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Design of reinforced concrete beams in a case of a change of cross section of composite strengthening reinforcement

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Abstract

The presented paper provides information on influence of such factors as type of composite reinforcement, percent ratios of steel and composite reinforcement of elements, presence or absence of anchoring hardware, which is anchoring ending parts of composite materials, on strength of normal cross-sections of strengthened reinforced concrete beams. On a basis of design equations of “Guidelines for strengthening of reinforced concrete structures with composite materials” in Russia, recommendations for evaluation of listed factors in design of strength of normal cross-sections of beams in a case of a change of cross-section area of composite strengthening reinforcement are presented.

Key words: Concrete, reinforced concrete, steel, composite reinforcement, glass fibre-reinforced plastic, carbon fibre-reinforced plastic, strength, load carrying capacity, anchor.

Introduction

Improvement of efficiency and durability civil engineering and building structures in the current stage of development of construction industry is impossible without use of modern types of composite materials.

Operation of many buildings and facilities is related with use or production-related emission of products, which are aggressive towards regular reinforced concrete. Improvement of durability of traditional reinforced concrete structures during their operation in a context of aggressive environment requires expensive and labor-intensive protection measures.

That problem can be solved in a case of implementation of construction materials that are initially inert towards aggressive environment at the aforementioned kind of buildings and facilities. Polymer concretes based on various types of resins can be attributed to that kind of materials.

Effectiveness of polymer concrete significantly increases in a case of an implementation, instead of traditional steel reinforcement, of bar-type or composite-type glass or carbon fibre-reinforced plastic reinforcement, which, according to western terminology, is specified, respectively, as GFRP and CFRP. Use of those types of reinforcement allows to create new types of structures, possessing increased corrosion resistance. Composite materials are irreplaceable in strengthening of reinforced concrete structures. Especially, they are useful in a case of strucutres having initial defects [1, 2]. However, many of the related problems haven't been signifficantly studied so far.

Methodology of the research

Plan of the research

Testing program was divided into two stages. At the first stage, strengthened reinforced concrete beams with main reinforcement, which has yield plateau (A500 class), and a relatively small percentage of steel reinforcement μ_s=0. 57%, were tested for normal cross-sections. At the second stage, similar beams were tested, but with main reinforcement of A600 class, which doesn't have yield plateau, percentage of steel reinforcement μ_s=1. 12% was close to the optimum.

For beams of both stages the study comprised an evaluation of an influence of a type of composite materials, percentage of composite reinforcement μ_j and anchoring hardware of various types on strength of normal cross-section, deformability and crack resistance of experimental specimens.

For that purpose beams of I and II stages had been divided into five additional series. Series"A" are reference concrete beams without strengthening; series "B" are beams strengthened with GFRP fabric; series "C" is the same, but with CFRP fabric; series "D"are beams with glued CFRP laminates; series "E" are beams of "D" type with anchoring hardware at the ends of composite materials.

Aforementioned issues were investigated using experimental specimens of a rectangular cross-section produced from regular concrete with designed B35 strength class.

All experimental beams had the same length, height and width, which were, respectively, 220, 25, and 12. 5 cm. Designed dimensions of beams' cross-sections, their reinforcement and methods of strengthening and testing by means of a short-term loading are presented in Figure 1-3.

According to the plan of the research 22 experimental specimen were tested, including 10 beams at the first stage and 12 beams at the second stage. Strength parameters of the experimental beams are presented in figure 1. Regulatory values of concrete's strength are R_{bn}^{exp} for uniaxial compression and R_{btn}^{exp} – for tension, they are accepted according to [3] depending on strength class of concrete obtained by means of testing of standard 150 mm cubes. Cylinder uniaxial compressive strength was obtained by means of multiplying of prismatic strength of concrete on the conversion factor equal to 0. 8 or 0. 787 according to table 3. 1 of EN 1992-1-1:2004 (E).

Design and methods of testing of experimental specimens

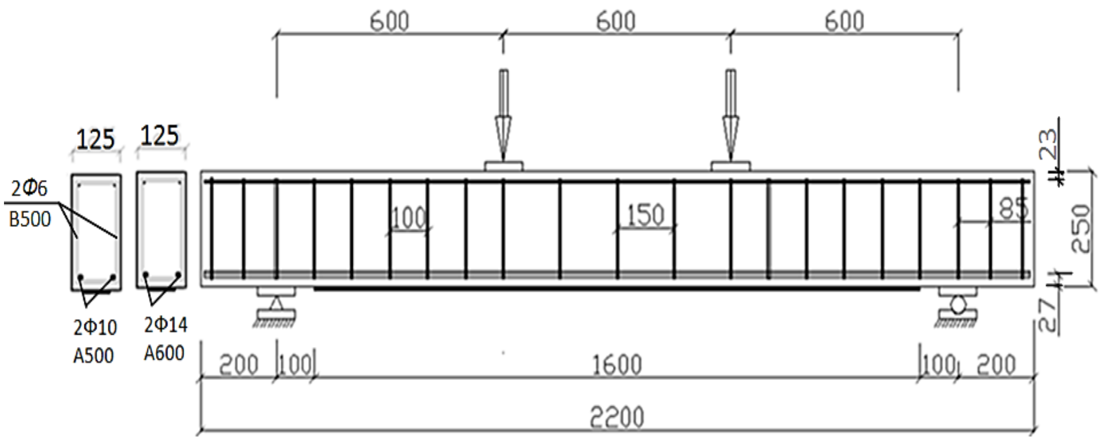


Figure 1. Scheme of reinforcement and testing method of the beams.

