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Time dependent slip flow of a micropolar fluid between two parallel plates through state space approach

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Abstract

The unsteady slip flow of a micropolar fluid between two infinite parallel plates is considered. The linear slip boundary conditions for both velocity and microrotation are applied on the two boundaries. One of the two plates is set in motion with time dependent velocity while the other is held fixed. Non-dimensional variables are introduced. The analytical solution of the problem is obtained using state space technique. The velocity and microrotation in the physical domain are obtained by inverting their Laplace transforms numerically. The effects of the micropolarity, velocity slip and microrotation slip parameters on the velocity and microrotation are discussed through graphs.

AMS subject classification: 76-XX, 00A69, 44A10.
Keywords: Micropolar fluid, Slip condition, State space approach.

1. Introduction

In 1966, Eringen introduced the theory of micropolar fluids to describe the motion of fluids with microstructure taking into consideration the local motion of the particles inside the volume element of the fluid. In his model of micropolar fluids, he developed the equations of motion together with appropriate constitutive equations. The theory of micropolar fluids has been receiving a great importance because of its applications in industry, such as extrusion of polymer fluids, solidification of liquids crystals, animal blood, unusual lubricants and engineering applications. The micropolar fluid is assumed to be described by two independent vectors; the classical velocity and the micro-rotation vector [1].

The no-slip boundary condition has been used extensively in fluid dynamics. It assumes that the tangential velocity of the fluid particles at a point on the boundary has the same value of the velocity of the boundary at the point of contact. A general boundary condition that permits the possibility of fluid slip along the boundary has been first introduced by Navier. This condition states that the tangential velocity of the fluid relative to the solid at a point on its surface is proportional to the tangential stress at that point. The constant of proportionality is termed slip coefficient and is assumed to depend only on the nature of the fluid and solid boundary. Several authors have used the slip boundary conditions in both viscous and micropolar fluids [2-6]. Moreover, the spin boundary condition which is responsible for rational motion of microelements has been applied.

The unsteady unidirectional Poiseuille flow of a micropolar fluid between two parallel plates with no-slip and no-spin boundary conditions was investigated by Faltas et al[7]. Ashmawy [8] discussed the effect of slip condition to the problem of Couette flow of an incompressible micropolar fluid. Devakar and Iyengar [9] applied the technique of state space approach to the flow of an incompressible micropolar fluid between parallel plates assuming no-slip and no-spin conditions. The same authors used the same method to discuss the motion of a micropolar fluid between two plates, one fixed and the other is moving, assuming no-slip and no-spin boundary conditions [10]. The unsteady motion of a micropolar fluid between two fixed plates due to the presence of time dependent pressure gradient has been investigated in [4].

In this work, we consider the micropolar fluid flow through a two infinite parallel plates, when the upper plate at $y = h$ is held fixed but the other plate at $y = 0$ is moving with some velocity. The effects of velocity slip and microrotation slip parameters on the flow field are studied. State space technique is utilized to obtain the analytical expression for the velocity and microrotation. A standard numerical inversion methods implemented to invert the Laplace transform of the velocity and microrotation. The numerical results discussed through graphs. If the slip parameters are assumed to tend to infinity, the solution of the problem with no-slip and no-spin conditions is recovered.

Formulation of the problem

The motion of an incompressible isothermal micropolar fluid is governed by the following differential equations;

Conservation of mass

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

Balance of Momentum

$$\rho \frac{\partial \vec{q}}{\partial t} = -\nabla p + \kappa \nabla \times \vec{\nu} - (\mu + \kappa) \nabla \times \nabla \times \vec{q} \tag{2}$$

Balance of Moment of Momentum

$$\rho j \frac{\partial \vec{\nu}}{\partial t} = -2\kappa \vec{\nu} + \kappa \nabla \times \vec{q} - \gamma_0 \nabla \times \nabla \times \vec{\nu} + (\alpha_0 + \beta_0 + \gamma_0) \nabla (\nabla \cdot \vec{\nu}) \tag{3}$$

