Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 2 (2016), pp. 1853-1871

http://www.ripublication.com/gjpam.htm

New System of a Parametric General Regularized Nonconvex Variational Inequalities in Banach Spaces

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Abstract

In this paper, we study the behaviour of solution sets for a new system of parametric general regularized nonconvex variational inequalities in q-uniformly smooth Banach spaces with locally relaxed (φ, ψ)-cocoercive mappings.

AMS subject classification: 49J40.
Keywords: New system of a parametric general regularized nonconvex variational inequalities, locally relaxed (φ, ψ)-cocoercive mapping, uniformly prox regular sets, q-uniformly smooth Banach spaces.

1. Introduction

Sensitivity analysis for the solutions of variational inequalities and inclusions have been studied by many authors via quite different methods. By the projection methods, Anastassiou et al. [2], Balooee and Kim [3], Chang et al. [7], Dafermos [9], Faraj and Salahuddin [10], Khan and Salahuddin [13], Lee and Salahuddin [18], Mohapatra and Verma [20], Pan [22], Qiu and Magnanti [23], Salahuddin [25, 26], Verma [27], and Yen [28] studied the sensitivity analysis for the solutions of some variational inequalities with single-valued mappings or set-valued mappings in finite dimensional spaces, or Hilbert spaces. By using the resolvent operator techniques, Agarwal et al. [1], Jeong [11, 12], Kim et al. [14], and Kim and Kim [16, 17] studied a new system of parametric generalized mixed quasi variational inclusions in Hilbert spaces and in $L_p(p \geq 2)$ spaces, respectively.

In this paper, we study the behaviour and sensitivity analysis of the solution set for a new system of parametric general regularized nonconvex variational inequalities with locally relaxed (φ, ψ)-cocoercive mappings in Banach spaces.

Let $\mathcal{X}$ be a real Banach space with dual space $\mathcal{X}^*$, the norm $\|\cdot\|$ and a dual pairing $\langle\cdot,\cdot\rangle$ between $\mathcal{X}$ and $\mathcal{X}^*$. Let $CB(\mathcal{X})$ denotes the family of all nonempty closed bounded subsets of $\mathcal{X}$ and let $\mathfrak{D}(\cdot,\cdot)$ be the Hausdorff metric on $CB(\mathcal{X})$, that is, for all $A, B \in CB(\mathcal{X})*$,

$$\mathfrak{D}(A, B) = \max\left\{\sup_{x\in A}\inf_{y\in B}||x-y||, \sup_{y\in B}\inf_{x\in A}||x-y||\right\}.$$

The generalized duality mapping $J_q : \mathcal{X} \to 2^{\mathcal{X}^*}$ is defined by

$$J_q(x) = \{f^* \in \mathcal{X}^* : \langle x, f^*\rangle = \|x\|^q, \|f^*\| = \|x\|^{q-1}\}, \ \forall x \in \mathcal{X}$$

where $q > 1$ is a constant. In particular, J_2 is a usual normalized duality mapping. It is known that in general $J_q(x) = \|x\|^{q-2}J_2(x)$ for all $x \neq 0$ and J_q is single-valued if $\mathcal{X}^*$ is strictly convex.

In the sequel, we always assume that $\mathcal{X}$ is a real Banach space such that J_q is single-valued. If $\mathcal{X}$ is a Hilbert space, then J_q becomes the identity mapping on $\mathcal{X}$. The modulus of smoothness of $\mathcal{X}$ is the function $\rho_{\mathcal{X}} : [0,\infty) \to [0,\infty)$ defined by

$$\rho_{\mathcal{X}}(t) = \sup\left\{\frac{1}{2}\big(\|x+y\| + \|x-y\|\big) - 1 : \|x\| \leq 1, \|y\| \leq t\right\}.$$

A Banach space $\mathcal{X}$ is called uniformly smooth if

$$\lim_{t\to 0}\frac{\rho_{\mathcal{X}}(t)}{t} = 0.$$

$\mathcal{X}$ is called q-uniformly smooth if there exists a constant $c > 0$ such that

$$\rho_{\mathcal{X}}(t) < ct^q, q > 1.$$

Note that J_q is single-valued if $\mathcal{X}$ is uniformly smooth. Concerned with the characteristic inequalities in q-uniformly smooth Banach spaces. Xu [29] proved the following results.

Lemma 1.1. [29] The real Banach space $\mathcal{X}$ is q-uniformly smooth if and only if there exists a constant $c_q > 0$ such that for all $x, y \in \mathcal{X}$

$$\|x + y\|^q \leq \|x\|^q + q\langle y, J_q(x)\rangle + c_q\|y\|^q.$$

Definition 1.2. Let $\mathcal{K}$ be a nonempty closed subset of a Banach space $\mathcal{X}$. The proximal normal cone of $\mathcal{K}$ at a point $u \in \mathcal{X}$ with $u \notin \mathcal{K}$ is given by

$$N_{\mathcal{K}}^P(u) = \{\zeta \in \mathcal{X} : u \in P_{\mathcal{K}}(u + \alpha\zeta) \text{ for some } \alpha > 0\},$$

where

$$P_{\mathcal{K}}(u) = \{v \in \mathcal{K} : d_{\mathcal{K}}(u) = \|u - v\|\}.$$

Here $d_{\mathcal{K}}(\cdot)$ is the usual distance function to the subset $\mathcal{K}$, *i.e.,*

$$d_{\mathcal{K}}(u) = \inf_{v \in \mathcal{K}} \|u - v\|.$$

We have the characterizations for the proximal normal cone $N_{\mathcal{K}}^P(u)$.

Lemma 1.3. [8] Let $\mathcal{K}$ be a nonempty closed subset in $\mathcal{X}$. Then $\zeta \in N_{\mathcal{K}}^P(u)$ if and only if there exists a constant $\alpha = \alpha(\zeta, u) > 0$ such that

$$\langle \zeta, j_q(v - u)\rangle \leq \alpha\|v - u\|^q, \quad \forall v \in \mathcal{K}.$$

Lemma 1.4. [8] Let $\mathcal{K}$ be a nonempty closed and convex subset in $\mathcal{X}$. Then $\zeta \in N_{\mathcal{K}}^P(u)$ if and only if

$$\langle \zeta, j_q(v - u)\rangle \leq 0, \quad \forall v \in \mathcal{K}.$$

The Clarke normal cone $N_{\mathcal{K}}^C(u)$ is defined by

$$N_{\mathcal{K}}^C(u) = \overline{co}\{N_{\mathcal{K}}^P(u)\},$$

where $\overline{co}$ is the closure of the convex hull. Clearly $N_{\mathcal{K}}^P(u) \subseteq N_{\mathcal{K}}^C(u)$, but the converse is not true in general. Note that $N_{\mathcal{K}}^C(u)$ is always closed and convex where as $N_{\mathcal{K}}^P(u)$ is always convex but may not be closed (see [4, 5, 6, 8, 24]).

Definition 1.5. [8, 24] For any $r \in (0, +\infty]$, a subset $\mathcal{K}_r$ of $\mathcal{X}$ is said to be normalized uniformly r-prox regular (or uniformly r-prox regular) if and only if every nonzero proximal normal to $\mathcal{K}_r$ can be realized by an r-ball, that is, for all $u \in \mathcal{K}_r$ and all $0 \neq \zeta \in N_{\mathcal{K}_r}^P(u)$ with $\|\zeta\| = 1$,

$$\langle \zeta, v - u\rangle \leq \frac{1}{2r}\|v - u\|^2, \quad v \in \mathcal{K}_r.$$

Lemma 1.6. [8] A closed set $\mathcal{K} \subseteq \mathcal{X}$ is convex if and only if it is proximally smooth of radius r for every $r > 0$.

If $r = \infty$ then uniformly prox regularity of $\mathcal{K}_r$ is equivalent to the convexity of $\mathcal{K}$. If $\mathcal{K}_r$ is a uniformly prox regular set, then the proximal normal cone $N^P_{\mathcal{K}_r}(u)$ is closed as a set-valued mapping. If we take $\eta = \frac{1}{2r}$, it is clear that $r \to \infty$ then $\eta = 0$.

Proposition 1.7. [24] Let $r > 0$ and $\mathcal{K}_r$ be a nonempty closed and uniformly r-prox regular subset of $\mathcal{X}$. Set

$$\mathcal{U}(r) = \{u \in \mathcal{X} : 0 \leq d_{\mathcal{K}_r}(u) < r\}.$$

Then the following statements hold.