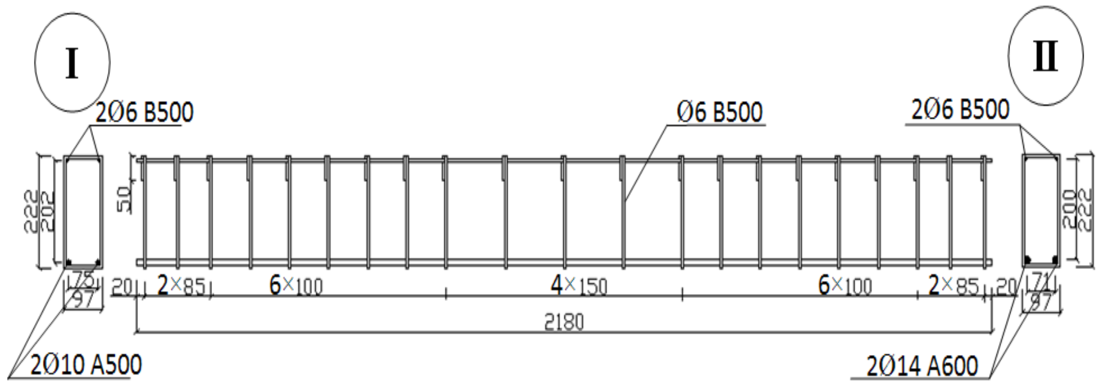


Figure 2. Design of frame of beams.

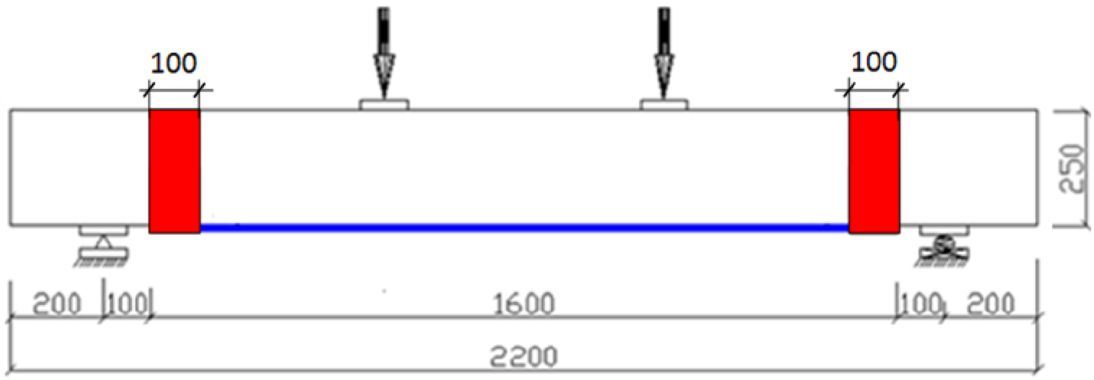


Figure 3. Scheme of strengthening and placement of anchors on the ends of laminates

Note:
U-shape end anchor was made from 4 layers of carbon fabric of 100 mm width.

Results of experiments
Strength of experimental specimens

Table 1. Results of strength tests of the beams

Stages of test of the beams by type of steel reinforcement	Series of the beams by a type of a composite material	Code of the beams	Area of a composite material, cm^2	Experimental strength of bars N_s^{exp}, N_f^{exp}, *kN*	Max. increase of load capacity N_f^{exp} / N_s^{exp} %	Load N_{ult}^{exp} at maximum allowable deflection $f_{ult}=l_o/200$ *kN*	Increase of maximum allowable load. $N_{ult}^{exp} / N_s^{exp}$ %
1	2	3	4	5	6	7	8
I stage	A Reference	**B-1-1**	-	57. 9	-	57. 7	-
		B-1-2	-	60. 6	-	60. 6	-
	B GFRP linen	**BSg-1-1**	0. 765	72. 5	22. 4	69. 5	17. 49
		BSg-1-2	1. 53	90. 8	53. 2	75. 5	27. 64
	C CFRP linen	**BSc-1-1**	0. 622	93. 6	58. 0	93. 6	58. 24
		BSc-1-2	1. 245	84. 0	41. 8	84	42

	D CFRP laminate	**BSL-1-1**	0. 7	96. 0	62. 0	96	62. 29
		BSL-1-2	1. 4	84. 0	41. 8	84	42
	E CFRP laminate+anchor	**BSL*-1-1**	0. 7	120. 0	102. 5	100	69. 06
		BSL*-1-2	1. 4	140. 1	136. 3	129	118. 1
II stage	A Reference	**BS-2-1**	-	125. 2	-	111	-
		BS-2-2	-	124. 6	-	110	-
	B GFRP linen	**BSg-2-1**	0. 765	140. 0	12. 1	113. 5	2. 7
		BSg-2-2	1. 53	151. 0	20. 9	116	4. 97
		BSg-2-3	1. 53	148. 3	18. 7	113	2. 26
	Same+half-anchor	**BSg*-2-4**	1. 53	155. 8	24. 7	116	4. 9
	C CFRP linen	**BSc-2-1**	0. 622	148. 0	18. 5	131	18. 55
		BSs-2-2	1. 245	134. 0	7. 3	131	18. 55
	D CFRP laminate	**B-CFRP L-2-1**	0. 7	133. 7	7. 04	127	14. 9
		B-CFRP L-2-2	1. 4	128. 0	2. 48	128	15. 84
	E CFRP laminate+anchor	**B-CFRP L*-2-1**	0. 7	166. 0	32. 9	132. 5	19. 9
		B-CFRP L*-2-2	1. 4	206. 0	64. 9	150	35. 75

Notes:

1) N_s^{exp} and N_f^{exp} designate a value of experimental load, applied on a traverse during testing of, respectively, reference and strengthened beams. 2) Strength of concrete was obtained according to [4].

Code of tested beams was accepted as follows: first letter «B» means that those are reference beams made from regular concrete; second letter «S» means that those are «B» type beams strengthened with various composite materials; third letter designates a type of a composite material («g» – GFRP linen; «c» – CFRP linen; «L» – CFRP laminates).

The first digit designates main reinforcement in a tension zone of the beams («1» – 2 A500 bars of 10 mm diameter; «2» – 2 A600 bars of 14 mm diameter). The second figure for reference specimens designates number of twin-beam and for strengthened beams – a second variant of a composite reinforcement. Asterisk (*) located next to the letters designates beams with anchoring hardware at the ends of strengthening elements.