The stress and couple stress tensors are evaluated, respectively, by

$$t_{ij} = (\lambda q_{r,r} - p)\delta_{ij} + \mu q_{i,j} + (\mu + \kappa) q_{j,i} - \kappa \epsilon_{ijk} \nu_k \tag{4}$$

$$m_{ij} = \alpha_0 \nu_{r,r} \delta_{ij} + \beta_0 \nu_{i,j} + \gamma_0 \nu_{j,i} \tag{5}$$

where, the scalar quantities ρ and j are, respectively, the fluid density and gyration parameters and are assumed to be constants. Also, δ_{ij} and ϵ_{ijk} are denoting, respectively, Kronecker delta function and the alternating tensor. The two vectors $\vec{q}$ and $\vec{\nu}$ are respresenting, respectively, the velocity and microrotation of the fluid flow. The fluid pressure at any point is denoted by p. The material constants (μ, κ) represent the viscosity coefficients and $(\alpha_0, \beta_0, \gamma_0)$ represent the gyro-viscosity coefficients. We now consider an incompressible micropolar fluid between two infinite horizontal parallel plates separated by a distance h. The lower plate starts to move suddenly by a time dependent velocity of magnitude $Uf(t)$, where U is a constant with dimensions of velocity, along x-direction while the upper plate is held fixed. The pressure gradient of the flow is assumed to be zero.

The components of velocity and microroation are taking the forms $\vec{q} = (u(y,t), 0, 0)$ and $\vec{\nu} = (0, 0, c(y,t))$, respectively.

The equation of continuity (1) is satisfied automatically while the two equations(2) and (3), describing the physical situation in the absence of body forces and body couples, reduce to

$$\rho \frac{\partial u}{\partial t} = \kappa \frac{\partial c}{\partial y} + (\mu + \kappa) \frac{\partial^2 u}{\partial y^2} \tag{6}$$

$$\rho j \frac{\partial c}{\partial t} = -2\kappa c - \kappa \frac{\partial u}{\partial y} + \gamma_0 \frac{\partial^2 c}{\partial y^2} \tag{7}$$

Initially, the fluid flow was at rest. Therefore

$$u(y, 0) = 0, \qquad c(y, 0) = 0 \tag{8}$$

The imposed slip boundary conditions for both velocity and microrotation are

$$\beta_1 (u(0,t) - Uf(t)) = \tau_{yx}(0,t), \tag{9}$$

$$\beta_2 u(h,t) = -\tau_{yx}(h,t), \tag{10}$$

$$\xi_1 c(0,t) = m_{yz}(0,t), \tag{11}$$

$$\xi_2 c(h,t) = -m_{yz}(h,t) \tag{12}$$

where β_1 and β_2 are the slip parameters of the lower and upper plates. These parameters are varying from zero to infinity and are assumed to depend only on the nature of the fluid and the boundaries. Also, the microrotation slip parameters ξ_1 and ξ_2 are varying from zero to infinity. Using equation (4) and (5), the non-vanishing stress and couple stress tensors takes the following forms

$$\tau_{yx} = (\mu+\kappa)\frac{\partial u}{\partial y} + \kappa c(y,t), \qquad m_{yz} = \gamma_0 \frac{\partial c}{\partial y} \tag{13}$$

The following non-dimensional variables are introduced

$$\hat{y} = \frac{y}{h}, \qquad \hat{u} = \frac{u}{U}, \qquad \hat{t} = \frac{U}{h}t, \qquad \hat{c} = \frac{h}{U}c,$$
$$\hat{t}_{yx} = \frac{h}{U\mu}t_{yx}, \qquad \hat{m}_{yz} = \frac{h^2}{\beta_0 U}m_{yz}$$

The governing equations can be written in the form

$$R\frac{\partial \hat{u}}{\partial \hat{t}} = m\frac{\partial \hat{c}}{\partial \hat{y}} + \frac{\partial^2 \hat{u}}{\partial \hat{y}^2} \tag{14}$$

$$\frac{R}{n_2}\frac{\partial \hat{c}}{\partial \hat{t}} = -2n\hat{c} - n\frac{\partial \hat{u}}{\partial \hat{y}} + \frac{\partial^2 \hat{c}}{\partial \hat{y}} \tag{15}$$

where

$$R = \frac{\rho U h}{(\mu+\kappa)}, \quad n_2 = \frac{2+K}{2(1+K)}, \quad m = \frac{K}{1+K}, \quad n = \frac{kh^2}{\gamma_0}, \quad K = \frac{\kappa}{\mu}$$