(i) For all $u \in \mathcal{K}_r$, we have $P_{\mathcal{K}_r}(u) \neq \emptyset$;

(ii) For all $r' \in (0, r)$, $P_{\mathcal{K}_r}$ is a Lipschitz continuous mapping with constant $\delta = \frac{r}{r - r'}$ on $\mathcal{U}(r') = \{u \in \mathcal{X} : 0 \leq d_{\mathcal{K}_r}(u) < r'\}$;

(iii) The proximal normal cone is closed as a set-valued mapping.

2. Sensitivity Analysis of Solution Sets

Now we consider a system of parametric general regularized nonconvex variational inequalities in a q-uniformly smooth Banach space $\mathcal{X}$. Let Ω and $\wedge$ be two nonempty open subsets of $\mathcal{X}$ in which the parameter λ and η take values, respectively. Let $h : \Omega \times \mathcal{X} \to \mathcal{X}$, $p : \wedge \times \mathcal{X} \to \mathcal{X}$ are single-valued mappings and $T : \wedge \times \mathcal{X} \to 2^{\mathcal{X}}$, $G : \Omega \times \mathcal{X} \to 2^{\mathcal{X}}$ be the set-valued mappings. For any constants $\rho > 0$ and $\mu > 0$, we consider the problem of finding $(u, v) \in \mathcal{X} \times \mathcal{X}$ and $x \in T(u, \eta)$, $y \in G(v, \lambda)$ such that $h(u, \lambda), p(v, \eta) \in \mathcal{K}_r$ and for all $(u, \lambda) \in \mathcal{X} \times \Omega$, $(v, \eta) \in \mathcal{X} \times \wedge$, $u^*, v^* \in \mathcal{K}_r$,

$$\begin{aligned} &\langle \rho U(u, y, \lambda) + h(u, \lambda) - u, u^* - h(u, \lambda)\rangle + \frac{1}{2r}\|u^* - h(u, \lambda)\|^2 \geq 0, \\ &\langle \mu V(x, v, \eta) + p(v, \eta) - v, v^* - p(v, \eta)\rangle + \frac{1}{2r}\|v^* - p(v, \eta)\|^2 \geq 0, \end{aligned} \tag{1}$$

where $U : \mathcal{X} \times \mathcal{X} \times \Omega \to \mathcal{X}$ and $V : \mathcal{X} \times \mathcal{X} \times \wedge \to \mathcal{X}$. The problem (1) is called a system of parametric general regularized nonconvex variational inequalities.

Definition 2.1. Let $h : \mathcal{X} \times \Omega \to \mathcal{X}$ be an operator. Then the operator $h(\cdot, \lambda)$ is said to be

(i) locally α_h-strongly accretive if there exists a constant $\alpha_h > 0$ such that for all $\lambda \in \Omega, u, v \in \mathcal{X}$,

$$\langle h(u, \lambda) - h(v, \lambda), j_q(u - v)\rangle \geq \alpha_h \|u - v\|^q,$$

(ii) locally β_h-Lipschitz continuous if there exists a constant $\beta_h > 0$ such that for all $\lambda \in \Omega, u, v \in \mathcal{X}$,

$$\|h(u, \lambda) - h(v, \lambda)\| \leq \beta_h \|u - v\|,$$

(iii) locally α_h-relaxed accretive if there exists a constant $\alpha_h > 0$ such that for all $\lambda \in \Omega, u, v \in \mathcal{X}$,

$$\langle h(u, \lambda) - h(v, \lambda), j_q(u - v)\rangle \geq -\alpha_h \|u - v\|^q.$$

Definition 2.2. A single-valued mapping $U : \mathcal{X} \times \mathcal{X} \times \Omega \to \mathcal{X}$ is said to be

(i) locally relaxed (φ_U, ψ_U)-cocoercive with respect to the first variable of U if there exist the constants $\varphi_U > 0$ and $\psi_U > 0$ such that for all $u_1, u_2, v \in \mathcal{X}, \lambda \in \Omega$,

$$\langle U(u_1, v, \lambda) - U(u_2, v, \lambda), j_q(u_1 - u_2)\rangle \geq -\varphi_U \|U(u_1, v, \lambda) - U(u_2, v, \lambda)\|^q + \psi_U \|u_1 - u_2\|^q,$$

(ii) locally ζ_U-Lipschitz continuous with respect to the first variable of U if there exists a constant $\zeta_U > 0$ such that for all $u_1, u_2, v \in \mathcal{X}, \lambda \in \Omega$,

$$\|U(u_1, v, \lambda) - U(u_2, v, \lambda)\| \leq \zeta_U \|u_1 - u_2\|,$$

(iii) locally κ_U-Lipschitz continuous with respect to the second variable of U if there exists a constant $\kappa_U > 0$ such that for all $v_1, v_2, u \in \mathcal{X}, \lambda \in \Omega$,

$$\|U(u, v_1, \lambda) - U(u, v_2, \lambda)\| \leq \kappa_U \|v_1 - v_2\|.$$

Similarly we can define the locally relaxed (φ_V, ψ_V)-cocoercivity and locally ζ_V-Lipschitz continuity of V.

Definition 2.3. Let $G : \mathcal{X} \times \Omega \to 2^{\mathcal{X}}$ be a set-valued mapping. Then G is called locally $\xi_G - \mathfrak{D}$-Lipschitz continuous in the first argument if there exists a constant $\xi_G > 0$ such that for all $u, v \in \mathcal{X}, \lambda \in \Omega$,

$$\mathfrak{D}(G(u, \lambda), G(v, \lambda)) \leq \xi_G \|u - v\|,$$

where $\mathfrak{D} : 2^{\mathcal{X}} \times 2^{\mathcal{X}} \to (-\infty, +\infty) \cup \{+\infty\}$ is the Hausdorff metric *i.e.,* for all $A, B \in 2^{\mathcal{X}}$,

$$\mathfrak{D}(A, B) = \max\left\{\sup_{u\in A}\inf_{v\in B} \|u - v\|, \sup_{u\in B}\inf_{v\in A} \|u - v\|\right\}.$$

Lemma 2.4. [19] Let $(\mathcal{X}, d)$ be a complete metric space and $T_1, T_2 : \mathcal{X} \to CB(\mathcal{X})$ be two set-valued contraction mappings with the same constant $\theta \in (0, 1)$ *i.e.,*

$$\mathfrak{D}(T_i(u), T_i(v)) \leq \theta d(u, v), \forall u, v \in \mathcal{X}, i = 1, 2.$$

Then

$$\mathfrak{D}(F(T_1), F(T_2)) \leq \frac{1}{1-\theta} \sup_{u \in \mathcal{X}} \mathfrak{D}(T_1(u), T_2(v)),$$

where $F(T_1)$ and $F(T_2)$ are fixed point sets of T_1, T_2, respectively.

Lemma 2.5. If $\mathcal{K}_r$ is a uniformly r-prox regular set, then problem (1) is equivalent to that of finding $(\lambda, \eta) \in \Omega \times \wedge$, $(u, v) \in \mathcal{X} \times \mathcal{X}$, $x \in T(u, \eta)$, $y \in G(v, \lambda)$ such that $h(u, \lambda), p(v, \eta) \in \mathcal{K}_r$ and

$$0 \in \rho U(u, y, \lambda) + h(u, \lambda) - u + N^P_{\mathcal{K}_r}(h(u, \lambda)),$$

$$0 \in \mu V(x, v, \eta) + p(v, \eta) - v + N^P_{\mathcal{K}_r}(p(v, \eta)), \tag{2}$$

where $N^P_{\mathcal{K}_r}(s)$ denotes the P-normal cone of $\mathcal{K}_r$ at s in the sense of nonconvex analysis.

Lemma 2.6. Let $U : \mathcal{X} \times \mathcal{X} \times \Omega \to \mathcal{X}$ and $V : \mathcal{X} \times \mathcal{X} \times \wedge \to \mathcal{X}$ be two mappings. Let $h : \Omega \times \mathcal{X} \to \mathcal{X}$, $p : \mathcal{X} \times \wedge \to \mathcal{X}$ be the single-valued mappings and let $T : \wedge \times \mathcal{X} \to 2^{\mathcal{X}}$, $G : \Omega \times \mathcal{X} \to 2^{\mathcal{X}}$ be the set-valued mappings. Then (u, v, x, y) with $u, v \in \mathcal{X}$, $h(u, \lambda) \in \mathcal{K}_r$ and $x \in T(u, \eta)$, $y \in G(v, \eta)$ is a solution of system (2) if and only if

$$\begin{aligned} h(u, \lambda) &= P_{\mathcal{K}_r}(u - \rho U(u, y, \lambda)), \\ p(v, \eta) &= P_{\mathcal{K}_r}(v - \mu V(x, v, \eta)), \end{aligned} \tag{3}$$

where $P_{\mathcal{K}_r}$ is the projection of $\mathcal{X}$ on the uniformly r-prox regular set $\mathcal{K}_r$ and $\rho, \mu > 0$ on $(\lambda, \eta) \in \Omega \times \wedge$.