Longitudinal reinforcement of compressed zone and transversal reinforcement of beams were the same, and that's why it is not reflected in the code. Beams' auxiliary reinforcement is made from 2 B500 class reinforcement wires of 6 mm diameter. Two legged stirrups, connected by means of tie wire, are accepted of the same wire class and diameter, and are placed with a step of 100 mm in shear zone and with a step of 150 mm in zone of simple bending (see Figure 1; 2). Mechanical properties of used steel reinforcement were identified according to [5], composite materials were identified according to [6].

Testing of two reference specimens (series I-A and II-A) was carried out using two twin-beams. Beams that were subjected to a strengthening («BS») were also produced with twin-beams from the same concrete mixture, but they were strengthened with a different percentage of a composite reinforcement. Let's call it strengthening consisting of one or two specific «linens» of composite materials. The first layer (the second digit of the code «1») is linen consisting of three layers of GFRP or CFRP fabric of 125 mm width or from one strip (laminate) of 1. 4××50 mm cross-section. The second layer (the second digit of the code «2») is linen of the same width made from six layers of GFRP or CFRP fabric or from 2 laminate strips on the basis CFRP with above mentioned cross-section.

The beams of «BS» type with asterisk «*» were equipped with anchors of 250 mm height and 100 mm width on the ends of the glued linens, the anchors were made from the same material as the strengthening elements.

Preliminary results

The analysis of the results allowed to note the following points.

The behavior of experiment specimens was directly dependent on an influence of all of factors mentioned above, which significantly affected the mode of failure of the tested specimens. Their total number was six.

Crushing of compressed concrete in a zone of simple bending, due to excessive development of a main normal crack;

Simultaneous crushing of concrete in a zone of simple bending and above the end an oblique crack – in a shear span due to a joint action of moments and lateral forces;

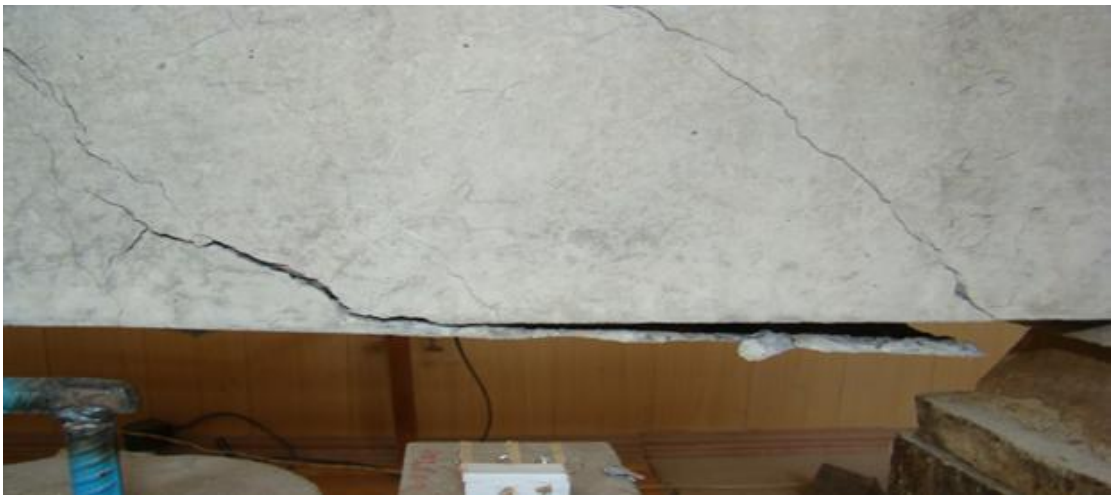
Simple failure in an oblique cross-section due to an action of transversal force because of crushing or cutting of concrete above the end an oblique crack;

Spalling of concrete cover due to an action of load in a case of significant deflections of beams;

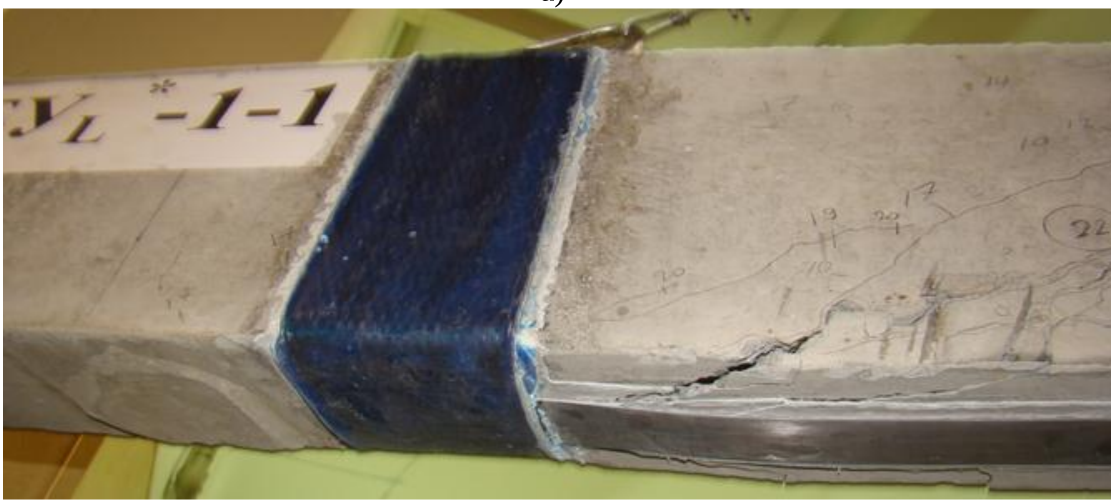
Failure of beams in a shear span in a case of an incomplete use of composite materials and load bearing capacity in total in normal and oblique cross-sections of the tested specimens. The reasons for those kinds of failures were: spalling of a composite material at its end (Figure 4a); slipping of a reinforcement under an anchor (Figure 4b); spalling of a concrete cover along main steel reinforcement (Figure 4c). The same mode of failure in a case of strength equal to 20 MPa is identified in the studies [7, 8, 9, 10].

