Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 5 (2016), pp. 3941-3947

http://www.ripublication.com/gjpam.htm

A Modified Group Chain Sampling Plans for Lifetimes Following a Rayleigh Distribution

Aiman Fikri Jamaludin, Zakiyah Zain and Nazrina Aziz

*Department of Mathematics and Statistics, School of Quantitative Sciences,
UUM College of Arts and Sciences, Universiti Utara Malaysia.
UUM Sintok 06010, Kedah, Malaysia.*

Abstract

In this paper, modified group chain sampling plan is developed for a truncated life test when the lifetime of an item follows Rayleigh distribution. Optimal number of groups and operating characteristic value are obtained by obeying the specified consumer's risk, test termination time and mean ratio.

Keyword: Truncated life test, Consumer's risk, Rayleigh distribution, Operating characteristic values, Number of Groups.

INTRODUCTION

Acceptance sampling is a quality control tool that has widely been used in industries. It considers inspection and making decision procedures to decide either to accept the products in the submitted lot or to reject them. Instead of testing every single item, the inspection process involves some samples from the entire lot which will save the cost and time. In addition, acceptance sampling provides protection to both producer and consumer. Many type of sampling plans have been introduced by researchers based on time truncated life test by using different kind of lifetime distributions such as Epstein [4], Baklizi [2], Lio, Tsai and Wu [6], Mohana and Rao [11], Aslam et al. [1], Rao [10], Mughal et al. [7] and Mughal and Ismail [8].

Dodge [3] suggested a chain sampling plan (ChSp-1) by using cumulative information of various samples. The acceptance condition of the current lot is based on the quality of immediately preceding sample. Chain sampling solved the shortcoming of the single sampling plan with zero acceptance number by allowing the current lot with one defective item to be accepted provided the preceding samples have zero defective item.

Recently, Mughal, Zain and Aziz [9] introduced group chain sampling plan where the procedure of chain sampling is applied with multiple items is tested simultaneously to save more time. The items in the tester can be distributed into groups of equal size and the number of items in a group is known as group size. Another improvement for chain sampling is established by Govindaraju and Lai [5] known as modified chain sampling plan which allows one defective item in any one of i preceding samples. This technique performed better than the existing chain sampling plans by increasing the probability of lot acceptance at good quality standard, while maintaining the probability of lot acceptance at poor quality standard. As a result, modified chain sampling offers better protection to the producer.

The main idea of this study is to combine both good concepts of the research by Mughal, Zain and Aziz [9] and Govindaraju and Lai [5] to develop a modified group chain sampling plan based on truncated life test for Rayleigh distribution as the lifetime distribution. This study is intended to determine the optimal number of groups which is proportional to the optimal sample size and to generate the operating characteristics values where various design parameters is taken into account.

GLOSSARY

g	:	Number of groups
r	:	Group size
n	:	Sample size
d	:	Number of defective items
i	:	Number of preceding samples
α	:	Producer's risk (Probability of rejecting a good lot)
β	:	Consumer's risk (Probability of accepting a bad lot)
$P_a(p)$	:	Probability of lot acceptance
μ/μ_0	:	Mean ratio
t_0	:	Test termination time
a	:	Test termination time multiplier

OPERATING PROCEDURE

By referring to the operating procedure of modified chain sampling plan in Govindaraju and Lai [5] and making an adjustment to involve number of groups and number of testers in each group, the proposed plan is constructed in the following steps:

1) For each of the submitted lots, find the minimal number of groups g for each and allocate r items to each group such that the sample size $n = r \times g$.
2) Specify the test termination time, t_0.
3) Inspect all the groups simultaneously and record the number of defective items, d found in each of the groups. Terminate the inspection at $t = t_0$.

4) Accept the lot if no defective items are found in the current under inspection sample provided the preceding *i* samples also contain no defective items. Also accept the lot if no defective items are found in the current under inspection sample provided any one of *i* preceding samples contains at most one defective item and the rest (i-1) samples have no defective items. Otherwise, reject the lot.

