4D^2 + 5D + 2)x_1 - (D^2 + 2(1+\varepsilon)D + 1)x_2 = 0$$
$$(D^2 + 4D + 4)x_1 - (D+1)x_2 = 0$$

Its characteristic polynomial is as follows:

$$(\lambda+2)(\varepsilon\lambda^2 + 2\varepsilon\lambda + \lambda^2 + 2 + 1)$$

In this case the function of this characteristic polynomial is illustrated in Fig. 2.

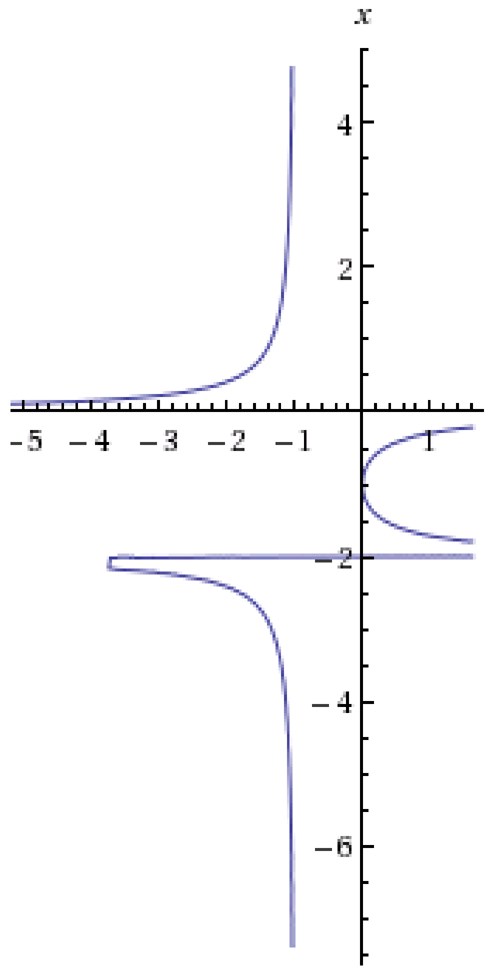

Figure 2. Graph of the characteristic polynomial

That is, at ε<-1 stability is lost. Contrary to the previous case the reserve of stability of this coefficient is sufficient and we cannot consider this case it as violation of stability criterion.

2) At x_2 Eqs. (2)–(3) are as follows:

$$(D^3 + 4D^2 + 5D + 2)x_1 - (D^2 + 2D + (1 + \varepsilon))x_2 = 0$$
$$(D^2 + 4D + 4)x_1 - (D + 1)x_2 = 0$$

The characteristic polynomial is as follows:

$$\varepsilon\lambda^2 + 4\varepsilon\lambda + 4\varepsilon + \lambda^3 + 4\lambda^2 + 5\lambda + 2$$

The diagram of the characteristic polynomial is illustrated in Fig. 3:

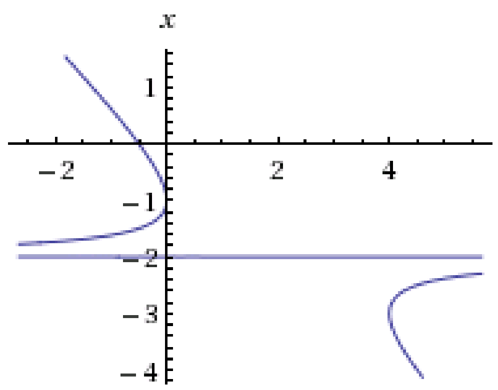

Figure 3. Graph of the characteristic polynomial

As can be seen in the diagram, at ε<-0.5 stability is lost. As in the previous case, the reserve of stability of this coefficient is sufficient and we cannot consider this case as violation of stability criterion.

3) At D^3x_1 Eqs. (2)–(3) are as follows:

$$((1+\varepsilon)D^3+4D^2+5D+2)x_1-(D^2+2D+1)x_2=0$$
$$(D^2+4D+4)x_1-(D+1)x_2=0$$

Its characteristic polynomial is as follows:

$$-\varepsilon\lambda^4-\varepsilon\lambda^3+\lambda^3+4\lambda^2+5\lambda+2$$

The diagram of the characteristic polynomial is illustrated in Fig. 4.

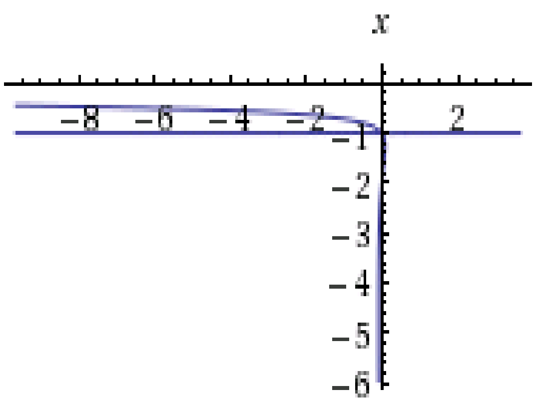

Figure 4. Graph of the characteristic polynomial

In this case the system of equations will preserve its stability at any variations of the coefficient. However, the solution pattern $x_1(t)$ of this system of equations will vary. One more term of $C_4 e^{\frac{t}{\varepsilon}}$ type appears here.