A change of design model of beams' performance with a shift from free support model to arch model. That mode of failure occurs in a case of U-shaped anchors are installed. The later allow not only significantly increase load bearing capacity of strengthen cross-sections, but also to increase effectiveness of an implementation of a composite material itself. Also, a positive effect of U-shaped anchors was described in the previous studies [10, 11].

A presence of different modes of failure, as well as different composite materials significantly influenced a change of strength of normal cross-sections of strengthen specimens.



a)



b)

c)

Figure 4. Critical modes of failure of beams strengthened with composite materials. $БУ_L$-1-2-BS_L-1-2

Discussion

Direct comparison of the results demonstrated the following:

– Effectiveness of a composite strengthening, especially in a case of GFRP application, is rapidly decreasing in a case of an increase of an area of main steel reinforcement, in particular, with an increase of a compressed zone of concrete.

– An exceed of a limiting value of relative height of a compressed zone of concrete for a strengthen cross-section, ζ_{Rf} in a case of an increase of an area of a composite reinforcement, leads to a rapid decrease of a strengthening effect.

– A presence of rigid U-shaped anchors, located at the ends of composite elements, significantly increases load bearing capacity of normally reinforced beams. It's also worthing mentioning that efficiency of that anchors is slightly decreased with an increase of a steel's strength and a percentage of steel reinforcement, especially for GFRP strengthened elements.

Conditions of analysis

In the following part of the paper an influence of the aforementioned factors is discussed on a basis of the author's experimental data. In order to do that the three conditions–inequalities were used, those inequalities are presented in Guidelines for strengthening of reinforced concrete structures with composite materials [12]; that approach in many aspects to the methodology used in the USA [13]. Foundations of design of structures reinforced with composite materials were also developed with help of other methods of calculation presented in the previous studies [14, 15].

The condition 1 – $\xi_{s,f} < \xi_{Rf} < \xi_{RS}$ (1) takes place, when a failure of a strengthen element takes place simultaneously both in steel and composite reinforcements.

The condition 2 – $\xi_{Rf} < \xi_{S,f} < \xi_{RS}$ (2) takes place, when a failure of a strengthen cross-section occurs in a compressed concrete on a condition that limiting stress levels, which are equal to a yield point, are reached in both tensioned and compressed steel reinforcement.

The condition 3 – $\xi_{Rf} < \xi_{RS} < \xi_{S,f}$ (3) takes place, when a stress in tensioned steel and composite reinforcements doesn't reach their limiting values, at the same time, a level of stress in concrete's compressed zone is reaching limiting values.

In order to simplify the analysis, in addition to main letter symbols for relative characteristics of concrete's compressed zone, which are designated in "Guidelines for strengthening of reinforced concrete structures with composite materials", we added the symbols, designating a specific type of reinforcement – *s* for a steel reinforcement and *f* for a composite reinforcement.

ξ_s (ξ) – a relative height of a compressed zone of concrete for reference and strengthened reinforced concrete beams with a consideration of a steel reinforcement.

$\xi_{s,f}$ (ξ_f) – a relative height of a compressed zone of concrete for a strengthened cross-section with consideration of steel and composite reinforcement.

ξ_{Rs} (ξ_R) – a limiting relative height of a compressed zone of concrete considering only a steel reinforcement's performance.

ξ_{Rf} – a limiting relative height of a compressed zone of concrete for a strengthened cross-section with consideration of only a composite reinforcement's performance.

Note: designations, specified in Code [3] and Guidelines… [12], are presented next to the proposed designations of relative values in brackets.

Evaluation of an influence of various factors

In order to obtain a quantitative evaluation of an influence of all aforementioned factors, on a basis of a comparison of testing results of reference and strengthen beams, table 2 is created.

Taking into account, that a load bearing capacity a steel main reinforcement in the beams of II stage of the experiment is considerably higher, it can be stated that, a percent ratio of the results of testing doesn't provide a complete solution to the problem of an influence of a composite reinforcement. Therefore, the following analysis of the data by a strength of the experimental specimens, was carried out also by means of a direct comparison of absolute values of increases of a strength for the strengthen specimens as compared to the reference specimens.

In the following, more detailed analysis of the most important factors is presented.

Table 2. Evaluation of varied factors, influencing strength of normal cross-sections of the beams, which are strengthened with a composite reinforcement

Stages of testing depending on a type of a composite reinforcement	Varied factors	Type of a composite material	Value of composite reinforcement $\mu_{f1,2}$%	An increase of moments $\Delta_i = M_f^{\exp} - M_s^{\exp}$ kN	Influence of μ_f on an increase of Δ, kN	$\frac{\Delta 2}{\Delta 1}$ Ratio $\frac{\Delta 2}{\Delta 1}$
I stage 2x10 mm A500 A_s=1. 57 cm^2 μ_s=0. 56% R_{sn}=500 MPa	An influence of a type of a composite	GFRP linen	0. 185	3. 46	5. 31	2. 53
			0. 3696	8. 77		
		CFRP linen	0. 199	8. 02	7. 14	1. 89
			0. 398	15. 16		
	An influence of U-shaped anchors	CFRP laminates	0. 224	20. 01	6. 33	1. 89
			0. 448	26. 34		
II stage 2x14 mm A600 A_s=3. 08 cm^2 μ_s=1. 11% R_{sn}=600 MPa	Influence of a type of a composite	GFRP linen	0. 185	6. 6	3. 46	1. 5
			0. 3696	10. 6		
		CFRP linen	0. 199	8. 34	14. 39	2. 72
			0. 398	22. 73		
	An influence of U-shaped anchors	CFRP laminates	0. 224	14. 54	12. 53	1. 86
			0. 448	27. 07		

Note:
Values of moments are obtained by means of multiplying of load N_i from table 2 (that load is transferred from a traverse to a beam) and a value of a shear span in Figure 2.