After dropping hats, the governing equations and the initial and boundary conditions in non-dimensional form are

$$R\frac{\partial u}{\partial t} = m\frac{\partial c}{\partial y} + \frac{\partial^2 u}{\partial y^2} \tag{16}$$

$$\frac{R}{n_2}\frac{\partial c}{\partial t} = -2nc - n\frac{\partial u}{\partial y} + \frac{\partial^2 c}{\partial y^2} \tag{17}$$

$$u(y,0) = 0 \qquad \text{and} \qquad c(y,0) = 0 \qquad \text{for all } y$$

$$\alpha_1(u(0,t) - f(t)) = \tau_{yx}(0,t) \tag{18}$$

$$-\alpha_2(u(1,t)) = \tau_{yx}(1,t) \tag{19}$$

$$\eta_1 c(0,t) = m_{yz}(0,t) \tag{20}$$

$$-\eta_2 c(1,t) = m_{yz}(1,t) \tag{21}$$

Also, the expressions of the non-dimensional stress and couple stress components are

$$\tau_{yx}(y,t) = (1+K)\frac{\partial u(y,t)}{\partial y} + Kc(y,t) \tag{22}$$

$$m_{yz}(y,t) = \frac{\gamma_0}{\beta_0}\frac{\partial c(y,t)}{\partial y}. \tag{23}$$

where

$$\alpha_1 = \frac{h\beta_1}{\mu} \qquad \alpha_2 = \frac{h\beta_2}{\mu} \qquad \eta_1 = \frac{h\xi_1}{\gamma_0} \qquad \eta_2 = \frac{h\xi_2}{\gamma_0}$$

Solution of the problem

We now introduce the Laplace transform defined by

$$\bar{F}(y,s) = \int_0^\infty e^{-st}F(y,t)dt \tag{24}$$

After applying Laplace transform to the equations (16)–(22), we obtain

$$\frac{\partial^2\bar{u}}{\partial y^2} + m\frac{\partial\bar{c}}{\partial y} - Rs\bar{u} = 0 \tag{25}$$

$$\frac{\partial^2\bar{c}}{\partial y^2} - n\frac{\partial\bar{u}}{\partial y} - a\bar{c} = 0 \tag{26}$$

where

$$a = (2n + \frac{Rs}{n_2}) \tag{27}$$

The boundary conditions are taking the forms

$$\alpha_1(\bar{u}(0,s) - \bar{f}(s)) = (1+K)\bar{u'}(0,s) + K\bar{c}(0,s) \tag{28}$$

$$-\alpha_2\bar{u}(1,s) = (1+K)\bar{u'}(1,s) + K\bar{c}(1,s) \tag{29}$$

$$\eta_1\bar{c}(0,s) = \bar{c'}(0,s) \tag{30}$$

$$-\eta_2\bar{c}(1,s) = \bar{c'}(1,s) \tag{31}$$

The notations $\bar{u'} = \frac{\partial\bar{u}}{\partial y}$ and $\bar{c'} = \frac{\partial\bar{c}}{\partial y}$ are used for simplicity. Now, we apply the state space approach, so that the governing equations can be witten in the matrix form

$$\frac{\partial}{\partial y}\begin{pmatrix}\bar{u}\\ \bar{c}\\ \bar{u'}\\ \bar{c'}\end{pmatrix} = \begin{pmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ Rs & 0 & 0 & -m\\ 0 & a & n & 0\end{pmatrix}\begin{pmatrix}\bar{u}\\ \bar{c}\\ \bar{u'}\\ \bar{c'}\end{pmatrix} \tag{32}$$

where

$$\frac{\partial}{\partial y}\bar{V}(y,s) = A(s)\bar{V}(y,s); \tag{33}$$

and

$$A(s)=\begin{pmatrix}0&0&1&0\\0&0&0&1\\Rs&0&0&-m\\0&a&n&0\end{pmatrix},\ \bar{V}(y,s)=\begin{pmatrix}\bar{u}(y,s)\\\bar{c}(y,s)\\\bar{u'}(y,s)\\\bar{c'}(y,s)\end{pmatrix}$$