Theorem 2.7. Let $U : \mathcal{X} \times \mathcal{X} \times \Omega \to \mathcal{X}$ and $V : \mathcal{X} \times \mathcal{X} \times \wedge \to \mathcal{X}$ be two mappings. Let $h : \Omega \times \mathcal{X} \to \mathcal{X}$, $p : \mathcal{X} \times \wedge \to \mathcal{X}$ be the single-valued mappings and let $T : \wedge \times \mathcal{X} \to 2^{\mathcal{X}}$, $G : \Omega \times \mathcal{X} \to 2^{\mathcal{X}}$ be the set-valued mappings. Assume that the mappings satisfy the following conditions:

(i) U is a locally relaxed (φ_U, ψ_U)-cocoercive mapping with respect to the first variable of U with constants $\varphi_U, \psi_U > 0$, respectively;

(ii) U is a locally ζ_U-Lipschitz continuous with respect to the first variable of U with constant $\zeta_U > 0$ and locally κ_U-Lipschitz continuous mapping with respect to the second variable of U with constant $\kappa_U > 0$;

(iii) V is a locally relaxed (φ_V, ψ_V)-cocoercive mapping with respect to the second variable of V with constants $\varphi_V, \psi_V > 0$, respectively;

(iv) V is a locally ζ_V-Lipschitz continuous with respect to the first variable of V with constant $\zeta_V > 0$ and locally κ_V-Lipschitz continuous mapping with respect to the second variable of V with constant $\kappa_V > 0$;

(v) T is a locally $\vartheta_T - \mathfrak{D}$-Lipschitz continuous mapping with constant $\vartheta_T > 0$;

(vi) G is a locally $\vartheta_G - \mathfrak{D}$-Lipschitz continuous mapping with constant $\vartheta_G > 0$;

(vii) h is a locally α_h-strongly accretive with respect to constant $\alpha_h > 0$ and locally β_h-Lipschitz continuous mapping with constant $\beta_h > 0$;

(viii) p is a locally α_p-relaxed accretive and locally β_p-Lipschitz continuous mapping with constants $\alpha_p > 0$ and $\beta_p > 0$, respectively.

If the constants $\rho > 0$ and $\mu > 0$ satisfy the following conditions:

$$\pi_h = \sqrt[q]{1 - q\alpha_h + \beta_h^q}, \ \ \pi_p = \sqrt[q]{1 + q\alpha_p + \beta_p^q},$$

$$\sigma_1 = 1 - \pi_h + \delta\mu\zeta_V\vartheta_T, \ \sigma_2 = 1 - \pi_p + \delta\rho\kappa_U\vartheta_G,$$

$$\sqrt[q]{1 - q\rho(\psi_U - \varphi_U\zeta_U^q) + c_q\rho^q\zeta_U^q} < \sigma_1\delta^{-1},$$

$$\sqrt[q]{1 - q\mu(\psi_V - \varphi_V\kappa_V^q) + c_q\mu^q\kappa_V^q} < \sigma_2\delta^{-1}, \tag{4}$$

where $r' \in (0, r)$, then for each $(\lambda, \eta) \in \Omega \times \wedge$, the system of parametric general regularized nonconvex variational inequalities (1) has a nonempty solution set $S(\lambda, \eta)$ which is a closed subset of $\mathcal{X} \times \mathcal{X}$.

Proof. From (3) we define $F_1 : \mathcal{X} \times \mathcal{X} \times \mathcal{X} \times \Omega \to \mathcal{X}$, $F_2 : \mathcal{X} \times \mathcal{X} \times \mathcal{X} \times \wedge \to \mathcal{X}$ as for all $(u, v, \lambda, \eta) \in \mathcal{X} \times \mathcal{X} \times \Omega \times \wedge$, $x \in T(u, \eta)$, $y \in G(v, \lambda)$,

$$F_1(u, v, y, \lambda) = u - h(u, \lambda) + P_{\mathcal{K}_r}(u - \rho U(u, y, \lambda)),$$

$$F_2(u, v, x, \eta) = v - p(v, \eta) + P_{\mathcal{K}_r}(v - \mu V(x, v, \eta)). \tag{5}$$

Now we define $\|\cdot\|_1$ on $\mathcal{X} \times \mathcal{X}$ by

$$\|(u, v)\|_1 = \|u\| + \|v\|, \forall (u, v) \in \mathcal{X} \times \mathcal{X}.$$

Then we know that $(\mathcal{X} \times \mathcal{X}, \|\cdot\|_1)$ is a Banach space. And also, for any $\rho > 0$, $\mu > 0$, we can define $F : \mathcal{X} \times \mathcal{X} \times \Omega \times \wedge \to 2^{\mathcal{X}} \times 2^{\mathcal{X}}$ by

$$F(u, v, \lambda, \eta) = \big\{(F_1(u, v, y, \lambda), F_2(u, v, x, \eta)) : x \in T(u, \eta), y \in G(v, \lambda)\big\}$$

for every $(u, v, \lambda, \eta) \in \mathcal{X} \times \mathcal{X} \times \Omega \times \wedge$. Since $T(u, \eta) \in CB(\mathcal{X})$, $G(v, \lambda) \in CB(\mathcal{X})$ and h, p, $P_{\mathcal{K}_r}$ are continuous mappings, we have

$$F(u, v, \lambda, \eta) \in CB(\mathcal{X} \times \mathcal{X}), \text{ for every } (u, v, \lambda, \eta) \in \mathcal{X} \times \mathcal{X} \times \Omega \times \wedge.$$

Now for each $(\lambda, \eta) \in \Omega \times \wedge$, we prove that $F(u, v, \lambda, \eta)$ is a multi-valued contractive mapping. In fact for any $(u_1, v_1, \lambda, \eta)$, $(u_2, v_2, \lambda, \eta) \in \mathcal{X} \times \mathcal{X} \times \Omega \times \wedge$ and $(a_1, a_2) \in F(u_1, v_1, \lambda, \eta)$ there exists $x_1 \in T(u_1, \eta)$, $y_1 \in G(v_1, \lambda)$ such that

$$a_1 = u_1 - h(u_1, \lambda) + P_{\mathcal{K}_r}(u_1 - \rho U(u_1, y_1, \lambda)),$$

$$a_2 = v_1 - p(v_1, \eta) + P_{\mathcal{K}_r}(v_1 - \mu V(x_1, v_1, \eta)).$$

From the Nadler Theorem [21], there exists $x_2 \in T(u_2, \eta)$, $y_2 \in G(v_2, \lambda)$ such that

$$\|x_1 - x_2\| \leq \mathfrak{D}(T(u_1, \eta), T(u_2, \eta)),$$

$$\|y_1 - y_2\| \leq \mathfrak{D}(G(v_1, \lambda), G(v_2, \lambda)). \tag{6}$$

Let

$$b_1 = u_2 - h(u_2, \lambda) + P_{\mathcal{K}_r}(u_2 - \rho U(u_2, y_2, \lambda)),$$

$$b_2 = v_2 - p(v_2, \eta) + P_{\mathcal{K}_r}(v_2 - \mu V(x_2, v_2, \eta)).$$

Then we have $(b_1, b_2) \in F(u_2, v_2, \lambda, \eta)$. Therefore, from Proposition 1.7, we have

$$\begin{aligned} \|a_1 - b_1\| &\leq \|u_1 - u_2 - (h(u_1, \lambda) - h(u_2, \lambda))\| \\ &\quad + \|P_{\mathcal{K}_r}(u_1 - \rho U(u_1, y_1, \lambda)) - P_{\mathcal{K}_r}(u_2 - \rho U(u_2, y_2, \lambda))\| \\ &\leq \|u_1 - u_2 - (h(u_1, \lambda) - h(u_2, \lambda))\| \\ &\quad + \delta\|u_1 - u_2 - \rho(U(u_1, y_1, \lambda) - U(u_2, y_2, \lambda))\| \\ &\leq \|u_1 - u_2 - (h(u_1, \lambda) - h(u_2, \lambda))\| \\ &\quad + \delta\|u_1 - u_2 - \rho(U(u_1, y_1, \lambda) - U(u_2, y_1, \lambda))\| \\ &\quad + \rho\|U(u_2, y_1, \lambda) - U(u_2, y_2, \lambda)\| \end{aligned} \tag{7}$$