4) At $4D^2x_1$ Eqs. (2)–(3) are as follows:

$$(D^3 + 4(1+\varepsilon)D^2 + 5D + 2)x_1 - (D^2 + 2D + 1)x_2 = 0$$
$$(D^2 + 4D + 4)x_1 - (D + 1)x_2 = 0$$

Its characteristic polynomial is as follows:

$$-4\varepsilon\lambda^3 + \lambda^3 + 4\varepsilon\lambda^2 + 4\lambda^2 + 5\lambda + 2$$

The diagram of the characteristic polynomial is illustrated in Fig. 5.

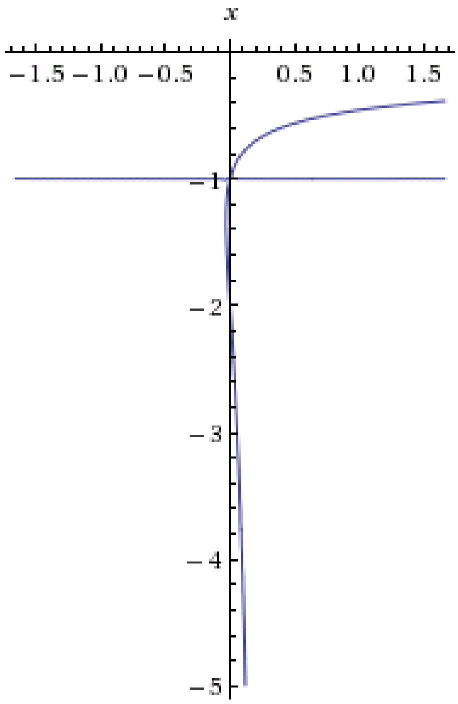

Figure 5. Graph of the characteristic polynomial

In this case variation of the coefficient will never result in loss of stability.

5) At $5Dx_1$ Eqs. (2)–(3) are as follows:

$$(D^3 + 4D^2 + 5(1+\varepsilon)D + 2)x_1 - (D^2 + 2D + 1)x_2 = 0$$
$$(D^2 + 4D + 4)x_1 - (D + 1)x_2 = 0$$

Its characteristic polynomial is as follows:

$$\lambda^3 - 5\varepsilon\lambda^2 + 4\lambda^2 - 5\varepsilon\lambda + 5\lambda + 2$$

The diagram of the characteristic polynomial is illustrated in Fig. 6.

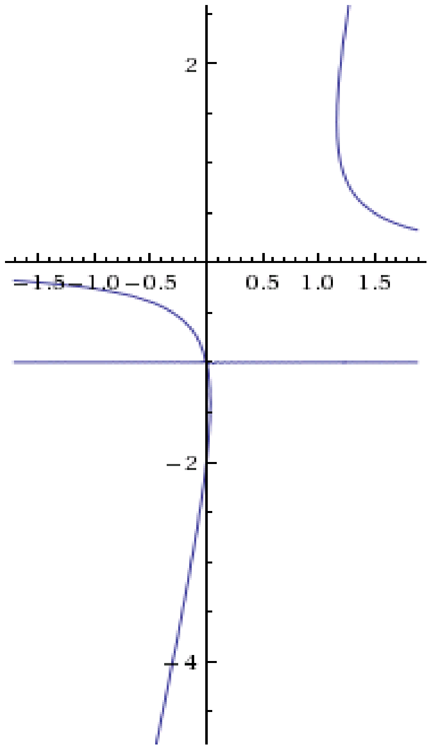

Figure 6. Graph of the characteristic polynomial (6)

It can be seen after calculations that at $\varepsilon > 1.1657$ the stability is lost. That is, the reserve of stability of the coefficient is sufficient and we cannot consider this case as violation of stability criterion.

6) At $2x_1$ Eqs. (2)–(3) are as follows:

$$(D^3 + 4D^2 + 5D + 2(1 + \varepsilon))x_1 - (D^2 + 2D + 1)x_2 = 0$$
$$(D^2 + 4D + 4)x_1 - (D + 1)x_2 = 0$$

Its characteristic polynomial is as follows:

$$\lambda^3 + 4\lambda^2 - 2\varepsilon\lambda + 5\lambda - 2\varepsilon + 2$$

The diagram of the characteristic polynomial is illustrated in Fig. 7.