The formal solution of the matrix differential equation (33) is taking the form

$$\bar{V}(y,s)=\exp[A(s)y]\bar{V}(0,s) \tag{34}$$

To determine the matrix $\exp[A(s)y]$, the following characteristic equation of the matrix A(s) is then used

$$k^4-(Rs+a-mn)k^2+Rsa=0 \tag{35}$$

where $\pm k_1$, $\pm k_2$ are the roots of the characteristic equation and they are taking the forms

$$k_1=\sqrt{\frac{(Rs+a-mn)+\sqrt{(Rs+a-mn)^2-4Rsa}}{2}} \tag{36}$$

$$k_2=\sqrt{\frac{(Rs+a-mn)-\sqrt{(Rs+a-mn)^2-4Rsa}}{2}} \tag{37}$$

The Maclaurin series expansion of $\exp[A(s)y]$ is given by

$$\exp[A(s)y]=\sum_{r=0}^{\infty}\frac{[A(s)y]^r}{r!} \tag{38}$$

Utilizing the Cayley-Hamilton theorem, we can write the infinite series(38) as

$$\exp[A(s)y]=L(y,s)=a_0I+a_1A+a_2A^2+a_3A^3, \tag{39}$$

where I is the unit matrix of order 4 and a_0, a_1,a_2,a_3 are parameters depending on y and s. Then, the characteristic roots $\pm k_1$ and $\pm k_2$ satisfy equation (39) and hence we obtain the following system of linear equations after replacing the matrix A with its characteristic roots

$$\exp[k_1y]=a_0+a_1k_1+a_2k_1^2+a_3k_1^3 \tag{40}$$

$$\exp[-k_1y]=a_0-a_1k_1+a_2k_1^2-a_3k_1^3 \tag{41}$$

$$\exp[k_2y]=a_0+a_1k_2+a_2k_2^2+a_3k_2^3 \tag{42}$$

$$\exp[-k_2y]=a_0-a_1k_2+a_2k_2^2-a_3k_2^3 \tag{43}$$

After solving this system, we determine a_0, a_1, a_2 and a_3 as

$$a_0=\frac{1}{F}\left[k_1^2\cosh(k_2y)-k_2^2\cosh(k_1y)\right] \tag{44}$$

$$a_1 = \frac{1}{F}\left[\frac{k_1^2}{k_2}\sinh(k_2 y) - \frac{k_2^2}{k_1}\sinh(k_1 y)\right] \quad (45)$$

$$a_2 = \frac{1}{F}\left[\cosh(k_1 y) - \cosh(k_2 y)\right] \quad (46)$$

$$a_3 = \frac{1}{F}\left[\frac{1}{k_1}\sinh(k_1 y) - \frac{1}{k_2}\sinh(k_2 y)\right] \quad (47)$$

where

$$F = k_1^2 - k_2^2 \quad (48)$$

The elements ($L_{ij}; i, j = 1, 2, 3, 4$) of the matrix $L(y, s)$ are obtained, after inserting A, A^2, A^3 into equation(39), in the forms

$$L_{11} = \frac{1}{F}\left\{(k_1^2 - Rs)\cosh(k_2 y) - (k_2^2 - Rs)\cosh(k_1 y)\right\},$$
$$L_{12} = \frac{ma}{F}\left\{\frac{1}{k_2}\sinh(k_2 y) - \frac{1}{k_1}\sinh(k_1 y)\right\},$$
$$L_{13} = \frac{1}{F}\left\{\left(\frac{a - k_2^2}{k_2}\right)\sinh(k_2 y) - \left(\frac{a - k_1^2}{k_1}\right)\sinh(k_1 y)\right\},$$
$$L_{14} = \frac{m}{F}\left\{\cosh(k_2 y) - \cosh(k_1 y)\right\},$$
$$L_{21} = \frac{nRs}{F}\left\{\frac{1}{k_1}\sinh(k_1 y) - \frac{1}{k_2}\sinh(k_2 y)\right\},$$
$$L_{22} = \frac{1}{F}\left\{(k_1^2 - a)\cosh(k_2 y) - (k_2^2 - a)\cosh(k_1 y)\right\},$$
$$L_{23} = \frac{-n}{m}L_{14},$$
$$L_{24} = \frac{1}{F}\left\{\left(\frac{Rs - k_2^2}{k_2}\right)\sinh(k_2 y) - \left(\frac{Rs - k_1^2}{k_1}\right)\sinh(k_1 y))\right\},$$
$$L_{31} = RsL_{13}, \qquad L_{32} = aL_{14}, \qquad L_{33} = L_{11} + nL_{14},$$
$$L_{34} = \frac{m}{F}\left\{k_2\sinh(k_2 y) - k_1\sinh(k_1 y)\right\}, \qquad L_{41} = -\frac{nRs}{m}L_{14},$$
$$L_{42} = aL_{24}, \qquad L_{43} = -\frac{n}{m}L_{34}, \qquad L_{44} = L_{22} + nL_{14}$$