and

$$\begin{aligned} \|a_2 - b_2\| &\leq \|v_1 - v_2 - (p(v_1, \eta) - p(v_2, \eta))\| \\ &\quad + \|P_{\mathcal{K}_r}(v_1 - \mu V(x_1, v_1, \eta)) - P_{\mathcal{K}_r}(v_2 - \mu V(x_2, v_2, \eta))\| \\ &\leq \|v_1 - v_2 - (p(v_1, \eta) - p(v_2, \eta))\| \\ &\quad + \delta\|v_1 - v_2 - \mu(V(x_1, v_1, \eta) - V(x_2, v_2, \eta))\| \\ &\leq \|v_1 - v_2 - (p(v_1, \eta) - p(v_2, \eta))\| \\ &\quad + \delta\|v_1 - v_2 - \mu(V(x_1, v_1, \eta) - V(x_1, v_2, \eta))\| \\ &\quad + \mu\|V(x_1, v_2, \eta) - V(x_2, v_2, \eta)\|. \end{aligned} \tag{8}$$

Since h is a locally α_h-strongly accretive and locally β_h-Lipschitz continuous mapping with constants $\alpha_h > 0$ and $\beta_h > 0$ respectively, we have

$$\begin{aligned} &\|u_1 - u_2 - (h(u_1, \lambda) - h(u_2, \lambda))\|^q \\ &\leq \|u_1 - u_2\|^q - q\langle h(u_1, \lambda) - h(u_2, \lambda), j_q(u_1 - u_2)\rangle + c_q\|h(u_1, \lambda) - h(u_2, \lambda)\|^q \\ &\leq \|u_1 - u_2\|^q - q\alpha_h\|u_1 - u_2\|^q + c_q\beta_h^q\|u_1 - u_2\|^q \\ &\leq (1 - q\alpha_h + c_q\beta_h^q)\|u_1 - u_2\|^q. \end{aligned} \tag{9}$$

Similarly, since p is a locally α_p-relaxed accretive with respect to constant $\alpha_p > 0$ and locally β_p-Lipschitz continuous mapping with respect to constant $\beta_p > 0$, we have

$$
\begin{aligned}
&\|v_1 - v_2 - (p(v_1, \eta) - p(v_2, \eta))\|^q \\
&\le \|v_1 - v_2\|^q - q\langle p(v_1, \eta) - p(v_2, \eta), j_q(v_1 - v_2)\rangle + c_q\|p(v_1, \eta) - p(v_2, \eta)\|^q \\
&\le \|v_1 - v_2\|^q + q\alpha_p\|v_1 - v_2\|^q + c_q\beta_p^q\|v_1 - v_2\|^q \\
&\le (1 + q\alpha_p + c_q\beta_p^q)\|v_1 - v_2\|^q. \qquad (10)
\end{aligned}
$$

Since U is a locally κ_U-Lipschitz continuous mapping with respect to the second variable with constant $\kappa_U > 0$ and G is a locally $\vartheta_G - \mathfrak{D}$-Lipschitz continuous mapping with constant $\vartheta_G > 0$, we have

$$
\begin{aligned}
\|U(u_2, y_1, \lambda) - U(u_2, y_2, \lambda)\| &\le \kappa_U\|y_1 - y_2\| \\
&\le \kappa_U\mathfrak{D}(G(v_1, \lambda) - G(v_2, \lambda)) \\
&\le \kappa_U\vartheta_G\|v_1 - v_2\|. \qquad (11)
\end{aligned}
$$

Since V is a locally ζ_V-Lipschitz continuous mapping with respect to the first variable with constant $\zeta_V > 0$ and T is a locally $\vartheta_T - \mathfrak{D}$-Lipschitz continuous mapping with constant $\vartheta_T > 0$, we have

$$
\begin{aligned}
\|V(x_1, v_2, \eta) - V(x_2, v_2, \eta)\| &\le \zeta_V\|x_1 - x_2\| \\
&\le \zeta_V\mathfrak{D}(T(u_1, \eta) - T(u_2, \eta)) \\
&\le \zeta_V\vartheta_T\|u_1 - u_2\|. \qquad (12)
\end{aligned}
$$

Since U is a locally relaxed (φ_U, ψ_U)-cocoercive mapping with respect to the first variable with constants $\varphi_U > 0$ and $\psi_U > 0$, respectively, we have

$$
\begin{aligned}
&\|u_1 - u_2 - \rho(U(u_1, y_1, \lambda) - U(u_2, y_1, \lambda))\|^q \\
&\le \|u_1 - u_2\|^q - q\rho\langle U(u_1, y_1, \lambda) - U(u_2, y_1, \lambda), j_q(u_1 - u_2)\rangle \\
&\quad + c_q\rho^q\|U(u_1, y_1, \lambda) - U(u_2, y_1, \lambda)\|^q \\
&\le \|u_1 - u_2\|^q - q\rho(-\varphi_U\|U(u_1, y_1, \lambda) - U(u_2, y_1, \lambda)\|^q + \psi_U\|u_1 - u_2\|^q) \\
&\quad + c_q\rho^q\zeta_U^q\|u_1 - u_2\|^q \\
&\le \|u_1 - u_2\|^q - q\rho(-\varphi_U\zeta_U^q\|u_1 - u_2\|^q + \psi_U\|u_1 - u_2\|^q) + c_q\rho^q\zeta_U^q\|u_1 - u_2\|^q \\
&\le (1 + q\rho\varphi_U\zeta_U^q - q\rho\psi_U + c_q\rho^q\zeta_U^q)\|u_1 - u_2\|^q. \qquad (13)
\end{aligned}
$$

Since V is a locally relaxed (φ_V, ψ_V)-cocoercive mapping with respect to the second variable with constants $\varphi_V > 0$ and $\psi_V > 0$, respectively, we have

$$\begin{aligned}
&\|v_1 - v_2 - \mu(V(x_1, v_1, \eta) - V(x_1, v_2, \eta))\|^q \\
&\le \|v_1 - v_2\|^q - q\mu\langle V(x_1, v_1, \eta) - V(x_1, v_2, \eta), j_q(v_1 - v_2)\rangle \\
&\quad + c_q\mu^q\|V(x_1, v_1, \eta) - V(x_1, v_2, \eta)\|^q \\
&\le \|v_1 - v_2\|^q - q\mu(-\varphi_V\|V(x_1, v_1, \eta) - V(x_1, v_2, \eta)\|^q + \psi_V\|v_1 - v_2\|^q) \\
&\quad + c_q\mu^q\kappa_V^q\|v_1 - v_2\|^q \\
&\le \|v_1 - v_2\|^q - q\mu(-\varphi_V\kappa_V^q\|v_1 - v_2\|^q + \psi_V\|v_1 - v_2\|^q) + c_q\mu^q\kappa_V^q\|v_1 - v_2\|^q \\
&\le (1 + q\mu\varphi_V\kappa_V^q - q\mu\psi_V + c_q\mu^q\kappa_V^q)\|v_1 - v_2\|^q. \qquad (14)
\end{aligned}$$