PROBABILITY OF DEFECTIVE

The cumulative distribution function (cdf) of Rayleigh Distribution is given by

$$\boldsymbol{F(t;\delta) = 1 - exp\left[-\frac{1}{2}\left(\frac{t}{\delta}\right)^{2}\right]} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (1)$$

$t > 0$, $\delta > 0$ where δ is the scale parameter.

When the lifetime follows the Rayleigh distribution, the true mean life is given by

$$\mu = \sigma\sqrt{\frac{\pi}{2}} \text{ Therefore } \boldsymbol{\sigma = \sqrt{2\mu^2/\pi}} \qquad (2)$$

The termination time, t_0 is a multiple of pre-assumed constant, a and specified mean life , μ_0.

$$\boldsymbol{t_0 = a\mu_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (3)$$

Due to the fact that lifetime distribution function depends only on time and scale parameter, the defective probability of an item is given by:

$$\boldsymbol{p = F(t_0\ ;\ \delta)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (4)$$

In the other word, p is the probability that a defective item is found in the sample before the test termination time, $\boldsymbol{t_0}$ reached.
We can rewrite equation (4) as a function of constant, a and mean ratio, (μ/μ_0).

$$\boldsymbol{p = F(a\mu_0 :\ \mu/\mu_0\ )} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (5)$$

Therefore by substituting equation (2) and (3) into equation (1), the probability of defective when we use Rayleigh as underlying distribution can be written as:

$$\boldsymbol{p = 1 - \exp\left[\left(\frac{-\pi a^2}{4}\right)\left(\frac{1}{\mu/\mu_0}\right)\right]} \quad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (6)$$

OPERATING CHARACTERISTIC

The Operating Characteristic (OC) function of modified chain sampling plan is given by:

$$\boldsymbol{P_a(p) = P_{0,n}\left[{P_{0,n}}^{i} + i\ {P_{0,n}}^{i-1}\, P_{1,n}\right]} \qquad (7)$$

where $\boldsymbol{P_{0,n}}$ and $\boldsymbol{P_{1,n}}$ are the probability of observing zero and one defective item in a sample of size, *n* respectively.

Since multiple item inspection is applied, n can be considered as g x r and under the condition of Binomial distribution, we can rewrite equation (7) as:

$$\boldsymbol{P_a(p) = (1-p)^{gr}\left[(1-p)^{gri} + i\,(1-p)^{gr(i-1)}\,grp(1-p)^{gr-1}\right]} \quad (8)$$

where p is the probability of defective obtained from (6).

CONSTRUCTION OF TABLES

Suppose μ is the true mean life and μ_0 is the specified mean life of a specific product. Both are used to investigate the quality standard of the product. A product is considered as good and acceptable when $\mu \geq \mu_0$ while it is considered as bad when $\mu < \mu_0$. Hence, the value of mean ratio,(μ/μ_0) must always more than 1 for conducting the experiment. The consumer's risk β is the probability of accepting a bad lot. In order to ensure that the consumer's risk does not exceed the specified β, we will determine the optimal number of groups, g through the inequality:

$$\boldsymbol{P_a(p_0) \leq \beta} \quad (9)$$

where p_0 is the probability of defective at $\mu = \mu_0$. In a time truncated life test, number of defective items is observed until a pre-specified termination time, t_0. If the number of defective observed exceeds the specified number of acceptance, then the submitted lot will rejected. Otherwise, the lot will be accepted. As it is convenient to specify the termination time as a multiple of the specified mean life, then we can have $t_0 = a\mu_0$. Table 1 shows the optimal group size, g obtained by obeying equation (9) for $\beta = 0.25, 0.10, 0.05, 0.01$; $a = 0.7, 0.8, 1.0, 1.2, 1.5, 2.0$; $r = 2(1)5$ and $i = 1(1)4$.

