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Electroviscous potential flow analysis for two bounded fluids streaming with mass and heat transfer in a porous medium

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Abstract

Using viscous potential flow theory, the linear stability analysis of two superposed dielectric fluids streaming with interfacial transfer of mass and heat through porous media is studied for two layers of finite thickness. The system is influenced by general applied horizontal or vertical (in absence or presence surface charges) electric fields. The method of multiple scales with appropriate boundary conditions are used to derive a quadratic dispersion relation in frequency with complex coefficients, separately. The stability of the system is conferred in detail and some limiting cases are recovered. It is observed that the horizontal electric field, porosity of a porous medium, fluid viscosities and surface tensions, increase the stability of the system while the vertical electric field, medium permeability, fluid velocities and fluid depths reduce the stability of the system. Lately the instability in the presence of surface charges at the interface occurs faster than the case of the absence of surface charges. Finally, the mass and heat transfer play a dual role on the stability of the system for vertical (in presence of surface charges) electric fields, while it increase the stability of the system for other cases.

AMS subject classification:
Keywords: Electrohydrodynamic stability, viscous potential flow, porous media, Heat and mass transfer, dielectric fluids.

1. Introduction

The unsteadiness of the plane interface between two superposed liquids moving with a relative horizontal velocity is called Kelvin-Helmholtz instability. This instability is essential in understanding a variety of space and astrophysical phenomena including sheared flow. On account of its significance to astrophysical, geophysical and laboratory circumstances [1], this problem has been dissected by many researchers. The linear instability of Kelvin-Helmholtz problem is examined in [2].

The subject of electrohydrodynamics has an extensive variety of significance in different physical circumstances [3]. In the linear electrohydrodynamic stability theory, it is known that the tangential electric field has a stabilizing effect, while the normal one always has a destabilizing influence [4–6]. Electrohydrodynamic instability problems of the interface between two fluids in plane and cylindrical geometries have been as of late examined by many authors [7–14].

The mechanism of heat and mass transfer across an interface have great significance in many environmental and industrial processes.The mathematical formulation for a liquid-vapor interface with heat and mass transfer and the dispersion relations for both Rayleigh-Taylor and Kelvin-Helmholtz instabilities for inviscid vapor and liquid derived by [15–17]. Also, for interface between two fluids in plane and cylindrical geometries in the presence of heat and mass transfer [18–27], flows through porous media [28–38], viscous potential flow to study various stability problem [39–44] are found in many recent works.

2. Problem Formulation

Let our system consists of two incompressible dielectric fluids with uniform densities ρ_1 and ρ_2 be separated by a horizontal interface $y = 0$ in its equilibrium state. The density

ρ_1 of the lower fluid be more than the density ρ_2 of the upper fluid so that in the absence of motion, the system is a stable one. The temperatures at rigid bounding surfaces $y = 0$, $y = -d_1$ and $y = d_2$ are T_0, T_1 and T_2, respectively. The two fluids are streaming along the x-axis with uniform velocities U_1 and U_2 through porous medium, and they are influenced by a constant horizontal electric field E_0 in the x-direction. The gravitational field g acting in the negative y-direction. The surface tension forces T are considered between the two fluids. Let ε_j, μ_j, $\mathbf{E}_j$, m and λ_1, $(j = 1, 2)$ denote, respectively, dielectric constants, viscosities coefficients, electric field components, porosity of porous medium and medium permeability, where the subscripts 1 and 2 refer to the lower and upper fluids, respectively. The interface is represented by

$$F(x, y, t) = y - \zeta(x, t) = 0, \tag{1}$$

where the disturbed interface $y = \zeta(x, t)$. The outward normal vector is written as

$$\mathbf{n} = \frac{\nabla F}{|\nabla F|} = \left[1 + \zeta_x^2\right]^{-\frac{1}{2}} (-\zeta_x, 1). \tag{2}$$

The equation of continuity is

$$\nabla.\mathbf{v} = 0. \tag{3}$$

Suppose the fluids motion is assumed to be irrotational, then there are velocity potentials $\Phi_j(x, y, t)$

$$v_j = U_j\mathbf{e}_x + \nabla\Phi_j, \quad j = 1, 2, \tag{4}$$

where $\mathbf{v}_j$ is the fluid velocity. For an incompressible fluids, the potential Φ_j satisfy the Laplace equation [33–36] i.e.

$$\nabla^2\Phi_1 = 0, \qquad \text{for} \qquad -d_1 < y < \zeta(x, t) \tag{5}$$

$$\nabla^2\Phi_2 = 0, \qquad \text{for} \qquad \zeta(x, t) < y < d_2 \tag{6}$$

In electrodynamics, it is supposed that the quasi-static approximation is valid.Thus the electrical equations are

$$\nabla \times \mathbf{E} = \mathbf{0}, \quad \text{and} \quad \nabla.(\varepsilon\,\mathbf{E}) = \mathbf{0}. \tag{7}$$

So, the electric field can be represented by an electrostatic potentials $\Psi_j(x, y, t)$

$$\mathbf{E}_j = E_0\mathbf{e}_x - \nabla\Psi_j, \quad j = 1, 2. \tag{8}$$

From (7) and (8) that the electrostatic potentials also satisfy the Laplace equation [35–36], i.e.

$$\nabla^2\Psi_1 = 0, \qquad \text{for} \qquad -d_1 < y < \zeta(x, t) \tag{9}$$

$$\nabla^2\Psi_2 = 0, \qquad \text{for} \qquad \zeta(x, t) < y < d_2 \tag{10}$$

3. Boundary conditions

The solutions for Φ_j and Ψ_j ($j = 1, 2$) have to satisfy the boundary conditions as follows:

On the rigid boundaries $y = -d_1$ and $y = d_2$, the normal fluid velocities vanish on both the bottom and top boundaries, i.e.

$$\frac{\partial \Phi_j}{\partial y} = 0 \qquad \text{on} \qquad y = (-1)^j d_j, \qquad j = 1, 2. \tag{11}$$

The normal components of the electric fields also vanish on these boundaries, i.e.

$$\frac{\partial \Psi_j}{\partial y} = 0 \qquad \text{on} \qquad y = (-1)^j d_j, \qquad j = 1, 2. \tag{12}$$