with these expressions, the solution (34) is obtained in the form

$$\bar{V}(y, s) = L(y, s)\bar{V}(0, s), \quad (49)$$

To determine the unknowns we proceed as follows. Using equation (49) and applying the boundary conditions satisfied at $y = 0$, we get

$$\bar{u}(0, s) = \frac{(1 + K)\bar{u'}(0, s) + K\bar{c'}(0, s)}{\alpha_1} + \bar{f}(s) \quad (50)$$

$$\bar{c}(0,s) = \frac{\bar{c'}(0,s)}{\eta_1} \tag{51}$$

Substituting the two equations (50) and (51) into the equation (49), we obtain the following expressions in the two unknowns $\bar{u'}(0,s)$ and $\bar{c'}(0,s)$

$$\bar{u}(y,s) = L_{11}\left(\frac{(1+K)\bar{u'}(0,s) + \frac{K\bar{c'}(0,s)}{\eta_1}}{\alpha_1} + \bar{f}(s)\right) + L_{12}\frac{\bar{c'}(0,s)}{\eta_1} + L_{13}\bar{u'}(0,s) + L_{14}\bar{c'}(0,s) \tag{52}$$

$$\bar{c}(y,s) = L_{21}\left(\frac{(1+K)\bar{u'}(0,s) + \frac{K\bar{c'}(0,s)}{\eta_1}}{\alpha_1} + \bar{f}(s)\right) + L_{22}\frac{\bar{c'}(0,s)}{\eta_1} + L_{23}\bar{u'}(0,s) + L_{24}\bar{c'}(0,s) \tag{53}$$

$$\bar{u'}(y,s) = L_{31}\left(\frac{(1+K)\bar{u'}(0,s) + \frac{K\bar{c'}(0,s)}{\eta_1}}{\alpha_1} + \bar{f}(s)\right) + L_{32}\frac{\bar{c'}(0,s)}{\eta_1} + L_{33}\bar{u'}(0,s) + L_{34}\bar{c'}(0,s) \tag{54}$$

$$\bar{c'}(y,s) = L_{41}\left(\frac{(1+K)\bar{u'}(0,s) + \frac{K\bar{c'}(0,s)}{\eta_1}}{\alpha_1} + \bar{f}(s)\right) + L_{42}\frac{\bar{c'}(0,s)}{\eta_1} + L_{43}\bar{u'}(0,s) + L_{44}\bar{c'}(0,s) \tag{55}$$

Applying the boundary conditions (19) and (21) at $y = 1$ to the equations (52)–(55), we get after some calculations and rearrangements

$$\begin{aligned}\bar{c'}(0,s) = \alpha_1\eta_1\bar{f}(s)(\alpha_2(\eta_2(L^1_{11}L^1_{23} - L^1_{13}L^1_{21}) + L^1_{11}L^1_{43} - L^1_{13}L^1_{41})\\ + \eta_2(K+1)(L^1_{23}L^1_{31} - L^1_{21}L^1_{33})\\ + K(L^1_{21}L^1_{43} - L^1_{23}L^1_{41} + L^1_{31}L^1_{43} - L^1_{33}L^1_{41})\\ + L^1_{31}L^1_{43} - L^1_{33}L^1_{41})/\Delta\end{aligned}$$

$$
\begin{aligned}
\bar{u}'(0,s) = {} & (-\alpha_1 \bar{f}(s)(\alpha_2(\eta_1(\eta_2(L^1_{11}L^1_{24} - L^1_{14}L^1_{21}) + L^1_{11}L^1_{44} - L^1_{14}L^1_{41}) \\
& + \eta_2(L^1_{11}L^1_{22} - L^1_{12}L^1_{21}) + L^1_{11}L^1_{42} - L^1_{12}L^1_{41}) \\
& - \eta_1(\eta_2(K+1)(L^1_{21}L^1_{34} - L^1_{24}L^1_{31}) - K(L^1_{21}L^1_{44} - L^1_{24}L^1_{41} \\
& + L^1_{31}L^1_{44} - L^1_{34}L^1_{41}) - L^1_{31}L^1_{44} + L^1_{34}L^1_{41}) + \eta_2(K+1)(L^1_{22}L^1_{31} - L^1_{21}L^1_{32}) \\
& + K(L^1_{21}L^1_{42} - L^1_{22}L^1_{41} + L^1_{31}L^1_{42} - L^1_{32}L^1_{41}) + L^1_{31}L^1_{42} - L^1_{32}L^1_{41}))/\Delta
\end{aligned}
$$