Influence of a type of a composite material
An estimation of that factor is obtained by means of a comparison of an actual load bearing capacity both $M^{\exp}$ for the reference and strengthened specimens.
The beams that were strengthen with GFRP, in a case of a percentage of composite reinforcement equal to $\mu_f = 0.185$, demonstrated an average increase of a bending moment $M^{\exp}$ of 19. 1% in a case of a percentage of a steel reinforcement equal to $\mu_s = 0.56\%$, and 17. 6% for a case of $\mu_s = 1.11\%$. In a case of $\mu_f = 0.369$, an increase $M^{\exp}$ was equal to, respectively, 49. 3% and 26. 8%.

The same results are obtained for a comparison of absolute values of an increase of a load bearing capacity. In particular, in a case of an increase of μ_f in two times, an increase of a load bearing capacity $M^{\exp}$ increase in 2. 53 times for a value of $\mu_s = 0.56\%$ and in 1. 5 times for a value of $\mu_s = 1.11\%$.
Thus, total effect of an increase of a load bearing capacity in a case of an increase of a steel reinforcement decreases by 68%. At the same time, a minimal increase of a bending moment was only 12%. It worth mentioning that a rational degree of a strengthening of a structure by means of fibre-reinforced plastics is in the range of 10-60% from an initial load bearing capacity of a strengthened element. In that connection, there are obvious reasons for a discussion about a rational area of an application of GFRP strengthening reinforcement.
Considering that in the presented experiments the used reinforcement was of A500 ($\mu_s = 0..56\%$) and A600 ($\mu_s = 1.11\%$) classes, it is not completely proper to directly compare the experiment results using only the value μ_s . It is logical to use a relative characteristics.

That kind of criterion can be a relative height of a compressed zone of concrete – ζ . Taking into account, that both steel and composite reinforcements are used in calculations, in order to simplify the analysis we substituted the designation ζ for $\zeta_S = {R_s \cdot A_s}/{R_b \cdot b \cdot x}$, which have been mentioned earlier.

A value of ζ_S in the experiments in a case of an implementation of a reinforcement of A600 class was 0. 242-0. 262 or 0. 252 in average. In a case of an implementation of other class of a reinforcement, a recommended value ζ_S can be defined using a ratio of a strength of a reinforcement of A600 class and a strength of another classes. The recommended values for various classes of a reinforcement are presented in the table 3.

Table 3. Recommended boundaries ζ_f in a case of an implementation of GFRP reinforcement

Reinforcement class	A400	A500	A600
ζ_S Rational value ζ_S	0. 375	0. 3	0. 25

The detailed analysis of efficiency of an implementation of CFRP reinforcement in a case of a strengthening of reinforced concrete beams allows to infer that with other conditions being equal, CFRP strengthening reinforcement demonstrates considerably bigger absolute effect on an increase of a load bearing capacity (table 2). Therefore, CFRP can be effectively implemented for a strengthening of reinforced concrete beams for a wider range of steel reinforcement. In our opinion, the reason for that is that elasticity modulus for CFRP is by 5-15% higher than for steel.

Influence of U-shaped anchors

In order to obtain quantitative data on an influence of U-shaped anchors on a strength of normal cross-sections, a load bearing capacity of the experiment specimens with anchoring hardware and without it and the absolutely identical steel and composite reinforcement was directly compared.

The comparison showed that, in a case of a strengthening with one conditional linen, which comprises three layers of a fabric ($\mu_f = 0.199\%$), an absolute increase of a value of a moment Δ for values $\mu_s = 0.56\%$ and $\mu_s = 1.11\%$ is, respectively, 8. 02 and 8. 34 kNm. In a case of a strengthening with two conditional linen (6 layers of a fabric) – $\mu_f = 0.398\%$, an analogous increase was equal to 15. 2 and 23. 7 kNm. That data indicates that, with other conditions equal, an increase of an area of a steel or CFRP composite strengthening reinforcement leads to an increase of effectiveness of a performance of anchors. That relationship is also correct for an area of a composite reinforcement, which exceeds a limiting value of a relative height of a compressed zone of concreter ζ_{Rf} for a strengthened composite cross-section.

According to the results of the experiments, the effect of an installation of U-shaped anchors on the ends of strengthening elements is not limited only by an increase of a load bearing capacity of normal cross-sections. In our opinion, those anchors change a designed model of a performance of a strengthened specimen in a form of a conditional provision of an additional elastic support.

That opinion is based on the fact that in a case of an installation of those anchors oblique cracks are formed not in a center of gravity of a cross-section and are extended to an axis of a support, but they appear from a side of a tensioned zone of a bending element and start from a border of an anchor from a side of a pure bending and not from a side of a support's axis. A comparison of values of an increase of bending moments in a case of an installation of anchor showed that for $\mu_f = 0.199\%$ they increase by 26. 9 and 19. 1% correspondingly for $\mu_s = 0.56\%$ and for 52. 3 and 54. 4% in a case of $\mu_f = 0.398\%$.

In our opinion, a role of an anchor consists in two points. In a case of normally steel reinforced cross-section and composite materials reinforced cross section, i. e. in a case of $\zeta_{Sf} < \zeta_{Rf}$, anchors act as an elastic support. In a case of excessive or overreinforced composite strengthening, an anchor prevents spalling of concrete cover along a reinforcement.

Evaluation of an influence of various factors on a change of strength

In order to justify a role of an anchor following prerequisites are used. In a normally reinforced cross-section a change of moment in a cross-section with an anchor can be presented in a form of two identical equations (4) and (5), which are different by coefficient k_f. In a case of an identical steel and composite reinforcement, an increase of a strength can be cause only by a decrease of a designed span. Agreeing with this proposition, formulating the following equations:

$$M^{\exp} = S \cdot k \cdot l_0^2 \quad (4)$$

$$M^{\exp} = k_f \cdot S \cdot q \cdot l_{01}^2 \quad (5)$$

In those equations: coefficient S considers loading mode; k_f is a coefficient of a change of a load bearing capacity due to a change of a conditional designed span. According to the results of the experiments, an average value of coefficient is $k_f = 1.23$. By means of equating of right parts of equations (4) and (5) obtaining a conditional designed span. $l_{01} = l_0^2 / k_f$ (6). In particular we obtained $l_{01} = 1.8^2 / 1.23 = 1.62m$. In the discussed case a distance between outer ends of anchors was 1. 6 m. Taking into account those facts, a value of k_f can be obtained from the formula $k_f = l_0^2 / l_{01}^2 \leq 1.2$ (7), in the condition (7) $l_{01}^2 = l_0 - 2b_f$, where b_f is a width of an anchor.