On the free interface $y = \zeta(x, t)$, The conservation of mass across the interface [15] is

$$\left\| \rho \left(\frac{\partial F}{\partial t} + \nabla\Phi.\nabla F \right) \right\| = 0, \tag{13}$$

or in a simplified form

$$m\frac{\partial \zeta}{\partial t} \|\rho\| + \frac{\partial \zeta}{\partial x} \left\| \rho(\frac{\partial \Phi}{\partial x} + U) \right\| - \left\| \rho \frac{\partial \Phi}{\partial y} \right\| = 0, \tag{14}$$

where $\|*\| = *_2 - *_1$ represents the jump across the interface. The tangential component of the electric field is assumed continuous at the interface implies that

$$\left\| \frac{\partial \Psi}{\partial x} \right\| + \frac{\partial \zeta}{\partial x} \left\| \frac{\partial \Psi}{\partial y} \right\| = 0. \tag{15}$$

The normal electric displacement is assumed to be continuous at the interface, and we get

$$\frac{\partial \zeta}{\partial x} \left\| \varepsilon \frac{\partial \Psi}{\partial x} \right\| - \left\| \varepsilon \frac{\partial \Psi}{\partial y} \right\| - E_0 \frac{\partial \zeta}{\partial x} \|\varepsilon\| = 0. \tag{16}$$

The conservation of energy [15, 16, 17]

$$\rho_1 \left[\frac{\partial \Phi_1}{\partial y} - m\frac{\partial \zeta}{\partial t} - \frac{\partial \zeta}{\partial x} \left(\frac{\partial \Phi_1}{\partial x} + U_1 \right) \right] = \alpha m(\zeta + \alpha_2 \zeta^2 + \alpha_3^3 \zeta^3) \tag{17}$$

where the coefficients of mass and heat transfer α, α_2 and α_3 are:

$$\alpha = \frac{G}{L}\left(\frac{1}{d_1} + \frac{1}{d_2} \right), \qquad \alpha_2 = \frac{1}{d_2} - \frac{1}{d_1}, \qquad \alpha_3 = \frac{1}{d_1^2} - \frac{1}{d_1 d_2} + \frac{1}{d_2^2}. \tag{18}$$

where L is the latent heat of transformation from one fluid to another, $G = \dfrac{K_2(T_0 - T_2)}{d_2} = \dfrac{K_1(T_1 - T_0)}{d_1}$ is the equilibrium heat flux and K_1 and K_2 are the heat conductivities of the two fluids.

The conservation of momentum balance [42]

$$\left\| \rho \left(\nabla\Phi.\nabla F \right) \left(\frac{\partial F}{\partial t} + \nabla\Phi.\nabla F \right) \right\| + \left\{ \frac{1}{2} \left\| \varepsilon \left(E_t^2 - E_n^2 \right) \right\| + \| P \| \right.$$
$$\left. -2 \left\| \mu \frac{\partial \mathbf{v}}{\partial y} \right\| + T \ \nabla.\mathbf{n} \right\} |\nabla F|^2 = 0, \quad \text{at} \quad y = \zeta(x, t) \tag{19}$$

where P is the pressure, E_t and E_n are the tangential and normal components of the electric field, respectively. By eliminating the pressure by using Bernoulli equation, and using equations (1), (2), (8), the condition (19) can be rewritten in linearized form as

$$\frac{1}{m} \left\| \rho \frac{\partial \Phi}{\partial t} \right\| + \frac{2}{m} \left\| \mu \frac{\partial^2 \Phi}{\partial y^2} \right\| + \frac{1}{m^2} \left\| \rho U \frac{\partial \Phi}{\partial x} \right\| + \frac{1}{\lambda_1} \| \mu \Phi \| + E_0 \left\| \varepsilon \frac{\partial \Psi}{\partial x} \right\|$$
$$+ g y \, \| \rho \| + T \frac{\partial^2 \zeta}{\partial x^2} = 0. \tag{20}$$

4. Multiple scales method and linear analysis

The set of the equations (5), (6), (9) and (10) with the conditions (11–17) is solved here by using the method of multiple scales [45]. Let us first expand Φ_j and Ψ_j, $(j = 1, 2)$ in the following asymptotic series

$$f(x, y, t) = \sum_{n=1}^{3} \epsilon^n f_n(x_0, x_1, x_2, y, t_0, t_1, t_2) + o(\epsilon^4) \tag{21}$$

where ϵ is a small dimensionless parameter showing the strength of the nonlinearity (Note that according to equation (21) ζ is independent of y). The multiple scales $x_n = \epsilon^n x$, and $t_n = \epsilon^n t$ are to satisfy the expansions:

$$\frac{\partial}{\partial \beta} = \sum_{n=0}^{3} \epsilon^n \frac{\partial}{\partial \beta_n} + o(\epsilon^4) \tag{22}$$

where β represent x and t. Here, the short scale x_0 represent the wavelength while the long scales x_1 and x_2 stand for the spatial modulations of the phase and the amplitude. The fast scale t_0 represents the frequency of the wave, whereas t_1 and t_2 denote respectively to the slow temporal scales of the phase and the amplitude. Now, expanding (11)-(17) and (20) into Taylor series around the undisturbed surface $y = 0$, then substituting (21) and (22) into (5), (6), (9), and (10) and the boundary conditions (11)-(17) and (20) and equating the coefficients of the same powers in ϵ.