It follows from (7), (9), (11) and (13) that

$$\begin{aligned}
\|a_1 - b_1\| &\le \sqrt[q]{1 - q\alpha_h + c_q\beta_h^q}\|u_1 - u_2\| \\
&\quad + \delta\sqrt[q]{1 - q\rho(\psi_U - \varphi_U\zeta_U^q) + c_q\rho^q\zeta_U^q}\|u_1 - u_2\| \\
&\quad + \delta\rho\kappa_U\vartheta_G\|v_1 - v_2\| \\
&= \left[\sqrt[q]{1 - q\alpha_h + c_q\beta_h^q} + \delta\sqrt[q]{1 - q\rho(\psi_U - \varphi_U\zeta_U^q) + c_q\rho^q\zeta_U^q}\right]\|u_1 - u_2\| \\
&\quad + \delta\rho\kappa_U\vartheta_G\|v_1 - v_2\| \\
&\le \theta_1\|u_1 - u_2\| + \theta_2\|v_1 - v_2\| \qquad (15)
\end{aligned}$$

and

$$\begin{aligned}
\|a_2 - b_2\| &\le \sqrt[q]{1 + q\alpha_p + c_q\beta_p^q}\|v_1 - v_2\| \\
&\quad + \delta\sqrt[q]{1 - q\mu(\psi_V - \varphi_V\kappa_V^q) + c_q\mu^q\kappa_V^q}\|v_1 - v_2\| \\
&\quad + \delta\mu\zeta_V\vartheta_T\|u_1 - u_2\| \\
&\le \left[\sqrt[q]{1 + q\alpha_p + \beta_p^q} + \delta\sqrt[q]{1 - q\mu(\psi_V - \varphi_V\kappa_V^q) + c_q\mu^q\kappa_V^q}\right]\|v_1 - v_2\| \\
&\quad + \delta\mu\zeta_V\vartheta_T\|u_1 - u_2\| \\
&\le \theta_3\|u_1 - u_2\| + \theta_4\|v_1 - v_2\|, \qquad (16)
\end{aligned}$$

where $\theta_1 = \sqrt[q]{1 - q\alpha_h + \beta_h^q} + \delta\sqrt[q]{1 - q\rho(\psi_U - \varphi_U\zeta_U^q) + c_q\rho^q\zeta_U^q}$,

$$\theta_2 = \delta\rho\kappa_U\vartheta_G, \qquad \theta_3 = \delta\mu\zeta_V\vartheta_T,$$

$$\theta_4 = \sqrt[q]{1 + q\alpha_p + c_q\beta_p^q} + \delta\sqrt[q]{1 - q\mu(\psi_V - \varphi_V\kappa_V^q) + c_q\mu^q\kappa_V^q}.$$

By (15) and (16), we have

$$\|a_1 - b_1\| + \|a_2 - b_2\| \le \theta(\|u_1 - u_2\| + \|v_1 - v_2\|), \qquad (17)$$

where $\theta = \max\{\theta_1 + \theta_3, \theta_2 + \theta_4\}$. Hence, we have

$$\begin{aligned} d((a_1, a_2), F(u_2, v_2, \lambda, \eta)) &= \inf_{(b_1,b_2)\in F(u_2,v_2,\lambda,\eta)} \Big(\|a_1 - b_1\| + \|a_2 - b_2\| \Big) \\ &\le \theta(\|u_1 - u_2\| + \|v_1 - v_2\|) \\ &= \theta\|(u_1, v_1) - (u_2, v_2)\|_1 \end{aligned}$$

and

$$d((b_1, b_2), F(u_1, v_1, \lambda, \eta)) \le \theta\|(u_1, v_1) - (u_2, v_2)\|_1.$$

From the definition of Hausdorff metric $\mathfrak{D}$ on $CB(\mathcal{X}\times\mathcal{X})$, we have, for all $u_1, u_2, v_1, v_2 \in \mathcal{X}$ and $(\lambda, \eta) \in \Omega \times \wedge$,

$$\begin{aligned} &\mathfrak{D}(F(u_1, v_1, \lambda, \eta), F(u_2, v_2, \lambda, \eta)) \\ &\quad = \max\Bigg\{ \sup_{(a_1,a_2)\in F(u_1,v_1,\lambda,\eta)} d((a_1, a_2), F(u_2, v_2, \lambda, \eta)), \\ &\qquad\qquad \sup_{(b_1,b_2)\in F(u_2,v_2,\lambda,\eta)} d((b_1, b_2), F(u_1, v_1, \lambda, \eta)) \Bigg\} \\ &\quad \le \theta\|(u_1, v_1) - (u_2, v_2)\|_1. \end{aligned} \tag{18}$$

We know that $\theta < 1$ from condition (4). Thus (18) implies that F is a contractive mapping which is uniform with respect to $(\lambda, \eta) \in \Omega \times \wedge$. By the Nadler fixed point Theorem [21], $F(u, v, \lambda, \eta)$ has a fixed point $(\overline{u}, \overline{v})$ for each $(\lambda, \eta) \in \Omega \times \wedge$. From the definition of F there exist $\overline{x} \in T(\overline{u}, \eta)$ and $\overline{y} \in G(\overline{v}, \lambda)$ such that (3) holds. By Lemma 2.6, $S(\lambda, \eta) \neq \emptyset$.

Now we have to prove that $S(\lambda, \eta)$ is closed. In fact, for each $(\lambda, \eta) \in \Omega \times \wedge$, let $(u_n, v_n) \in S(\lambda, \eta)$ and $u_n \to u_0$, $v_n \to v_0$ as $n \to \infty$. Then we have

$$(u_n, v_n) \in F(u_n, v_n, \lambda, \eta), n = 1, 2, \cdots.$$

And also, we have

$$\mathfrak{D}(F(u_n, v_n, \lambda, \eta), F(u_0, v_0, \lambda, \eta)) \le \theta\|(u_n, v_n) - (u_0, v_0)\|_1.$$

It follows that

$$\begin{aligned} d((u_0, v_0), F(u_0, v_0, \lambda, \eta)) &\le \|(u_0, v_0) - (u_n, v_n)\|_1 \\ &\quad + d((u_n, v_n), F(u_n, v_n, \lambda, \eta)) \\ &\quad + \mathfrak{D}(F(u_n, v_n, \lambda, \eta), F(u_0, v_0, \lambda, \eta)) \\ &\le (1 + \theta)\|(u_n, v_n) - (u_0, v_0)\|_1 \\ &\to 0, \text{ as } n \to \infty. \end{aligned}$$

Hence, we have $(u_0, v_0) \in F(u_0, v_0, \lambda, \eta)$. From Lemma 2.6, we have $(u_0, v_0) \in S(\lambda, \eta)$. Therefore $S(\lambda, \eta)$ is a nonempty closed subset of $\mathcal{X} \times \mathcal{X}$. This completes the proof. ■

Theorem 2.8. The hypothesises of Theorem 2.7 are hold and assume that for any $u, v \in \mathcal{X}$, the mappings $\lambda \to U(u, v, \lambda)$, $\eta \to V(u, v, \eta)$, $\lambda \to h(u, \lambda)$, $\eta \to p(v, \eta)$ are locally Lipschitz continuous with constants $\ell_U, \ell_V, \ell_p, \ell_h$, respectively. Let $\eta \to T(u, \eta)$ be a locally $\ell_T - \mathfrak{D}$-Lipschitz continuous mapping and $\lambda \to G(v, \lambda)$ be a locally $\ell_G - \mathfrak{D}$-Lipschitz continuous mapping for $u, v \in \mathcal{X}$. Let $P_{\mathcal{K}_r}$ be a Lipschitz continuous operator with constant $\delta = \dfrac{r}{r - r'}$. Then the solution $S(\lambda, \eta)$ for a system of parametric general regularized nonconvex variational inequalities is locally Lipschitz continuous from $\Omega \times \wedge$ to $\mathcal{X} \times \mathcal{X}$.

Proof. By Theorem 2.7, for any $(t, \lambda, \overline{\lambda}) \in \mathcal{X} \times \Omega \times \Omega$ and $(z, \eta, \overline{\eta}) \in \mathcal{X} \times \wedge \times \wedge$, $S(\lambda, \eta)$ and $S(\overline{\lambda}, \overline{\eta})$ are nonempty closed subsets. Also, for each $(\lambda, \eta), (\overline{\lambda}, \overline{\eta}) \in \Omega \times \wedge$, $F(u, v, \lambda, \eta)$ and $F(u, v, \overline{\lambda}, \overline{\eta})$ are contractive mappings with some constant $\theta \in (0, 1)$ and have fixed points $(u(\lambda, \eta), v(\lambda, \eta))$ and $(u(\overline{\lambda}, \overline{\eta}), v(\overline{\lambda}, \overline{\eta}))$, respectively. Hence, by Lemma 2.4, for any fixed $(\lambda, \eta), (\overline{\lambda}, \overline{\eta}) \in \Omega \times \wedge$, we have

$$\begin{aligned}
&\mathfrak{D}(S(\lambda, \eta), S(\overline{\lambda}, \overline{\eta})) \\
&\leq \frac{1}{1 - \theta} \sup_{(u,v) \in \mathcal{X} \times \mathcal{X}} \mathfrak{D}(F(u(\lambda, \eta), v(\lambda, \eta), \lambda, \eta), F(u(\overline{\lambda}, \overline{\eta}), v(\overline{\lambda}, \overline{\eta}), \overline{\lambda}, \overline{\eta})). \qquad (19)
\end{aligned}$$