where

$$
\begin{aligned}
\Delta = {} & (\alpha_1(\alpha_2(\eta_1(\eta_2(L^1_{13}L^1_{24} - L^1_{14}L^1_{23}) + L^1_{13}L^1_{44} - L^1_{14}L^1_{43}) \\
& + \eta_2(L^1_{13}L^1_{22} - L^1_{12}L^1_{23}) - L^1_{12}L^1_{43} + L^1_{13}L^1_{42}) \\
& - \eta_1(\eta_2(K+1)(L^1_{23}L^1_{34} - L^1_{24}L^1_{33}) - K(L^1_{23}L^1_{44} - L^1_{24}L^1_{43} \\
& + L^1_{33}L^1_{44} - L^1_{34}L^1_{43}) - L^1_{33}L^1_{44} \\
& + L^1_{34}L^1_{43}) + \eta_2(K+1)(L^1_{22}L^1_{33} - L^1_{23}L^1_{32}) - K(L^1_{22}L^1_{43} - L^1_{23}L^1_{42} \\
& + L^1_{32}L^1_{43} - L^1_{33}L^1_{42}) - L^1_{32}L^1_{43} + L^1_{33}L^1_{42}) + \alpha_2(\eta_1(K+1)(\eta_2(L^1_{11}L^1_{24} - L^1_{14}L^1_{21}) \\
& + L^1_{11}L^1_{44} - L^1_{14}L^1_{41}) + \eta_2(K(L^1_{11}(L^1_{22} - L^1_{23}) - L^1_{21}(L^1_{12} - L^1_{13})) \\
& + L^1_{11}L^1_{22} - L^1_{12}L^1_{21}) + K(L^1_{11}(L^1_{42} - L^1_{43} - L^1_{41}(L^1_{12} - L^1_{13})L^1_{11}L^1_{42} - L^1_{12}L^1_{41}) \\
& - \eta_1(K+1)(\eta_2(K+1)(L^1_{21}L^1_{34} - L^1_{24}L^1_{31}) - K(L^1_{21}L^1_{44} - L^1_{24}L^1_{41} \\
& + L^1_{31}L^1_{44} - L^1_{34}L^1_{41}) - L^1_{31}L^1_{44} + L^1_{34}L^1_{41}) \\
& - \eta_2(K+1)(K(L^1_{21}(L^1_{32} - L^1_{33}) - L^1_{31}(L^1_{22} - L^1_{23})) \\
& + L^1_{21}L^1_{32} - L^1_{22}L^1_{31}) + K^2(L^1_{21}(L^1_{42} - L^1_{43}) - L^1_{22}L^1_{41} \\
& + L^1_{23}L^1_{41} + L^1_{31}(L^1_{42} - L^1_{43}) - L^1_{41}(L^1_{32} - L^1_{33})) \\
& + K(L^1_{21}L^1_{42} - L^1_{22}L^1_{41} + L^1_{31}(2L^1_{42} - L^1_{43}) - L^1_{41}(2L^1_{32} - L^1_{33})) \\
& + L^1_{31}L^1_{42} - L^1_{32}L^1_{41})
\end{aligned}
$$

where L^1_{ij}'s are denoting the values of $L(y,s)$ at $y = 1$. Then, by inserting the expressions for $\bar{u}'(0,s)$, $\bar{c}'(0,s)$ into equations (52-55), we obtain the velocity and microrotation in the Laplace domain.

The numerical inversion of Laplace transform

To obtain the components of the velocity and microrotation, namely $u(y,t)$ and $c(y,t)$, in the physical domain, we use a numerical inversion technique developed by Honig and Hirdes [11] to invert Laplace transform.