The second component of an anchor's performance is starting to work in a case of an excessive cross-section of CFRP reinforcement. Let's designate it as α_{sf} and define it by means of the formula $\alpha_{sf} = \zeta_{Sf} / \zeta_{Rf} \leq 1.25$ (8). A limitation of a ratio by 1, 25 was accepted on a basis of the experiment results and so far doesn't have a sufficient statistical justification.

In a whole, an influence of an anchor on a change of a strength of beams can be presented in a form of a product of two coefficients $\alpha_{sf} \cdot k_f \leq 1.5$ (9), where k_f and α_{sf} are calculated by means of the equations (7) and (8). In the expression (8) in a case of a normally reinforced cross-section the coefficient α_{sf} for $\zeta_{Sf} \leq \zeta_{Rf}$ (10) will be gradually decreasing approaching one. In a case of an opposite sign in the inequality (10), it will be increasing.

An influence of the coefficient k_f on a change of a limiting moment received by a composite reinforcement is proposed to consider in a case the design condition (1) is satisfied, at the same time, a combined use of coefficients $\alpha_{sf} \cdot k_f$ – in a case of a satisfaction of the condition (2), by means of their introduction into formulas of Guidelines... corresponding to specified conditions.

The condition 3 – $\xi_{Rf} < \xi_{RS} < \xi_{S,f}$. It also takes place in a case of a strengthening of a structure. However, Guidelines... doesn't consider those variant of failure and design, which do not fully utilize mechanical properties of a steel bar-type reinforcement and an external composite reinforcement. Let us call that a case of failure by overreinforeced cross-section.

In the same time, a value of deformations of an external composite reinforcement is limited in design formulas by means of k_m coefficient in order to avoid its spalling. A value of the coefficient is defined form the two conditions:

$$k_m = \frac{1}{60 \cdot \varepsilon_{ft}} \cdot \left(1 - \frac{n \cdot E_{ft} \cdot t_f}{360000}\right) \le 0.9 \qquad (11)$$

$$k_m = \frac{1}{60 \cdot \varepsilon_{ft}} \cdot \left(\frac{90000}{n \cdot E_{ft} \cdot t_f}\right) \le 0.9 \qquad (12)$$

The identical limitation is presented in the USA codes (Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening Concrete Structures). [13]. Essentially, the equations (11) and (12) limit maximum allowable stress taking place in an external strengthening reinforcement.

The expression (11) is used on the condition, when $n \cdot E_{ft} \cdot t_f \le 180000$, and (12) – for $n \cdot E_{ft} \cdot t_f > 180000$.

Allowable maximum design deformations of fibre-reinforced plastics are equal to:

$$\varepsilon_{fu} \le k_m \cdot \varepsilon_{ft} \qquad (13)$$

A value of reached maximum deformations in an external reinforcement in a limiting state is defined by means of the equation:

$$\varepsilon_{fu} = \varepsilon_{cu} \cdot \left(\frac{h - x}{x}\right) - \varepsilon_{bi} \le k_m \cdot \varepsilon_{ft} \qquad (14)$$

An allowable level of stresses in a composite reinforcement is defined by means of Hook law:

$$\sigma_{fu} \le E_f \cdot \varepsilon_{fu} \qquad (15)$$

In th equations (11) and (12) all letter designations are accepted according to currently valid regulations of the Russian Federation [3, 12].

In a design practice a strengthening of reinforced concrete structures by the condition (3) exists. For example, in a case of a strengthening of thin-walled structures of T-shaped cross-section. For that kind of elements with a big compressed zone of concrete an increased cross-section of an external composite reinforcement is necessary. Finally, a percentage of steel and composite reinforcement, calculated using a section of a rib will increase its limiting values and a variant of strengthening will correspond exactly to the condition (3).

That fact facilitated experiments with overreinforced cross-sections. Experimental specimens of B, C, D series both for I and II stages of the study belong to that group (table 1).

Results of the experiments demonstrated that clearly overreinforced beams of the abovementioned series on the second stage of the study failed because of spalling of concrete cover along a steel reinforcement, but not because of spalling of a composite reinforcement (fig, 4c). That results appeared in spite of the fact that the strength of concrete of the tested beams exceed values of B30 strength class.

A preliminary analysis shows that in a case of an area of a composite reinforcement exceeding its limiting value, an increase of its load bearing capacity, as well as efficiency of an implementation of a composite reinforcement is vanishing. At the same time, a degree of a decrease of efficiency is directly dependent on a percentage of a reinforcement and a class of steel reinforcement.

It is also worth mentioning, that spalling of concrete cover, which leads to a failure of strengthen elements, occurs for loads not less than a strength of reference specimens. Thus, an excessive strengthening reinforcement doesn't lead to a negative result.

A direct comparison of the experimental and theoretical values of a bending moment shows, that design formulas [12] overestimate a load bearing capacity of the experimental specimens at the first stage of the study for $\mu_S = 0.56$ % and, backwards, underestimate it at the second stage, when $\mu_S = 1.11$ %. The best convergence is demonstrated by the experimental specimens strengthened by CFRP independently of a percentage of steel and composite reinforcement and beams, strengthened by CFRP laminates with an implementation of U-shaped anchors at the ends.

Less discrepancy of the results is demonstrated by the experimental specimens, for which coefficient k_m in the expression (12) was accepted equal to 0. 9.

It is also worth noting, that in the beams, strengthened with CFRP fabric and two layers of CFRP laminates, design stresses were minimal and laid in the limits of 522. 7-1071. 42 MPa, which indicates that the experimental specimens are clearly overreinforced.