Since the problem used here is linear, the effect of nonlinearity will not be discussed. Therefore the solutions of the first order problem for the traveling waves with respect to the variables x_0 and t_0 are

$$\zeta = A e^{i\theta} + c.c., \tag{23}$$

$$\Phi_1 = \frac{1}{k\rho_1}\left[m\alpha + i(kU_1 - m\omega)\rho_1\right]\frac{\cosh k(y+d_1)}{\sinh kd_1}Ae^{i\theta} + c.c., \tag{24}$$

$$\Phi_2 = -\frac{1}{k\rho_2}\left[m\alpha + i(kU_2 - m\omega)\rho_2\right]\frac{\cosh k(y-d_2)}{\sinh kd_2}Ae^{i\theta} + c.c., \tag{25}$$

$$\Psi_1 = \frac{iE_0(\varepsilon_2-\varepsilon_1)}{(\varepsilon_1\sigma_1+\varepsilon_2\sigma_2)}\frac{\cosh k(y+d_1)}{\cosh kd_1}Ae^{i\theta} + c.c., \tag{26}$$

$$\Psi_2 = \frac{iE_0(\varepsilon_2-\varepsilon_1)}{(\varepsilon_1\sigma_1+\varepsilon_2\sigma_2)}\frac{\cosh k(y-d_2)}{\cosh kd_2}Ae^{i\theta} + c.c. \tag{27}$$

where $\sigma_j = \tanh kd_j$, $(j = 1, 2)$, $\theta = kx_0 - \omega t_0$ is the phase of the carrier wave , k is the wavenumber along $x-$direction while ω, A, i, $c.c$ stands the angular frequency, the complex amplitude of the surface elevation, the imaginary unit and the complex conjugate of the preceding terms respectively.The wavenumber k and the frequency ω must satisfy the following characteristic equation

$$\begin{aligned} S(\omega,k) &= \frac{1}{km^2}\left\{\frac{\rho_1(m\omega-kU_1)^2}{\sigma_1} + \frac{\rho_2(m\omega-kU_2)^2}{\sigma_2}\right\} \\ &\quad -\frac{\alpha}{k}\left\{\left(\frac{2\lambda_1k^2+m}{\lambda_1}\right)\left(\frac{\mu_1}{\rho_1\sigma_1}+\frac{\mu_2}{\rho_2\sigma_2}\right)\right. \\ &\quad \left. -\frac{i}{m}\left(\frac{m\omega-kU_1}{\sigma_1}+\frac{m\omega-kU_2}{\sigma_2}\right)\right\} \\ &\quad +\frac{i\left(2\lambda_1k^2+m\right)}{mk\lambda_1}\left\{\frac{\mu_1(m\omega-kU_1)}{\sigma_1}+\frac{\mu_2(m\omega-kU_2)}{\sigma_2}\right\} \\ &\quad -\frac{kE_0^2(\varepsilon_2-\varepsilon_1)^2}{(\varepsilon_1\sigma_1+\varepsilon_2\sigma_2)} + g(\rho_2-\rho_1) - T\,k^2 = 0 \end{aligned} \tag{28}$$

The characteristic equation (28) is reduced to the dispersion relation as follows:

$$a_0\omega^2 + (a_1 + ib_1)\omega + (a_2 + ib_2) = 0 \tag{29}$$

where

$$\begin{aligned} a_0 &= \left(\frac{\rho_1}{\sigma_1}+\frac{\rho_2}{\sigma_2}\right), \\ a_1 &= -\frac{2k}{m}\left(\frac{\rho_1U_1}{\sigma_1}+\frac{\rho_2U_2}{\sigma_2}\right), \\ b_1 &= \frac{1}{\lambda_1}\left(\frac{\mu_1}{\sigma_1}+\frac{\mu_2}{\sigma_2}\right)\left(2\lambda_1k^2+m\right) + \alpha\left(\frac{1}{\sigma_1}+\frac{1}{\sigma_2}\right), \end{aligned}$$

$$a_2 = \frac{k^2}{m^2}\left(\frac{\rho_1 U_1^2}{\sigma_1} + \frac{\rho_2 U_2^2}{\sigma_2}\right) - \frac{\alpha}{\lambda_1}\left(\frac{\mu_1}{\rho_1\sigma_1} + \frac{\mu_2}{\rho_2\sigma_2}\right)\left(2\lambda_1 k^2 + m\right)$$
$$-\frac{k^2 E_0^2(\varepsilon_2 - \varepsilon_1)^2}{(\varepsilon_1\sigma_1 + \varepsilon_2\sigma_2)} + gk(\rho_2 - \rho_1) - T\,k^3$$
$$b_2 = -\frac{k}{\lambda_1 m}\left(\frac{\mu_1 U_1}{\sigma_1} + \frac{\mu_2 U_2}{\sigma_2}\right)(2\lambda_1 k^2 + m) - \frac{\alpha k}{m}\left(\frac{U_1}{\sigma_1} + \frac{U_2}{\sigma_2}\right),$$

Note that, in the limiting case of semi-infinite fluids and absence of mass and heat transfer, the linear dispersion relation (29) reduces to the same dispersion relation obtained earlier in [33], and in the limiting case of non-porous media, it reduces to the corresponding equation given in [4]; while in the absence of both electric field and porous medium it tends to the same equation given by Chandrasekhar [2]. Therefore, their results are recovered.

5. Effect of vertical electric fields

Here, we shall consider the viscous potential flow analysis for two superposed dielectric finite fluids streaming with mass and heat transfer in a porous medium under the effect of constant applied electric fields E_{01} and E_{02} acting in the positive y-direction arise from the presence of electric potential difference between the upper and lower boundaries. The total electric fields in this case will be given by

$$\mathbf{E}_j = E_{0j}\mathbf{e}_y - \nabla\Psi_j,\ \ j = 1, 2. \tag{30}$$

we shall distinguish between the following two cases:

5.1. Absence of surface charges

Here we suppose that there are no surface charges (in the equilibrium state) at the surface of separation, hence the electric displacement is continuous at the interface [35], i.e., $\varepsilon_1 E_{01} = \varepsilon_2 E_{02}$. Now, the boundary conditions (12), (15), (16) and (20) will be replaced by the conditions:

The tangential components of the electric fields vanish on the rigid boundaries $y = -d_1$ and $y = d_2$, i.e.

$$\frac{\partial\Psi_j}{\partial x} = 0 \quad \text{on} \quad y = (-1)^j d_j\,, \quad j = 1, 2. \tag{31}$$

The tangential component of the electric field is supposed continuous at the interface,

$$\left\|\frac{\partial\Psi}{\partial x}\right\| + \frac{\partial\zeta}{\partial x}\left\|\frac{\partial\Psi}{\partial y}\right\| - \frac{\partial\zeta}{\partial x}\|E_0\| = 0,\ \text{at}\ \ y = \zeta(x, t). \tag{32}$$