For any $(a_1, a_2) \in F(u(\lambda, \eta), v(\lambda, \eta), \lambda, \eta)$, there exists $x(\lambda, \eta) \in T(u(\lambda, \eta), \eta)$, $y(\lambda, \eta) \in G(v(\lambda, \eta), \lambda)$ such that

$$\begin{aligned}
a_1 &= u(\lambda, \eta) - h(u(\lambda, \eta), \lambda) + P_{\mathcal{K}_r}(u(\lambda, \eta) - \rho U(u(\lambda, \eta), y(\lambda, \eta), \lambda)) \\
a_2 &= v(\lambda, \eta) - p(v(\lambda, \eta), \eta) + P_{\mathcal{K}_r}(v(\lambda, \eta) - \mu V(x(\lambda, \eta), v(\lambda, \eta), \eta)). \qquad (20)
\end{aligned}$$

From the Nadler Theorem [21], there exists $x(\overline{\lambda}, \overline{\eta}) \in T(u(\overline{\lambda}, \overline{\eta}), \overline{\eta})$, $y(\overline{\lambda}, \overline{\eta}) \in G(v(\overline{\lambda}, \overline{\eta}), \overline{\lambda})$ such that

$$\begin{aligned}
\|x(\lambda, \eta) - x(\overline{\lambda}, \overline{\eta})\| &\leq \mathfrak{D}(T(u(\lambda, \eta), \eta), T(u(\overline{\lambda}, \overline{\eta}), \overline{\eta})), \\
\|y(\lambda, \eta) - y(\overline{\lambda}, \overline{\eta})\| &\leq \mathfrak{D}(G(v(\lambda, \eta), \lambda), G(v(\overline{\lambda}, \overline{\eta}), \overline{\lambda})). \qquad (21)
\end{aligned}$$

Let

$$\begin{aligned}
b_1 &= u(\overline{\lambda}, \overline{\eta}) - h(u(\overline{\lambda}, \overline{\eta}), \overline{\lambda}) + P_{\mathcal{K}_r}(u(\overline{\lambda}, \overline{\eta}) - \rho U(u(\overline{\lambda}, \overline{\eta}), y(\overline{\lambda}, \overline{\eta}), \overline{\lambda})), \\
b_2 &= v(\overline{\lambda}, \overline{\eta}) - p(v(\overline{\lambda}, \overline{\eta}), \overline{\eta}) + P_{\mathcal{K}_r}(v(\overline{\lambda}, \overline{\eta}) - \mu V(x(\overline{\lambda}, \overline{\eta}), v(\overline{\lambda}, \overline{\eta}), \overline{\eta})). \qquad (22)
\end{aligned}$$

Then, we have

$$(b_1, b_2) \in F(u(\overline{\lambda}, \overline{\eta}), v(\overline{\lambda}, \overline{\eta}), \overline{\lambda}, \overline{\eta}).$$

From (20), (22) and Proposition 1.7, we have

$$
\begin{aligned}
&\|a_1 - b_1\| \\
&\le \|u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta}) - (h(u(\lambda,\eta),\lambda) - h(u(\overline{\lambda},\overline{\eta}),\lambda))\| \\
&\quad + \|h(u(\overline{\lambda},\overline{\eta}),\lambda) - h(u(\overline{\lambda},\overline{\eta}),\overline{\lambda})\| \\
&\quad + \|P_{\mathcal{K}_r}(u(\lambda,\eta) - \rho U(u(\lambda,\eta), y(\lambda,\eta),\lambda)) \\
&\quad - P_{\mathcal{K}_r}(u(\overline{\lambda},\overline{\eta}) - \rho U(u(\overline{\lambda},\overline{\eta}), y(\overline{\lambda},\overline{\eta}),\overline{\lambda}))\| \\
&\le \|u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta}) - (h(u(\lambda,\eta),\lambda) - h(u(\overline{\lambda},\overline{\eta}),\lambda))\| \\
&\quad + \|h(u(\overline{\lambda},\overline{\eta}),\lambda) - h(u(\overline{\lambda},\overline{\eta}),\overline{\lambda})\| \\
&\quad + \|P_{\mathcal{K}_r}(u(\lambda,\eta) - \rho U(u(\lambda,\eta), y(\lambda,\eta),\lambda)) \\
&\quad - P_{\mathcal{K}_r}(u(\overline{\lambda},\overline{\eta}) - \rho U(u(\overline{\lambda},\overline{\eta}), y(\overline{\lambda},\overline{\eta}),\lambda))\| \\
&\quad + \|P_{\mathcal{K}_r}(u(\overline{\lambda},\overline{\eta}) - \rho U(u(\overline{\lambda},\overline{\eta}), y(\overline{\lambda},\overline{\eta}),\lambda)) \\
&\quad - P_{\mathcal{K}_r}(u(\overline{\lambda},\overline{\eta}) - \rho U(u(\overline{\lambda},\overline{\eta}), y(\overline{\lambda},\overline{\eta}),\overline{\lambda}))\| \\
&\le \|u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta}) - (h(u(\lambda,\eta),\lambda) - h(u(\overline{\lambda},\overline{\eta}),\lambda))\| \\
&\quad + \|h(u(\overline{\lambda},\overline{\eta}),\lambda) - h(u(\overline{\lambda},\overline{\eta}),\overline{\lambda})\| \\
&\quad + \delta\|u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta}) - \rho(U(u(\lambda,\eta), y(\lambda,\eta),\lambda) - U(u(\overline{\lambda},\overline{\eta}), y(\overline{\lambda},\overline{\eta}),\lambda))\| \\
&\quad + \delta\|u(\overline{\lambda},\overline{\eta}) - u(\overline{\lambda},\overline{\eta}) - \rho(U(u(\overline{\lambda},\overline{\eta}), y(\overline{\lambda},\overline{\eta}),\lambda) - U(u(\overline{\lambda},\overline{\eta}), y(\overline{\lambda},\overline{\eta}),\overline{\lambda}))\| \\
&\le \|u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta}) - (h(u(\lambda,\eta),\lambda) - h(u(\overline{\lambda},\overline{\eta}),\lambda))\| \\
&\quad + \|h(u(\overline{\lambda},\overline{\eta}),\lambda) - h(u(\overline{\lambda},\overline{\eta}),\overline{\lambda})\| \\
&\quad + \delta\|u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta}) - \rho(U(u(\lambda,\eta), y(\lambda,\eta),\lambda) - U(u(\overline{\lambda},\overline{\eta}), y(\lambda,\eta),\lambda))\| \\
&\quad + \delta\rho\|U(u(\overline{\lambda},\overline{\eta}), y(\lambda,\eta),\lambda) - U(u(\overline{\lambda},\overline{\eta}), y(\overline{\lambda},\overline{\eta}),\lambda)\| \\
&\quad + \delta\rho\|U(u(\overline{\lambda},\overline{\eta}), y(\overline{\lambda},\overline{\eta}),\lambda) - U(u(\overline{\lambda},\overline{\eta}), y(\overline{\lambda},\overline{\eta}),\overline{\lambda})\| \\
&\le \|u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta}) - (h(u(\lambda,\eta),\lambda) - h(u(\overline{\lambda},\overline{\eta}),\lambda))\| + \ell_h\|\lambda - \overline{\lambda}\| \\
&\quad + \delta\|u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta}) - \rho(U(u(\lambda,\eta), y(\lambda,\eta),\lambda) - U(u(\overline{\lambda},\overline{\eta}), y(\lambda,\eta),\lambda))\| \\
&\quad + \delta\rho\|U(u(\overline{\lambda},\overline{\eta}), y(\lambda,\eta),\lambda) - U(u(\overline{\lambda},\overline{\eta}), y(\overline{\lambda},\overline{\eta}),\lambda)\| \\
&\quad + \delta\rho\ell_U\|\overline{\lambda} - \overline{\lambda}\| \\
&\le \|u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta}) - (h(u(\lambda,\eta),\lambda) - h(u(\overline{\lambda},\overline{\eta}),\lambda))\| + \ell_h\|\lambda - \overline{\lambda}\| \\
&\quad + \delta\|u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta}) - \rho(U(u(\lambda,\eta), y(\lambda,\eta),\lambda) - U(u(\overline{\lambda},\overline{\eta}), y(\lambda,\eta),\lambda))\| \\
&\quad + \delta\rho\|U(u(\overline{\lambda},\overline{\eta}), y(\lambda,\eta),\lambda) - U(u(\overline{\lambda},\overline{\eta}), y(\overline{\lambda},\overline{\eta}),\lambda)\| \\
&\quad + \delta\rho\ell_U\|\overline{\lambda} - \overline{\lambda}\| \qquad (23)
\end{aligned}
$$