Utilizing this numerical technique, the inverse Laplace transform of the function $\bar{g}(s)$ is approximated by the formula

$$
g(t) = \frac{\exp(bt)}{T}\left[\frac{1}{2}\bar{g}(b) + Re\left(\sum_{k=1}^{N} \bar{g}\left(b + \frac{ik\pi}{T}\right)\exp(\frac{ik\pi t}{T}\right)\right], \qquad 0 < t < 2T
$$

N is sufficiently large integer chosen such that,

$$\exp(bt)Re\left[\bar{g}(b+\frac{iN\pi}{T})\exp(\frac{iN\pi t}{T})\right] < \epsilon,$$

where ϵ is a small positive number that corresponds to the degree of accuracy required. The parameter b is a positive free parameter that must be greater than real parts of all singularities of $\bar{g}(s)$.

Numerical results and discussions

In this section, we implement the above mentioned technique to the obtained results by assuming that the moving plate is suddenly moved with constant velocity; i.e. $f(t) = H(t)$, where $H(t)$ is the Heaviside step function. Also, the slip and spin of the upper and lower plates are assumed to be equal, this means that we take $\alpha_1 = \alpha_2 = \alpha$ and $\eta_1 = \eta_2 = \eta$. The following figures show the behaviors of the velocity and microrotation versus the distance for different values of the time, slip and spin parameters, respectively. It can be observed that the velocity slip coefficient has a considerable influence on both velocity and microrotation functions while the spin coefficient affects the microrotation only. Also, the steady state solution can be deduced from this case when the time is assigned a large value.

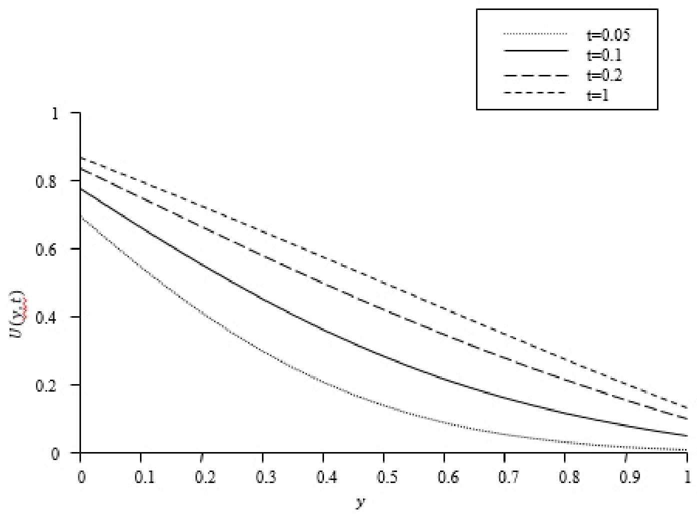

Figure 1: variation of velocity versus distance for partial slip with $\alpha = \eta = 10$ and $K = 1$.

Conclusion

The unsteady flow of a micropolar fluid between two infinite parallel plates with velocity slip and microrotation slip boundary conditions is investigated. The techniques of

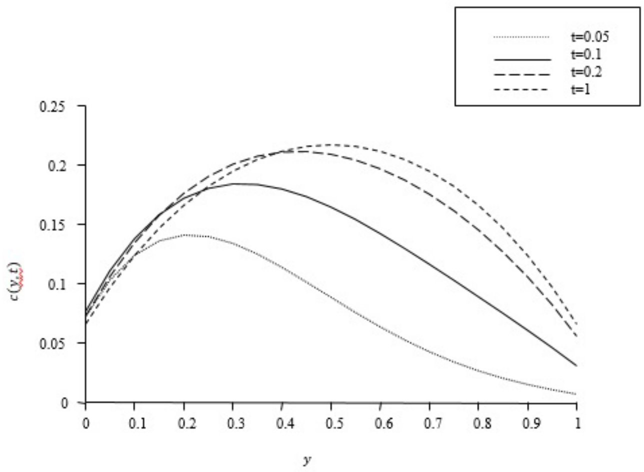

Figure 2: variation of microrotation versus distance at $K = 1$ and $\alpha = \eta = 10$.