Consequently, the operating characteristic (OC) at different levels of β can be determined by using the values of optimal g obtained for a fixed r and i values. Different a and (μ/μ_0) are also taken into consideration to see the pattern of the outcomes. Table 2 shows the OC values generated in details.

Table 1: Number of optimal group size for the proposed plan.

			a					
β	*r*	*i*	0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	1	2	2	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.1	2	1	3	2	2	1	1	1
	3	2	2	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1

0.05	2	1	3	3	2	2	1	1
	3	2	2	2	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	5	4	3	2	1	1
	3	2	2	2	1	1	1	1
	4	3	2	1	1	1	1	1
	5	4	1	1	1	1	1	1

Table 2: Operating Characteristics values for the proposed plan.

			μ/μ0						
β	g	a	1	2	4	6	8	10	12
0.25	1	0.7	0.01405	0.47449	0.87937	0.95148	0.97401	0.98379	0.98891
	1	0.8	0.00284	0.34908	0.83660	0.93474	0.96536	0.97852	0.98536
	1	1.0	0.00005	0.15573	0.73122	0.89204	0.94348	0.96536	0.97658
	1	1.2	0.00000	0.05338	0.60637	0.83660	0.91497	0.94837	0.96536
	1	1.5	0.00000	0.00653	0.41020	0.73122	0.85887	0.91497	0.94348
	1	2.0	0.00000	0.00005	0.15573	0.51838	0.73122	0.83660	0.89204
0.1	1	0.7	0.01405	0.47449	0.87937	0.95148	0.97401	0.98379	0.98891
	1	0.8	0.00284	0.34908	0.83660	0.93474	0.96536	0.97852	0.98536
	1	1.0	0.00005	0.15573	0.73122	0.89204	0.94348	0.96536	0.97658
	1	1.2	0.00000	0.05338	0.60637	0.83660	0.91497	0.94837	0.96536
	1	1.5	0.00000	0.00653	0.41020	0.73122	0.85887	0.91497	0.94348
	1	2.0	0.00000	0.00005	0.15573	0.51838	0.73122	0.83660	0.89204
0.05	1	0.7	0.01405	0.47449	0.87937	0.95148	0.97401	0.98379	0.98891
	1	0.8	0.00284	0.34908	0.83660	0.93474	0.96536	0.97852	0.98536
	1	1.0	0.00005	0.15573	0.73122	0.89204	0.94348	0.96536	0.97658
	1	1.2	0.00000	0.05338	0.60637	0.83660	0.91497	0.94837	0.96536
	1	1.5	0.00000	0.00653	0.41020	0.73122	0.85887	0.91497	0.94348
	1	2.0	0.00000	0.00005	0.15573	0.51838	0.73122	0.83660	0.89204
0.01	2	0.7	0.00006	0.15756	0.73377	0.89350	0.94437	0.96595	0.97700
	1	0.8	0.00284	0.34908	0.83660	0.93474	0.96536	0.97852	0.98536
	1	1.0	0.00005	0.15573	0.73122	0.89204	0.94348	0.96536	0.97658
	1	1.2	0.00000	0.05338	0.60637	0.83660	0.91497	0.94837	0.96536
	1	1.5	0.00000	0.00653	0.41020	0.73122	0.85887	0.91497	0.94348
	1	2.0	0.00000	0.00005	0.15573	0.51838	0.73122	0.83660	0.89204