and

$$
\begin{aligned}
&\|a_2 - b_2\| \\
&\leq \|v(\lambda,\eta) - v(\overline{\lambda},\overline{\eta}) - (p(v(\lambda,\eta),\eta) - p(v(\overline{\lambda},\overline{\eta}),\eta))\| \\
&\quad + \|p(v(\overline{\lambda},\overline{\eta}),\eta) - p(v(\overline{\lambda},\overline{\eta}),\overline{\eta})\| \\
&\quad - \|P_{\mathcal{K}_r}(v(\lambda,\eta) - \mu V(x(\lambda,\eta), v(\lambda,\eta),\eta)) \\
&\quad - P_{\mathcal{K}_r}(v(\overline{\lambda},\overline{\eta}) - \mu V(x(\overline{\lambda},\overline{\eta}), v(\overline{\lambda},\overline{\eta}),\overline{\eta}))\| \\
&\leq \|v(\lambda,\eta) - v(\overline{\lambda},\overline{\eta}) - (p(v(\lambda,\eta),\eta) - p(v(\overline{\lambda},\overline{\eta}),\eta))\| \\
&\quad + \|p(v(\overline{\lambda},\overline{\eta}),\eta) - p(v(\overline{\lambda},\overline{\eta}),\overline{\eta})\| \\
&\quad + \|P_{\mathcal{K}_r}(v(\lambda,\eta) - \mu V(x(\lambda,\eta), v(\lambda,\eta),\eta)) \\
&\quad - P_{\mathcal{K}_r}(v(\overline{\lambda},\overline{\eta}) - \mu V(x(\overline{\lambda},\overline{\eta}), v(\overline{\lambda},\overline{\eta}),\eta))\| \\
&\quad + \|P_{\mathcal{K}_r}(v(\overline{\lambda},\overline{\eta}) - \mu V(x(\overline{\lambda},\overline{\eta}), v(\overline{\lambda},\overline{\eta}),\eta)) \\
&\quad - P_{\mathcal{K}_r}(v(\overline{\lambda},\overline{\eta}) - \mu V(x(\overline{\lambda},\overline{\eta}), v(\overline{\lambda},\overline{\eta}),\overline{\lambda}))\| \\
&\leq \|v(\lambda,\eta) - v(\overline{\lambda},\overline{\eta}) - (p(v(\lambda,\eta),\eta) - p(v(\overline{\lambda},\overline{\eta}),\eta))\| \\
&\quad + \|p(v(\overline{\lambda},\overline{\eta}),\eta) - p(v(\overline{\lambda},\overline{\eta}),\overline{\eta})\| \\
&\quad + \delta\|v(\lambda,\eta) - v(\overline{\lambda},\overline{\eta}) - \mu(V(x(\lambda,\eta), v(\lambda,\eta),\eta) - V(x(\lambda,\eta), v(\overline{\lambda},\overline{\eta}),\eta))\| \\
&\quad + \delta\|v(\overline{\lambda},\overline{\eta}) - v(\overline{\lambda},\overline{\eta}) - \mu(V(x(\overline{\lambda},\overline{\eta}), v(\overline{\lambda},\overline{\eta}),\eta) - V(x(\overline{\lambda},\overline{\eta}), v(\overline{\lambda},\overline{\eta}),\overline{\eta}))\| \\
&\leq \|v(\lambda,\eta) - v(\overline{\lambda},\overline{\eta}) - (p(v(\lambda,\eta),\eta) - p(v(\overline{\lambda},\overline{\eta}),\eta))\| + \ell_p\|\eta - \overline{\eta}\| \\
&\quad + \delta\|v(\lambda,\eta) - v(\overline{\lambda},\overline{\eta}) - \mu(V(x(\lambda,\eta), v(\lambda,\eta),\eta) - V(x(\lambda,\eta), v(\overline{\lambda},\overline{\eta}),\eta))\| \\
&\quad + \delta\mu\|V(x(\lambda,\eta), v(\overline{\lambda},\overline{\eta}),\eta) - V(x(\overline{\lambda},\overline{\eta}), v(\overline{\lambda},\overline{\eta}),\eta)\| + \delta\mu\ell_V\|\eta - \overline{\eta}\|. \qquad (24)
\end{aligned}
$$

Now, we know that

$$
\begin{aligned}
&\|u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta}) - (h(u(\lambda,\eta),\lambda) - h(u(\overline{\lambda},\overline{\eta}),\lambda))\|^q \\
&\leq \|u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta})\|^q - q\langle h(u(\lambda,\eta),\lambda) - h(u(\overline{\lambda},\overline{\eta}),\lambda), j_q(u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta}))\rangle \\
&\quad + c_q\|h(u(\lambda,\eta),\lambda) - h(u(\overline{\lambda},\overline{\eta}),\lambda)\|^q \\
&\leq (1 - q\alpha_h + c_q\beta_h^q)\|u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta})\|^q, \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
&\|u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta}) - \rho(U(u(\lambda,\eta), y(\lambda,\eta),\lambda) - U(u(\overline{\lambda},\overline{\eta}), y(\lambda,\eta),\lambda))\|^q \\
&\leq \|u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta})\|^q \\
&\quad - q\rho\langle U(u(\lambda,\eta), y(\lambda,\eta),\lambda) - U(u(\overline{\lambda},\overline{\eta}), y(\lambda,\eta),\lambda), j_q(u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta}))\rangle \\
&\quad + c_q\rho^q\|U(u(\lambda,\eta), y(\lambda,\eta),\lambda) - U(u(\overline{\lambda},\overline{\eta}), y(\lambda,\eta),\lambda)\|^q \\
&\leq \|u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta})\|^q \\
&\quad - q\rho(-\varphi_U\|U(u(\lambda,\eta), y(\lambda,\eta),\lambda) - U(u(\overline{\lambda},\overline{\eta}), y(\lambda,\eta),\lambda)\|^q \\
&\quad - \psi_U\|u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta})\|^q) + c_q\rho^q\zeta_U^q\|u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta})\|^q \\
&\leq (1 - q\rho(\psi_U - \varphi_U\zeta_U^q) + c_q\rho^q\zeta_U^q)\|u(\lambda,\eta) - u(\overline{\lambda},\overline{\eta})\|^q \qquad (26)
\end{aligned}
$$

and

$$\begin{aligned}
&\|U(u(\overline{\lambda}, \overline{\eta}), y(\lambda, \eta), \lambda) - U(u(\overline{\lambda}, \overline{\eta}), y(\overline{\lambda}, \overline{\eta}), \lambda)\| \\
&\leq \kappa_U \|y(\lambda, \eta) - y(\overline{\lambda}, \overline{\eta})\| \\
&\leq \kappa_U \mathfrak{D}(G(v(\lambda, \eta), \lambda) - G(v(\overline{\lambda}, \overline{\eta}), \overline{\lambda})) \\
&\leq \kappa_U \big[\mathfrak{D}(G(v(\lambda, \eta), \lambda) - G(v(\overline{\lambda}, \overline{\eta}), \lambda)) + \mathfrak{D}(G(v(\overline{\lambda}, \overline{\eta}), \lambda) - G(v(\overline{\lambda}, \overline{\eta}), \overline{\lambda}))\big] \\
&\leq \kappa_U \big[\vartheta_G \|v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta})\| + \ell_G \|\lambda - \overline{\lambda}\|\big].
\end{aligned} \tag{27}$$