The normal electric displacement is supposed continuous at the interface,

$$\frac{\partial\zeta}{\partial x}\left\|\varepsilon\frac{\partial\Psi}{\partial x}\right\| - \left\|\varepsilon\frac{\partial\Psi}{\partial y}\right\| = 0 \quad \text{at} \quad y = \zeta(x, t). \tag{33}$$

The conservation of momentum balance

$$\frac{1}{m}\left\|\rho\frac{\partial\Phi}{\partial t}\right\|+\frac{1}{m^2}\left\|\rho U\frac{\partial\Phi}{\partial x}\right\|+\frac{2}{m}\left\|\mu\frac{\partial^2\Phi}{\partial y^2}\right\|+\frac{1}{\lambda_1}\left\|\mu\Phi\right\|-\left\|\varepsilon E_0\frac{\partial\Psi}{\partial y}\right\|$$
$$+gy\left\|\rho\right\|+T\frac{\partial^2\zeta}{\partial x^2}=0. \tag{34}$$

Using the new set of boundary conditions, we get

$$\Psi_1=\frac{E_{01}(\varepsilon_2-\varepsilon_1)}{(\varepsilon_1\sigma_2+\varepsilon_2\sigma_1)}\frac{\sinh k(y+d_1)}{\cosh kd_1}Ae^{i\theta}+c.c., \tag{35}$$

$$\Psi_2=\frac{E_{02}(\varepsilon_2-\varepsilon_1)}{(\varepsilon_1\sigma_2+\varepsilon_2\sigma_1)}\frac{\sinh k(y-d_2)}{\cosh kd_2}Ae^{i\theta}+c.c. \tag{36}$$

Note that k and ω must satisfy the dispersion relation as:

$$a_0\omega^2+(a_1+ib_1)\omega+(a_2+ib_2)=0, \tag{37}$$

where

$$\begin{aligned}
a_0 &= \left(\frac{\rho_1}{\sigma_1}+\frac{\rho_2}{\sigma_2}\right),\\
a_1 &= -\frac{2k}{m}\left(\frac{\rho_1U_1}{\sigma_1}+\frac{\rho_2U_2}{\sigma_2}\right),\\
b_1 &= \frac{1}{\lambda_1}\left(\frac{\mu_1}{\sigma_1}+\frac{\mu_2}{\sigma_2}\right)\left(2\lambda_1k^2+m\right)+\alpha\left(\frac{1}{\sigma_1}+\frac{1}{\sigma_2}\right),\\
a_2 &= \frac{k^2}{m^2}\left(\frac{\rho_1U_1^2}{\sigma_1}+\frac{\rho_2U_2^2}{\sigma_2}\right)-\frac{\alpha}{\lambda_1}\left(\frac{\mu_1}{\rho_1\sigma_1}+\frac{\mu_2}{\rho_2\sigma_2}\right)\left(2\lambda_1k^2+m\right)\\
&\quad+\frac{k^2V_E(\varepsilon-1)^2}{(\varepsilon\sigma_1+\sigma_2)}+gk(\rho_2-\rho_1)-T\,k^3\\
b_2 &= -\frac{k}{\lambda_1m}\left(\frac{\mu_1U_1}{\sigma_1}+\frac{\mu_2U_2}{\sigma_2}\right)(2\lambda_1k^2+m)-\frac{\alpha k}{m}\left(\frac{U_1}{\sigma_1}+\frac{U_2}{\sigma_2}\right),
\end{aligned}$$

where $\varepsilon=\varepsilon_2/\varepsilon_1$, and$V_E=\varepsilon_1E_{01}E_{02}$.The relation (41) reduces to the same equation given in [2] in the limiting case of non-porous medium and absence of both electric field and mass and heat transfer. Also, It reduces to equations given in [46], and [35] in the limiting cases of semi-infinite fluids in absence and presence of electric field, respectively.

5.2. Presence of surface charges

Let us consider now surface charges present at the surface of separation and hence, our condition in this case will be $\varepsilon_1E_{01}\neq\varepsilon_2E_{02}$ and last boundary condition (36) will replaced by the following boundary condition the tangential component of the stress tensor is continuous at the interface [7], and hence we obtain

$$\frac{\partial \zeta}{\partial x} \left\| \varepsilon E_0^2 \right\| - \left\| \varepsilon E_0 \frac{\partial \Psi}{\partial x} \right\| = 0. \tag{38}$$

Then, we have

$$\Psi_1 = E_{01} \left(\frac{\sinh k(y + d_1)}{\sinh k d_1} \right) A e^{i\theta} + c.c., \tag{39}$$

$$\Psi_2 = -E_{02} \left(\frac{\sinh k(y - d_2)}{\sinh k d_2} \right) A e^{i\theta} + c.c. \tag{40}$$

Similarly,

$$a_0 \omega^2 + (a_1 + i b_1)\omega + (a_2 + i b_2) = 0, \tag{41}$$

where

$$\begin{aligned}
a_0 &= \left(\frac{\rho_1}{\sigma_1} + \frac{\rho_2}{\sigma_2} \right), \\
a_1 &= -\frac{2k}{m} \left(\frac{\rho_1 U_1}{\sigma_1} + \frac{\rho_2 U_2}{\sigma_2} \right), \\
b_1 &= \frac{1}{\lambda_1} \left(\frac{\mu_1}{\sigma_1} + \frac{\mu_2}{\sigma_2} \right) \left(2\lambda_1 k^2 + m \right) + \alpha \left(\frac{1}{\sigma_1} + \frac{1}{\sigma_2} \right), \\
a_2 &= \frac{k^2}{m^2} \left(\frac{\rho_1 U_1^2}{\sigma_1} + \frac{\rho_2 U_2^2}{\sigma_2} \right) - \frac{\alpha}{\lambda_1} \left(\frac{\mu_1}{\rho_1 \sigma_1} + \frac{\mu_2}{\rho_2 \sigma_2} \right) \left(2\lambda_1 k^2 + m \right) \\
&\quad + k \left(\frac{\varepsilon_1 E_{01}^2}{\sigma_1} + \frac{\varepsilon_2 E_{02}^2}{\sigma_2} \right) + gk(\rho_2 - \rho_1) - T\, k^3 \\
b_2 &= -\frac{k}{\lambda_1 m} \left(\frac{\mu_1 U_1}{\sigma_1} + \frac{\mu_2 U_2}{\sigma_2} \right) (2\lambda_1 k^2 + m) - \frac{\alpha k}{m} \left(\frac{U_1}{\sigma_1} + \frac{U_2}{\sigma_2} \right),
\end{aligned}$$

The dispersion relation (41) reduces to the same equation given in [35] in the absence of both porous medium and mass and heat transfer. Also, It reduces to the same linear dispersion equations achieved in [7] in the limiting cases of semi-infinite fluids in absence of both velocities (Rayleigh-Taylor case) and mass and heat transfer, respectively.