A presence of a significant discrepancies between the results of the calculation and the experiment also indicates that it is necessary to correct calculation methodology, in the first place, the relationship (12) must be defined more precisely, that relationship must directly or indirectly consider a performance of a steel reinforcement, as well as an influence of U-shaped anchors on a load bearing capacity of strengthened cross-sections.

A positive influence of U-shaped anchors is also observed in the following studies [8, 16, 17]. In the study of D. N. Smerdov [18], on the contrary, it is stated that ending anchors doesn't influence a change of a load bearing capacity. In our opinion, this is obviously related with the that a transversal reinforcement of the experimental specimens wasn't of an equal strength with a normal cross-section.

In order to improve a convergence of results of a calculation and an experiment in a case of a clearly overreinforced cross-sections and without anchors, it is recommended to add the coefficient Δ_f , defined by means of an equation (16), to the equation (12).

$$\Delta_f = (1 - \zeta_{Rf} / \zeta_{Sf}), \tag{16}$$

in which boundary values of a relative height of compressed zone concrete for a regular reinforced concrete element ζ_{Rs} and an element reinforced with composite materials ζ_{Rf} is determined from the expression (17) and (18).

$$\xi_{RS} = \frac{x_{RS}}{h_0} = \frac{0,8}{1+\varepsilon_{s,el}/\varepsilon_{b,ult}} = \frac{0,8}{1+R_s/700} \quad (17)$$

$$\xi_{Rf} = \frac{x_{Rf}}{h} = \frac{\omega}{1+\frac{R_{fu}}{\varepsilon_{bu1}\cdot E_f}\cdot\left(1-\frac{\omega}{1,1}\right)} \quad (18)$$

$\omega = 0.85 - 0.008\cdot R_b$ where: $\varepsilon_{bu1} = \varepsilon_{b0}$ in a case of short term load and $\varepsilon_{bu1} = \varepsilon_{b2}$ in a case of long-term load. ($\varepsilon_{b0} =$ 0. 02 ; $\varepsilon_{b2} =$ 0. 0035); $\omega = 0.85 - 0.008\cdot R_b$

Because steel flowing deformations $R_s / R_b = 0.2\%$ are, in the vast majority of cases, less than the ratio $R_{fu} / E_f = (0.3-1)\%$ for composite materials, then the value is $\xi_{Rf} < \xi_{RS}$.

In a case of a presence of U-shaped anchors coefficients k_f and α_{sf} are used, which have been already mentioned above.

A selection of an area of a composite reinforcement – A_f . Guidelines... offer to carry out in iterations, starting from a certain initial value. After that the value A_f is corrected according to the result of calculations of a strength and an action of bending moments.

Considering, that it is quite difficult to select a reliable value of an area of a composite reinforcement from a first try, in order to reduce the calculation of iterations it is proposed to use previous works on a calculation of a strengthening of structures with the regular method. In a form adapted for a case of composite materials, design equations are, according to [19] are as follows:

$$A_f = -A/2 - \sqrt{A^2/4 - B} \quad (19)$$

where: coefficients A and B are defined from the expressions:

$$A = [R_s\cdot A_s - R_{sc}\cdot A_s' - R_b\cdot b\cdot(h_0 + a_0)]/0.5R_{ft} \quad (20)$$

$$B = \{2\cdot[M + (R_{sc}\cdot A_s' - R_s\cdot A_s\cdot h_0)]\cdot R_b\cdot b + [R_s\cdot A_s - R_{sc}\cdot A_s']^2\}/R^2_{ft}, \quad (21)$$

where the value a_0 is a distance between centers gravity steel and composite reinforcements. Considering that a composite reinforcement is glued onto a lower tensioned part of the beams and its thickness is only 1-2 mm, in order to simplify the calculations it is suggested to accept the value of a_0 equal to a distance from a center of gravity of a steel reinforcement to an ending fiber of a beam, i.

$$a_0 = a_в + 0.5d_s \tag{22}$$

R_{ft} – designed strength of a composite material is defined taking into account a reliability coefficient of a material – γ_f and a coefficient of operation conditions – C_E .

Conclusion

The presented analysis of design relationships of Guidelines... [12] used in Russia shows, that, in a whole, there is a good convergence between results of a calculation and an experiment. The experimental specimens, which failed because of spallling of concrete cover starting form an end of a composite materials, were an exception.

However, quality of calculation methodology, in our opinion, can be improved, if the following suggestions of authors will be considered.

1. An area of a strengthening composite reinforcement, which is unknown before a start of a calculation. is recommended to define, in a first approximation, using the following equations:
2. It is recommended for strengthened structures to restrict an area of application of GFRP reinforcement by means of values of relative height of a compressed zone of concrete – ξ_S, which are defined by means of table 3. 3. The restrictions are based on a low elasticity modulus of GFRP reinforcement, because in a case of an increase of a percentage of steel reinforcement a decrease of a level of an increase of a load bearing capacity takes place.
3. It is recommended to take into account a performance of U-shaped anchors in a performance of structures, which have an excessive cross-section of a composite reinforcement, using suggested coefficients k_f and $\alpha_{s,f}$, which are defined using the equations (7) and (8). Their values, depending on the ratio of $\xi_{S,f}$ and $\xi_{R,,f,}$ which are defined using the equations (17) and (18), must be introduced into design equations defining limiting moments taken by a strengthened cross-sections.