And also, we know that

$$\begin{aligned}
&\|v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta}) - (p(v(\lambda, \eta), \eta) - p(v(\overline{\lambda}, \overline{\eta}), \eta))\|^q \\
&\leq \|v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta})\|^q - q\langle p(v(\lambda, \eta), \eta) - p(v(\overline{\lambda}, \overline{\eta}), \eta), j_q(v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta}))\rangle \\
&\quad + c_q \|p(v(\lambda, \eta), \eta) - p(v(\lambda, \eta), \eta)\|^q \\
&\leq \|v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta})\|^q + q\alpha_p \|v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta})\|^q + c_q \beta_p^q \|v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta})\|^q \\
&\leq (1 + q\alpha_p + c_q \beta_p^q) \|v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta})\|^q,
\end{aligned} \tag{28}$$

$$\begin{aligned}
&\|v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta}) - \mu(V(x(\lambda, \eta), v(\lambda, \eta), \eta) - V(x(\lambda, \eta), v(\overline{\lambda}, \overline{\eta}), \eta))\|^q \\
&\leq \|v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta})\|^q \\
&\quad - q\mu\langle V(x(\lambda, \eta), v(\lambda, \eta), \eta) - V(x(\lambda, \eta), v(\overline{\lambda}, \overline{\eta}), \eta), j_q(v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta}))\rangle \\
&\quad + c_q \mu^q \|V(x(\lambda, \eta), v(\lambda, \eta), \eta) - V(x(\lambda, \eta), v(\overline{\lambda}, \overline{\eta}), \eta)\|^q \\
&\leq \|v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta})\|^q \\
&\quad - q\mu(-\varphi_V \|V(x(\lambda, \eta), v(\lambda, \eta), \eta) - V(x(\lambda, \eta), v(\overline{\lambda}, \overline{\eta}), \eta)\|^q \\
&\quad + \psi_V \|v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta})\|^q) + c_q \mu^q \kappa_V^q \|v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta})\|^q \\
&\leq \|v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta})\|^q - q\mu(-\varphi_V \kappa_V^q \|v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta})\|^q \\
&\quad + \psi_V \|v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta})\|^q) + c_q \mu^q \kappa_V^q \|v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta})\|^q \\
&\leq (1 - q\mu(\psi_V - \varphi_V \kappa_V^q) + c_q \mu^q \kappa_V^q) \|v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta})\|^q
\end{aligned} \tag{29}$$

and

$$\begin{aligned}
&\|V(x(\lambda, \eta), v(\overline{\lambda}, \overline{\eta}), \eta) - V(x(\overline{\lambda}, \overline{\eta}), v(\overline{\lambda}, \overline{\eta}), \eta)\| \\
&\leq \zeta_V \|x(\lambda, \eta) - x(\overline{\lambda}, \overline{\eta})\| \\
&\leq \zeta_V \mathfrak{D}(T(u(\lambda, \eta), \eta), T(u(\overline{\lambda}, \overline{\eta}), \overline{\eta})) \\
&\leq \zeta_V \big[\mathfrak{D}(T(u(\lambda, \eta), \eta), T(u(\overline{\lambda}, \overline{\eta}), \eta)) + \mathfrak{D}(T(u(\overline{\lambda}, \overline{\eta}), \eta), T(u(\overline{\lambda}, \overline{\eta}), \overline{\eta}))\big] \\
&\leq \zeta_V \big[\vartheta_T \|u(\lambda, \eta) - u(\overline{\lambda}, \overline{\eta})\| + \ell_T \|\eta - \overline{\eta}\|\big].
\end{aligned} \tag{30}$$

Therefore, from (23)–(30), we have

$$
\begin{aligned}
&\|a_1 - b_1\| + \|a_2 - b_2\| \\
&\leq \left[\sqrt[q]{1 - q\alpha_h + c_q\beta_h^q} + \delta \sqrt[q]{1 - q\rho(\psi_U - \varphi_U \zeta_U^q) + c_q\rho^q\zeta_U^q} + \delta\mu\zeta_V\vartheta_T \right] \\
&\quad \times \|u(\lambda, \eta) - u(\overline{\lambda}, \overline{\eta})\| \\
&\quad + \left[\sqrt[q]{1 + q\alpha_p + c_q\beta_p^q} + \delta \sqrt[q]{1 - q\mu(\psi_V - \varphi_V \kappa_V^q) + c_q\mu^q\kappa_V^q} + \delta\rho\kappa_U\vartheta_G \right] \\
&\quad \times \|v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta})\| \\
&\quad + \big[\ell_h + \delta\rho\ell_U + \mu\delta\kappa_U\ell_G\big]\|\lambda - \overline{\lambda}\| + \big[\ell_p + \delta\mu\ell_V + \rho\delta\zeta_V\ell_T\big]\|\eta - \overline{\eta}\| \\
&= \theta_1\|u(\lambda, \eta) - u(\overline{\lambda}, \overline{\eta})\| + \theta_2\|v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta})\| + \jmath_1\|\lambda - \overline{\lambda}\| + \jmath_2\|\eta - \overline{\eta}\| \\
&\leq \theta\big[\|u(\lambda, \eta) - u(\overline{\lambda}, \overline{\eta})\| + \|v(\lambda, \eta) - v(\overline{\lambda}, \overline{\eta})\|\big] + \jmath_1\|\lambda - \overline{\lambda}\| + \jmath_2\|\eta - \overline{\eta}\| \\
&\leq \theta\big[\|a_1 - b_1\| + \|a_2 - b_2\|\big] + \jmath_1\|\lambda - \overline{\lambda}\| + \jmath_2\|\eta - \overline{\eta}\|, \qquad (31)
\end{aligned}
$$

where

$$
\begin{aligned}
\theta_1 &= \sqrt[q]{1 - q\alpha_h + c_q\beta_h^q} + \delta \sqrt[q]{1 - q\rho(\psi_U - \varphi_U \zeta_U^q) + c_q\rho^q\zeta_U^q} + \delta\mu\zeta_V\vartheta_T, \\
\theta_2 &= \sqrt[q]{1 + q\alpha_p + c_q\beta_p^q} + \delta \sqrt[q]{1 - q\mu(\psi_V - \varphi_V \kappa_V^q) + c_q\mu^q\kappa_V^q} + \delta\rho\kappa_U\vartheta_G, \\
\jmath_1 &= \ell_h + \delta\rho\ell_U + \mu\delta\kappa_U\ell_G, \\
\jmath_2 &= \ell_p + \delta\mu\ell_V + \rho\delta\zeta_V\ell_T, \\
\theta &= \max\{\theta_1, \theta_2\}.
\end{aligned}
$$

It follows from (4) and (31) that

$$
\begin{aligned}
\|a_1 - b_1\| + \|a_2 - b_2\| &\leq \frac{1}{1-\theta}\left[\jmath_1\|\lambda - \overline{\lambda}\| + \jmath_2\|\eta - \overline{\eta}\|\right] \\
&\leq \frac{1}{1-\theta}\max\{\jmath_1, \jmath_2\}\Big(\|\lambda - \overline{\lambda}\| + \|\eta - \overline{\eta}\|\Big) \\
&\leq \wp(\|\lambda - \overline{\lambda}\| + \|\eta - \overline{\eta}\|),
\end{aligned}
$$

where $\wp = \dfrac{1}{1-\theta}\max\{\jmath_1, \jmath_2\}$. Hence, we have

$$
\begin{aligned}
&d((a_1, a_2), F(u(\overline{\lambda}, \overline{\eta}), v(\overline{\lambda}, \overline{\eta}), \overline{\lambda}, \overline{\eta})) \\
&\quad = \inf_{(b_1, b_2) \in F(u(\overline{\lambda}, \overline{\eta}), v(\overline{\lambda}, \overline{\eta}), \overline{\lambda}, \overline{\eta})} \Big(\|a_1 - b_1\| + \|a_2 - b_2\|\Big) \\
&\quad \leq \wp(\|\lambda - \overline{\lambda}\| + \|\eta - \overline{\eta}\|) \\
&\quad = \wp\|(\lambda, \eta) - (\overline{\lambda}, \overline{\eta})\|_1. \qquad (32)
\end{aligned}
$$

Similarly, we have

$$d((b_1, b_2), F(u(\lambda, \eta), v(\lambda, \eta), \lambda, \eta)) \leq \wp\|(\lambda, \eta) - (\bar{\lambda}, \bar{\eta})\|_1. \tag{33}$$

Hence from (19),(32) and (33), we have

$$\begin{aligned}
&\mathfrak{D}(S(\lambda, \eta), S(\bar{\lambda}, \bar{\eta})) \\
&\leq \frac{1}{1-\theta} \sup_{(u,v)\in H\times H} \mathfrak{D}(F(u(\lambda, \eta), v(\lambda, \eta), \lambda, \eta), F(u(\bar{\lambda}, \bar{\eta}), v(\bar{\lambda}, \bar{\eta}), \bar{\lambda}, \bar{\eta})) \\
&\leq \frac{\wp}{1-\theta}\|(\lambda, \eta) - (\bar{\lambda}, \bar{\eta})\|.
\end{aligned}$$

This means that $S(\lambda, \eta)$ is Lipschitz continuous with respect to $(\lambda, \eta) \in \Omega \times \wedge$. ■

Acknowledgments

This work was supported by the Basic Science Research Program through the National Research Foundation (NRF) Grant funded by Ministry of Education of the republic of Korea (2014046293).

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