DESCRIPTION OF TABLES AND EXAMPLES

The consumer' risk, β is related to consumer's confidence level. If the confidence level denoted by p^*, then $\beta = 1 - p^*$. Assume that the lifetime of the product follows Rayleigh distribution and the experimenter is interested to have at least 700 hours of true mean life with 99% confidence level. He has an advantage to inspect multiple items on a tester simultaneously. Let say, the experimenter choose the values of a = 0.7, r = 4 and i = 3 for conducting the experiment. Then, the optimal group size required is g = 2 as shown in Table 1. Hence, the design parameters are $(a, r, i, g) = (0.7, 4, 3, 2)$ which means he must draw a random sample of size 8 where 2 groups in that sample must have 4 items each to put on the tester. If no defective items is found in the current under inspection sample during 700 hours provided that all 3 preceding samples are free from any defective, the lot will be accepted. If no defective item is found in the current under inspection sample during 700 hours provided that one of the preceding samples has one defective item while the rest two samples are free from any defective, the lot will also be accepted. Otherwise, the lot will be rejected. By using the same value of design parameters, the value of operating characteristic will increase from 0.15756 to 0.97700 when the mean ratio increases from 2 to 12 as recorded in Table 2.

CONCLUSION

In this paper, a time truncated modified group chain sampling plan is developed when the life time of the product follows Rayleigh distribution. For this proposed plan, the number of inspected items is directly proportional to the number of group. It is observed that, the optimal number of group decreases when the test termination time multiplier increases. It is also recorded that the operating characteristics increases as the mean ratio increases. However, the operating characteristics decreases as the test termination time multiplier increases. In modified group chain sampling plan, the experimenter has an advantage to inspect multiple items on a tester simultaneously. In addition, it has shown that modified group chain sampling plan provides extra chances of acceptance as the acceptance condition of a product is based on the quality of preceding samples. Hence, it would be beneficial in terms of cost, time and protection to the producer.

REFERENCES

[1] Aslam, M. , Mughal, A. R. , Ahmad, M. & Yab, Z. (2010). Group Acceptance Sampling Plan for Pareto Distribution of the Second Kind. *Journal of Testing and Evaluation, 38*(2), 1-8.

[2] Baklizi, A. (2003). Acceptance sampling based on truncated life tests in the Pareto distribution of the Second Kind. *Advances and Applications in Statistics, 3*(1), 33-48.

[3] Dodge, H. F. (1955). Chain Sampling Inspection Plan. *Industry Quality Control, 11*(4), 10-13.

[4] Epstein, B. (1954). Truncated Life Tests in the Exponential Case. *The Annals of Mathematical Statistics, 25*(3), 555-564.

[5] Govindaraju, K. & Lai, C. D. (1998). A modified ChSP-1 chain sampling plan, M ChSP-1, with very small sample sizes. *American Journal of Mathematics and Management Sciences, 18*, 346-358.

[6] Lio, Y. L. , Tsai, T-R. & Wu, S-J. (2010). Acceptance Sampling Plans from truncated life tests based on the Burr Type XII Percentiles. *Journal of the Chinese Institute of Industrial Engineers, 27*(4), 270-280.

[7] Mughal, A. R. , Hanif, M. , Ahmad, M. & Rahman, A. (2011). Economic Reliability Acceptance Sampling Plans from Truncated Life Tests based on the Burr Type XII Percentiles. *Pakistan Journal of Commerce and Social Sciences, 5*(1), 166-176.

[8] Mughal, A. R. & Ismail, M. (2013). An Economic Reliability Efficient Group Acceptance Sampling Plans for Family Pareto Distributions. *Research Journal of Applied Sciences, Engineering and Technology, 6*(24), 4646-4652.

[9] Mughal, A. R., Zain, Z., & Aziz, N. (2015). Time truncated Group Chain Sampling Strategy for Pareto Distribution of the 2^{nd} Kind. *Research Journal of Applied Sciences, Engineering and Technology, 10*(4), 471-474.

[10] Rao, G. S. (2013). Acceptance Sampling Plans for percentiles based on the Marshall-Olkin Extended Lomax Distribution. *International Journal of Statistics and Economics, 11*(2), 83-96.

[11] Rao, P. R. M. & Rao, G. S. (2013). Percentiles Based Construction of Acceptance Sampling Plans for the Truncated Type-1 Generalized Logistic Distribution. *American Journal of Mathematics and Statistics, 3*(4), 194-203.