6. Numerical analysis and stability discussion

The dispersion relations (29), (37) and (41) obtained when the applied electric fields are horizontal and vertical (in the absence or presence of surface charges), respectively, represent quadratic equations in ω with complex coefficients. Applying the Routh-Hurwitz stability criterion [42, 47] to relations (29), (37) and (41), we get the necessary and sufficient conditions for stability as

$$b_1 > 0 \quad \text{and} \quad a_2 b_1^2 - a_1 b_1 b_2 + a_0 b_2^2 \leq 0, \tag{42}$$

Since λ_1, m and α are always positive, then the first condition in (42) is trivially satisfied. Now we will show numerically the linear stability of our considered system by drawing (using Mathematica 7.0) the transition curves represented by

$$F_1 = a_2 b_1^2 - a_1 b_1 b_2 + a_0 b_2^2 = 0, \tag{43}$$

in the $F_1 - k$ plane for various values of the parameters including in the analysis. These transition curves that are presented in Figs.1–4, separate the stable S from unstable U regions, for the system having $\rho_1 = 0.9956$ gr/cm^3, $\rho_2 = 0.0012$ gr/cm^3(potentially stable configuration), $\varepsilon_1 = 1.7$ farad/cm, $\varepsilon_2 = 1.007$ farad/cm, $d_1 = 0.5$ cm, $d_2 = 0.3$ cm, $\mu_1 = 0.4$ cm^2/sec, $\mu_2 = 0.1$ cm^2/sec, $U_1 = 5$ cm/sec, $U_2 = 3$ cm/sec, $\lambda_1 = 0.9$ cm^2, $\alpha = 0.5$ gr/cm^3, $g = 980$ cm/sec^2, $T = 76$ dyn/cm, $m = 0.09$ sec/cm. Hence, the stability criterion occurs when $F_1 \leq 0$, otherwise, instability holds when $F_1 > 0$. Now, Figs. 1–4 are drawn to compare between three cases: (a) horizontal applied electric field E_0 and (b) vertical applied electric fields V_E in the absence of surface charges at the interface while (c) vertical applied electric fields $E_{01}$and E_{02} in the presence of surface charges at the interface.

Figs. (1a, b, c) show the variation of F_1 with k for different values of the electric field (a) horizontal applied electric field $E_0 = 0, 30, 50, 70$, (b) vertical applied electric fields $V_E = 0, 300, 700, 1000$ (c) vertical applied electric fields $E_{01} = 0, 10, 12, 14$ and $E_{02} = 0, 5, 7, 9$ respectively. It is found that in the absence of electric field ($E_0 = 0$, $V_E = 0$, or $E_{01} = 0$,and $E_{02} = 0$) for all cases, the system is stable, As E_0 increases in (a) from $E_0 = 30$ to $E_0 = 70$, it indicates that at any value of E_0, the system is always stable, and also that this stability is less effective for the wavenumber range $1 < k < 3$. It is obvious also that this stability behavior increases by increasing the the electric field parameter E_0. While in case (b) as V_E increases $V_E = 300, 700, 1000$, it indicates that the stability decreased by increasing the vertical electric field parameter V_E and this stability is less effective for the wavenumber range $0.8 < k < 3.2$, and this reduction in the stability of the system occurs faster in case (c) when E_{01} and E_{02} are increasing $E_{01} = 10, 12, 14$ and $E_{02} = 5, 7, 9$.

Figs. (2a, b, c) show the variation of $F_1$with k for different values of the fluid depths $d_1$and d_2, for the same system in Figs. (1a, b, c) in the cases of (a) horizontal electric field $E_0 = 10$ V/cm (b) vertical electric field in the absence of surface charges $V_E = 100$ V/cm, and (c) vertical electric field in the presence of surface charges, $E_{01} = 15$ V/cm, $E_{02} = 7$ V/cm, respectively. It is found that for small values of d_1 and d_2 in all cases (a, b, c) e.g. when ($d_1 = 0.7$, $d_2 = 0.4$), the system is always stable. By increasing the values of the fluid depths, i.e. when ($d_1 = 0.9$, $d_2 = 0.6$) and also ($d_1 = 1.1$, $d_2 = 0.8$) the system is still stable and also that this stability is decreased by increasing the fluid depths. For two semi-infinite fluids, i.e. when ($d_1 \to -\infty$, $d_2 \to \infty$), the stability also occurs, and this stability is less effective compared with finite fluids. Therefore, the fluid depths d_1 and d_2 tends to reduce the stability of the system. Similarly, it is found that the fluid velocities U_1 and U_2 have the same effects on the stability of the system as the effect of fluid depths $d_1$and d_2 discussed in Figs. (2a, b, c) and it is more stable in the absence of fluid velocities case (Rayleigh-Taylor instability), but the corresponding