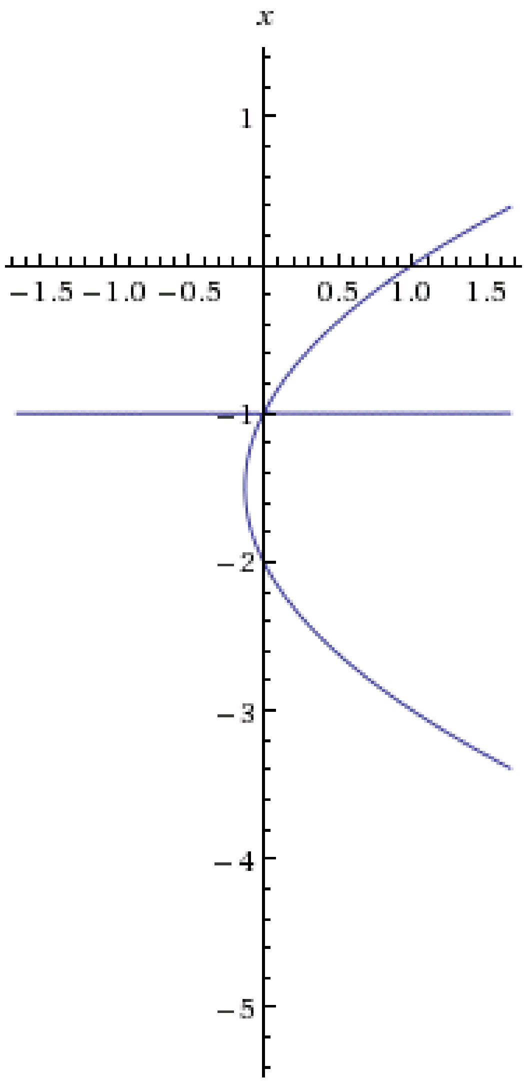

Figure 7. Graph of the characteristic polynomial (6)

As seen in the diagram, at $\varepsilon > 1$ stability is lost. As in the previous case, reserve of stability of this coefficient is sufficient and we cannot consider this case as violation of stability criterion.

While analyzing, we can conclude that arbitrarily small variation of coefficients only in one case leads to violation of stability criterion. In the third example this variation leads to strong deviation of the solution from initial one, which also negatively influence on control task.

On the basis of this example it should be taken into account that more profound attention should be given to stability verification, higher care should be applied to equivalent transformations, variation of stability properties should be monitored at each step.
The example with Eqs. (2)–(3) reveals the reason why the existence of the Lyapunov function was considered as stability guarantee and the theorem on continuous dependence of solutions of differential equations on parameters was believed to be valid. These provisions were proven for systems in normal form, and then, since nearly any system of differential equations can be normalized by equivalent transformations, these provisions were assumed as valid for all systems already without proving.
The example with Eqs. (2)–(3) is not unique. The class of problems exists which vary their correctness in the course of equivalent transformations used for their solution. If previously it was assumed that the problems of physics and engineering could be separated into two classes, correct problems and ill-posed problems [41], then, after revealing of new properties of equivalent transformations, it was required to highlight the new third class of problems, intermediate between correct and ill-posed ones [39]. Peculiar properties of the problems of the third class were described elsewhere [5], [39], including numerous examples.

The most important practical consequence of these studies into solution of problems of stability or instability of program operation was detection of existence of peculiar objects which:

1. Are capable to vary radically their behavior (to lose stability, in particular) at minor variations of parameters;
2. This dangerous property is not detected by means of conventional procedures of analysis and design.

An example is the system of automatic control which mathematical model can be described in the form of Eqs. (2)–(3), where $x_1(t)$ is the deviation of rotation frequency from the required value, $x_2(t)$ is the control. In Eqs. (2)–(3) the coefficients, for convenience of calculation of characteristic polynomials and verification of equivalence of transformations, are rounded to integers. In general, mathematical models, similar to Eqs. (2)–(3), describe quite real control systems.

CONCLUSIONS

Control systems, identical in specified sense to those described by Eqs. (2)–(3), are peculiar objects: they lose stability not only at minor but also at arbitrarily small variations of parameters. At the same time, conventional procedures of stability verification will not discover this fact. Indeed, calculating characteristic polynomial and its roots, either directly or automatically, we inevitably arrive at false conclusion about high reserve of stability of designed system and about possibility of its fabrication and application for control of real objects.
Further on, since upon fabrication of real control systems small deviations of real values of parameters from calculations are possible and their sign cannot be predicted,

then it is possible that $\varepsilon > 0$ and the system will be stable upon testing. Thus, it would be possible to believe that conventional analysis of stability provided correct result. In fact the stability reserve will be very small (about ε). Upon drift of parameters in operation the stability reserve can be rapidly exhausted, the system will lose its stability, overrun, emergency situation occurs which can result in incident or even catastrophe.

As shown in [5], [39], some recent incidents and catastrophes resulted from this very reason: incompleteness of commonly applied design procedures and calculations, not considering for new properties of equivalent transformations and not taking into account existence of peculiar objects.

In order to prevent incidents and catastrophes it is necessary to apply methods of detection of peculiar objects. These methods are described in details elsewhere [5], [39].

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