With a consideration of the recommendations, it is suggested to formulate the design equations as follows:

$(\xi_{Sf} < \xi_{Rf} < \xi_{RS})$ For the condition 1 $(\xi_{Sf} < \xi_{Rf} < \xi_{RS})$

$$M_{ult} = k_l \cdot \lfloor R_{fu} A_f \cdot (h - 0.5x) \rfloor + R_s A_s \cdot (h_0 - 0.5x) + R_{sc} A'_s \cdot (0.5x - a') \tag{23}$$

For the condition 2 $(\xi_{Rf} < \xi_{s,f} < \xi_{RS})$.

$$M_{ult} = k_l \cdot \alpha_{st} \cdot \left[\sigma_{fu} A_s \cdot (h_0 - 0.5x)\right] + R_s A_s \cdot (h_0 - 0.5x) + R_{sc} A_s' \cdot (0.5x - a') \qquad (24)$$

4. In a case of a design of an over reinforced with an external reinforcement structures, i. e. for th condition 3 – $\xi_{Rf} < \xi_{RS} < \xi_{S,f}$, the design equation (12) should be formulated as follows in order to improve a convergence of results of a calculation and an experiment (on a conditions of no anchors):

$$k_m = \frac{1}{60 \cdot \varepsilon_{ft}} \cdot \left[\frac{90000}{n \cdot E_{ft} \cdot t_f}\right] \cdot \Delta_f \leq 0.9 , \qquad (25)$$

where Δ_f is defined by means of the equation (13).

5. The term ending anchor should be understood as U-shaped rigid anchor of 150-200 mm width made of lines, which consists of not less than 4 layers of fabric of the same time with strengthening linens and laminates.

For a definition of a convergence of experimental and theoretical values of strength, taking into account the methodology of data treatment [20], an additional checking calculation with a consideration of suggestions of the authors was carried out. That calculation was conducted for the beams of C series independently on value of μ_f for the both stages of the research and for the beams of D and E series, which were strengthened with 6 layers of a fabric or by one of two types of CFRP laminates for the both stages of the study.

On a basis of the obtained results it can be concluded that coefficients k_f, $\alpha_{S,f}$ and Δ_f allow to significantly increase a convergence of the results of a calculation and an experiment for an increased or an excessive percentage of an external reinforcement.

The experiments will be continued in order to verify statistical reliability of the proposed recommendations.

References

[1] M. Arduini, A. Tommaso, A. Nanni, " Brittle failure in FRP plate and sheet bonded beams", ACI Struct. J., vol. 94(4), pp. 363-370, 1997. DOI: 10. 14359/487

[2] E. David, C. Djelal, F. Buyle-Bodin, "Repair and strengthening of reinforced concrete beams using composite", 7th Int. conf. on Struct. Faults and Repair, vol. 2, pp. 169-173, 1997. DOI: 10. 1061/(ASCE)CC. 1943-5614. 0000290

[3] SP 63. 13330. 2012, "Concrete and reinforced concrete structures. General provisions, Actualized version of SNIP 52-01-2003". -Moscow: FAI"FCS", 2012.

[4] GOST 10180-90, “Concretes. Methods for strength determination using reference specimens. (1990)”. -Moscow: Regulations publishing house, 1991.
[5] GOST 12004-81, “Reinforcing-bar steel. Tensile test methods. (1981)”. -Moscow: Regulations publishing house, 1983.
[6] GOST 25. 601-80, “Design calculation and strength testings. Methods of mechanical testing of polymeric composite materials. Test for tensile properties on plane specimens at normal, elevated and low temperatures”, 1980.
[7] N. Grace, G. Abdel-Sayed, W. Raghed, “Strengthening of concrete beams using innovative ductile fiber-fiber reinforced polymer fabric”, ACI Structural Journal, vol. 5(99), pp. 692-700, 2002. DOI: 10. 14359/12309
[8] P. Ritchie, D. Thomas, L. Lu, G. Connelly, “External reinforcement of concrete beams using fiber reinforced plastics”, ACI Structural Journal, vol. 4(88), pp. 490-499, 1991. DOI: 10. 14359/2723
[9] H. Toutanji, “Stress strain characteristics of concrete columns externally confined with advanced fiber composite sheet”, ACI Structural Journal, vol. 3(96), pp. 397-405, 1999. DOI: 10. 14359/639
[10] P. Akishin, A. Kovalovs, V. Kulakov, A. Arnautov, “Finite element modeling of reinforced concrete structures strengthened with FRP laminates”, final report SPR. -Oregon department of transportation, 2001. DOI: 10. 7250/iscconstrs. 2014. 01
[11] Ph. Mukhopadhyaya, N. Swamy, C. Lynsdale, “Optimizing structural response of beams strengthened with GFRP plates”, Journal Of Composites For Construction, pp. 78-95, 1998 DOI: 10. 1061/(ASCE)1090-0268(1998)2:2(87)
[12] V. Klevcov, “Guidelines for strengthening of reinforced concrete structures with composite materials”. -Moscow: NIIZhB, 2006.
[13] ACI 440. 2R-02, ” Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening Concrete Structures”, American Concrete Institute. DOI: 10. 1061/40753(171)159
[14] A. Shilin, V. Pshenichnyj, D. Kartuzov, “External strengthening of reinforced concrete strucutres woth composite materials”. -Moscow: JSC "Publishing house Stroiizdat", 2007.
[15] CNR-DT 200/2004, “Guide for the design and construction of externally bonded FRP systems for strengthening existing structures”, 2004.
[16] SG. padea, F. Bencardino, R. Swamy, “Structural Behavior of Composite RC beams with externally bonded CFRP”, J. Comp. Constr., ASCE, vol. 2(3), pp. 132-137, 1997. DOI: 10. 1061/(ASCE)1090-0268(1998)2:3(132).
[17] G. Tumialan, P. Serra, A. Nanni, A. Belarbi, “Concrete cover Delamination in RC beams strengthened with FRP sheets”, 4th International symposium on FRP for reinforcement of concrete structures, Balfimore, pp. 725-735, 1999. DOI: 10. 14359/639
[18] D. Smerdov, “Evaluation of load-bearing capacity of reinforced concrete bridge spans strengthened with composite material, PhD thesis, Novosibirsk, 2010.

[19] P. Pol'skoj, “Design and calculation f reinforced concrete structures strengthened by means of increase of cross-section. Textbook”. -Rostov-on-Don: RSCEU, 2012.

[20] L. Rumshiskij, “Mathematical treatment of experiments' results”. -Moscow: Science, 1971.