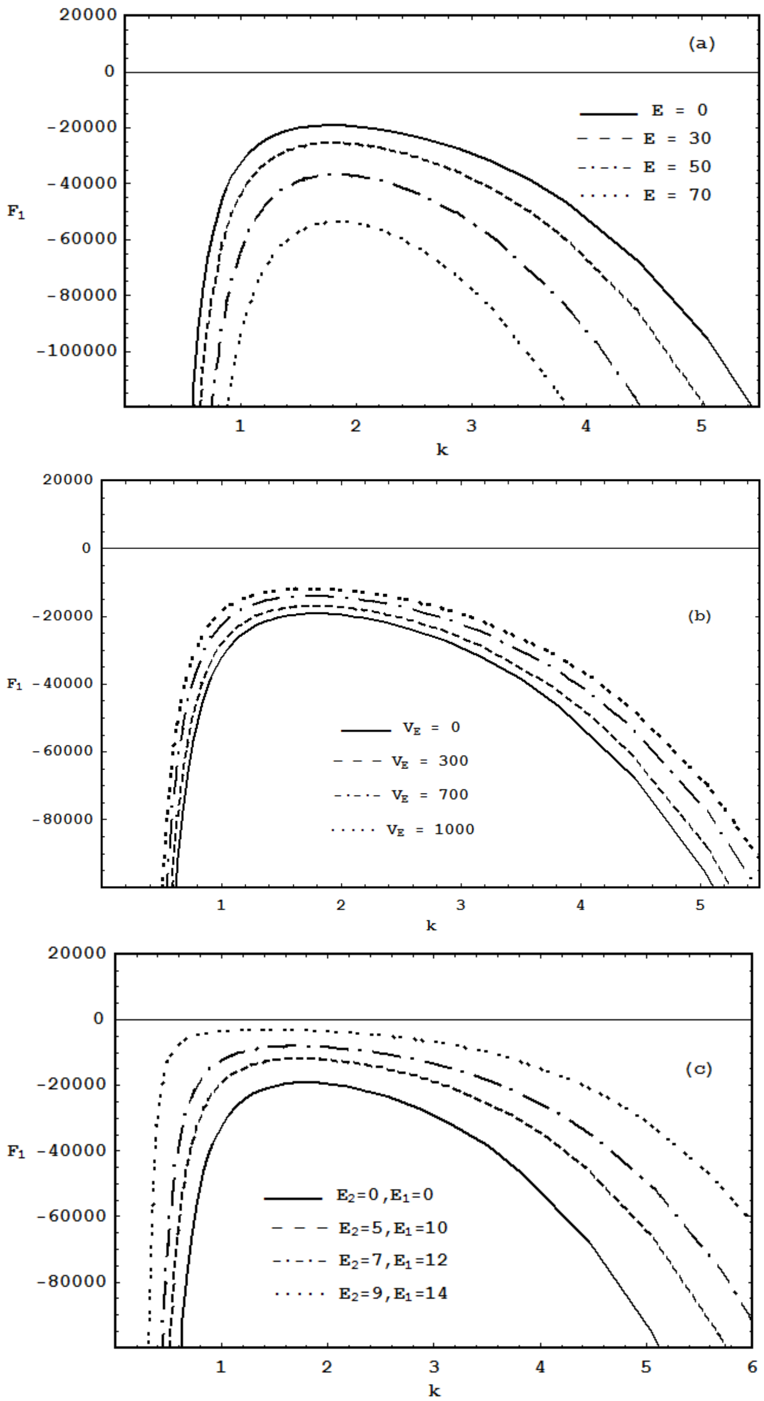

Figure 1: variation of F_1 with k for different values of the electric field

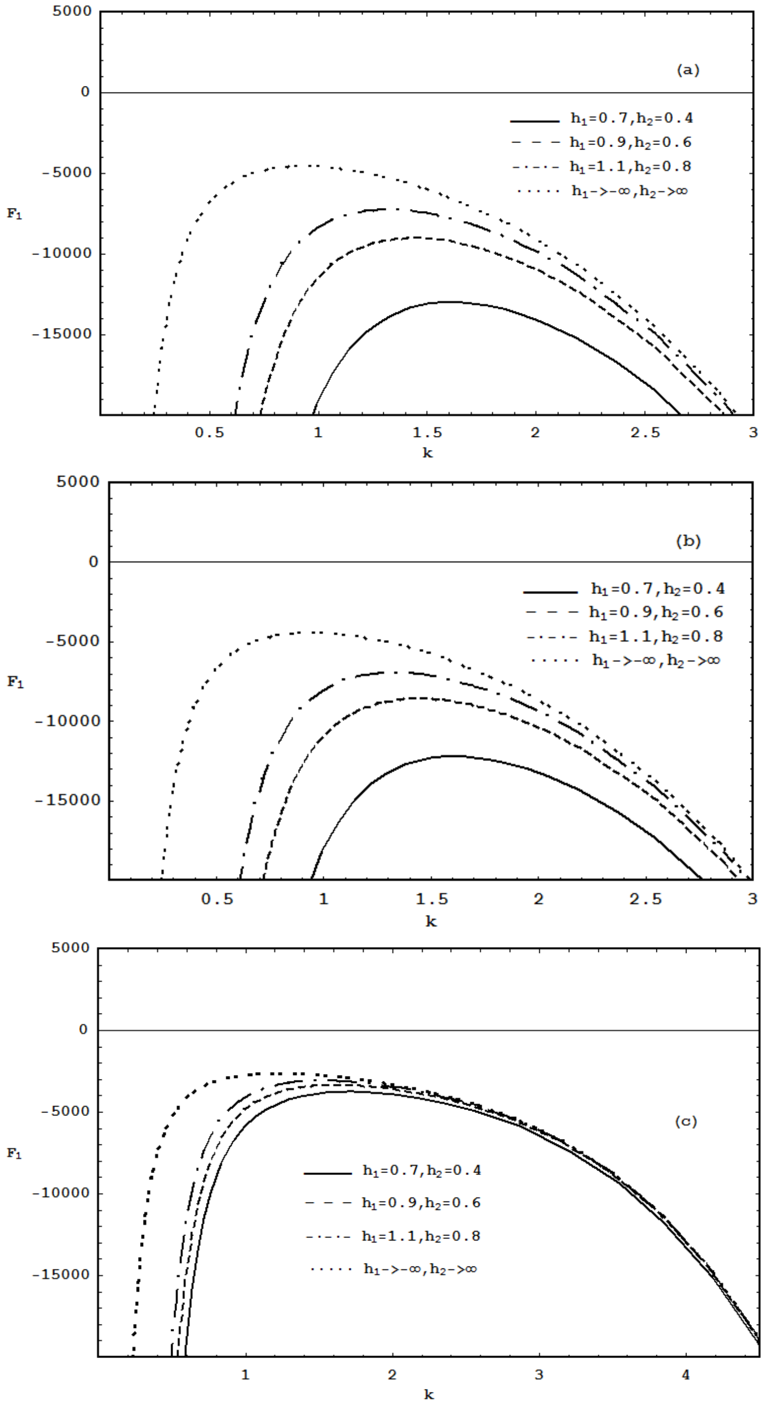

Figure 2: variation of F_1 with k for different values of the fluid depths d_1 and d_2, for the same system

figure is excluded to save a space. It should be noted also that the effect of surface tension T on the stability of the system are found to be opposite to the effects of fluid depths $d_1$and d_2 shown in Figs. (2a, b, c) and stability for the cases (a, b) occurs faster and stronger than the corresponding case (c), but figures will not be given here to avoid any repetition.