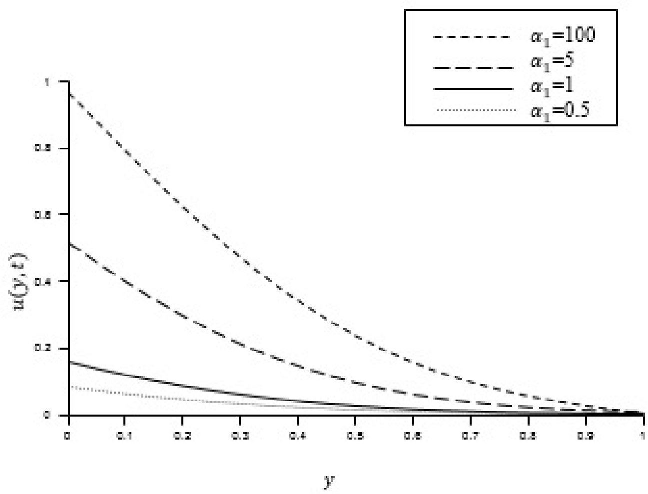

Figure 3: Variation of velocity versus distance at at $t = 0.05$, $K = 1$ and $\eta = 10$.

Laplace transform together with the state space approach are utilized to obtain the analytical solution in the Laplace domain. The inversion of Laplace transforms are carried out numerically. The results indicate that the micropolarity parameter has an increasing effect on both velocity and microrotation. In addition, it is concluded that the velocity slip parameters increase considerably the values of both velocity and microrotation. Also, the microrotation slip coefficient has considerable contribution to the microrotation function only.

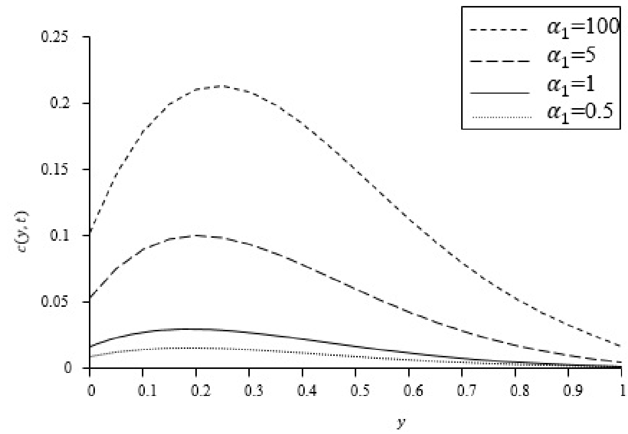

Figure 4: Variation of microrotation versus distance at at $t = 0.05$, $K = 1$ and $\eta_1 = 10$.

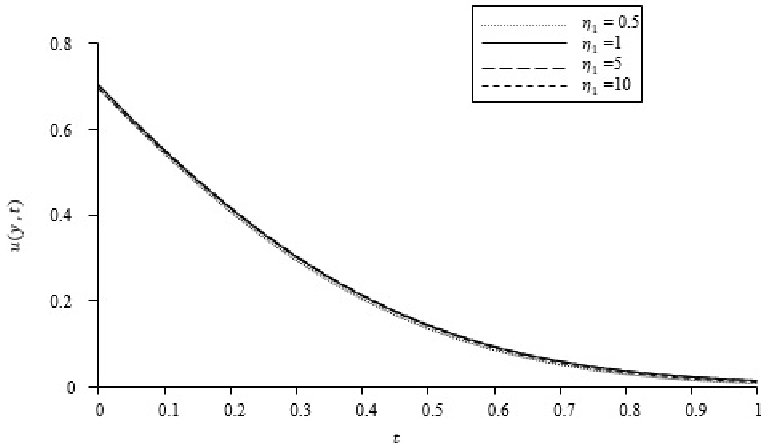

Figure 5: Variation of velocity versus distance at $t = 0.05$, $K = 1$ and $\alpha_1 = 10$.

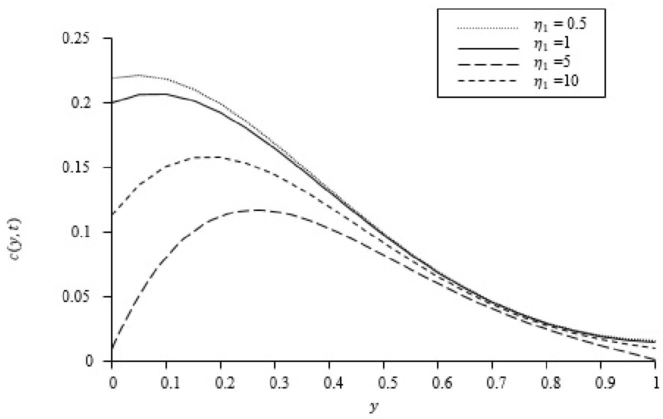

Figure 6: Variation of microrotation versus distance at $t = 0.05$, $K = 1$ and $\alpha_1 = 10$.

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