Figs. (3a, b, c) show the variation of $F_1$with k for different values of the fluid viscosities μ_1 and μ_2. It is obtained that for non-porous medium when $\mu_1 = \mu_2 = 0$, (and also $m = 1$ and $\lambda_1 \to \infty$), the system is stable for wavenumber range $k \leq 2.4$ for (a,b) and stable for wavenumber range $k \leq 1$ for (c), otherwise it has been found to be neutrally stable in all cases. While, for definite values of the fluid viscosities μ_1 and μ_2, i.e. when ($\mu_1 = 0.4$, $\mu_2 = 0.1$), the system is always stable. As μ_1 and μ_2 increases, e.g. when ($\mu_1 = 0.5$, $\mu_2 = 0.2$) and also ($\mu_1 = 0.6$, $\mu_2 = 0.3$), the stability occurs, the system is always stable, and also that this stability is the least effective for case (c) compared with other cases (a, b). It should be mentioned here that similar effects is found for the porosity of porous medium m, on the stability of the system as illustrated in Figs. (3a, b, c), but the corresponding figure is not given here. While opposite effects is found for the medium permeability λ_1 on the stability of the system to that given in Figs. (3a, b, c), but its figure is also removed to save a space.

Figs. (4a, b, c) represent the variation of $F_1$with k for different values of the mass and heat transfer coefficient α. It is found that in the cases (a, b) for absence of mass and heat transfer coefficient $\alpha = 0$ the system is always stable for any wavenumber value k and as α increases, e.g. when ($\alpha = 0.9$, $\alpha = 1.5$ and also $\alpha = 3$) the stability occurs and this stability increases by increasing α. While in case (c) when $\alpha = 0$, it is found that for small wavenumber range $k \leq 0.5$ the system is unstable, otherwise the system is stable for wavenumber range $k > 0.5$. By increasing the coefficient α e.g. when ($\alpha = 0.9$, $\alpha = 1.5$ and $\alpha = 3$) the system is always stable and this stability increases by increasing α. Therefore, we conclude that the mass and heat transfer α increase the stability of the system when the applied electric fields is horizontal or vertical (in absence of surface charges) and a dual role for the case of vertical (in presence of surface charges) electric fields. Finally, from Figs. (1a, b, c)-(4a, b, c), we notice that the effects of various parameters on the stability of the system is more strong than the case of vertical electric field in presence of surface charges at the interface.

7. Conclusions

Using viscous potential flow theory, the linear electrohydrodynamic stability of the plane interface between two uniform superposed streaming dielectric fluids of finite depths through a porous medium is considered under the influence of general applied horizontal or vertical (in absence and presence of surface charges) electric fields. The obtained results can be summarized as follows:

Case I: If the applied electric field acted tangentially to the interface between the two fluids, we found that

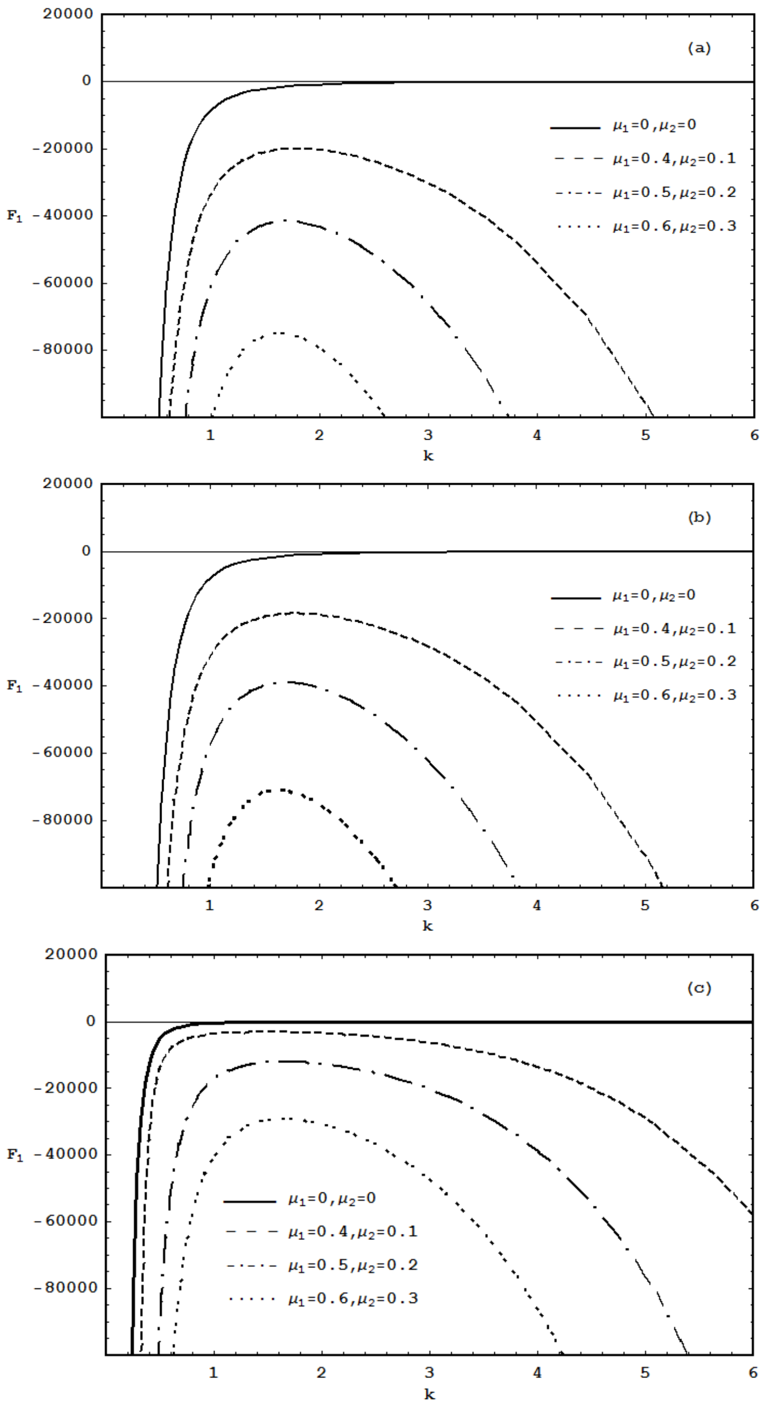

Figure 3: variation of F_1 with k for different values of the fluid viscosities μ_1 and μ_2.

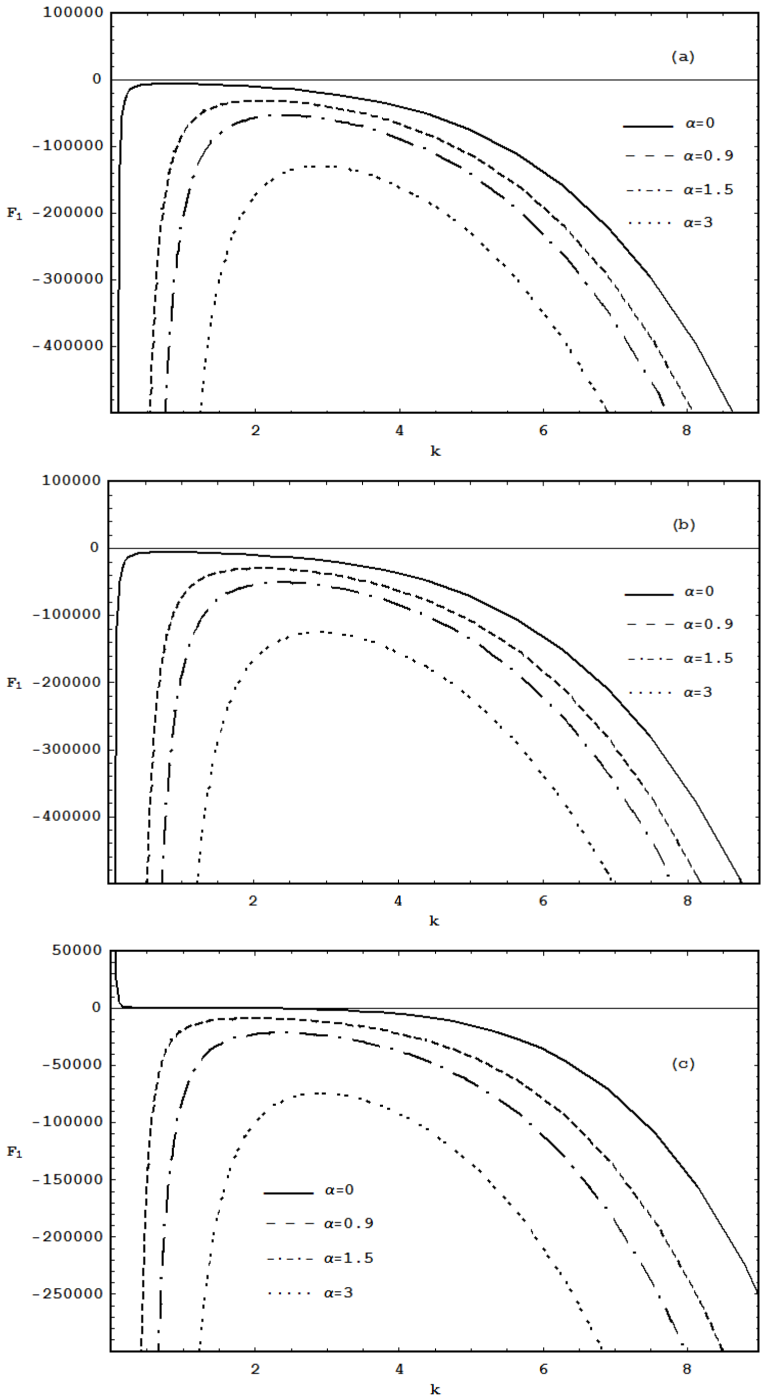

Figure 4: variation of F_1 with k for different values of the mass and heat transfer coefficient α.

1. The horizontal electric field E_0, porosity of porous medium m, surface tension T, fluid viscosities μ_1, μ_2 and mass and heat transfer α have stabilizing effects or increase the stability of the system.

2. The fluid velocities U_1, U_2, medium permeability λ_1, and fluid depths d_1, d_2 (including the case of semi-infinite fluids) have destabilizing effects or tends to reduce the stability of the system.

Case II: If the applied electric fields acted normally to the interface between the two fluids, we found that the instability in the case of the presence of surface charges at the interface is stronger than its effect in the corresponding case of the absence of surface charges, and also that

1. The vertical electric field parameterV_E or (E_{01} and E_{02}), fluid velocities U_1, U_2, medium permeability λ_1, and fluid depths d_1, d_2 (including the case of semi-infinite fluids) have destabilizing effects or tends to reduce the stability of the system.

2. The porosity of porous medium m, surface tension T, and fluid viscosities μ_1, μ_2 have stabilizing effects or increase the stability of the system.

3. The mass and heat transfer coefficient α has a dual role (stabilizing as well as destabilizing) for the case of vertical (in presence of surface charges) electric fields, while it has a stabilizing effect on the system for the case of vertical (in absence of surface charges) electric fields.

For the previous cases:

1. In non-porous media, (when $\lambda_1 \to \infty$, μ_1, $\mu_2 \to 0$, and $m = 1$) the system has been found to be neutrally stable after a fixed wavenumber value.

2. The system has been found to be stable in absence of fluid velocities $U_1 = 0$ and $U_2 = 0$ (Rayleigh-Taylor stability).

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