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An identical period-doubling route to chaos in a family of 3-D sinusoid discrete maps

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Abstract

In this paper, we have proposed a new family of sinusoid discrete chaotic map with two parameters, this family can exhibits a chaotic attractors form same typical period-doubling bifurcation route to chaos. This physical phenomenon is justified by numerical investigation.

AMS subject classification:
Keywords: Route to chaos, period-doubling, 3-D sinusoid discrete map, chaotic attractors.

1. Introduction

It is well-known in the theoretical research the sinusoid map play an important role in mathematics, sinusoid map or sinusoidal map is generally the sine wave map which is related to the oscillations, can describe many oscillating phenomena. This map is very commonly used in pure and applied mathematics [3], [4], in addition to mathematics, sinusoid map occur in other fields of study such as science, physics and engineering [5], [7]. This map also occurs in nature as seen in ocean waves, sound waves, light waves and many other fields. Some authors have described chaotic map with sine map, such as: [5], [8], [9], [10], [11]. In recent years, many documents have described 3-D chaotic maps such as with quadratic inverse [12], [13], [14], [15], [16]. Doubtless, the study of 3-D discrete maps such as with sine and cosine functions and especially the discrete maps which shows the same route to chaos of bifurcation is a very interesting contribution to the development of the theory of dynamical systems. This short paper

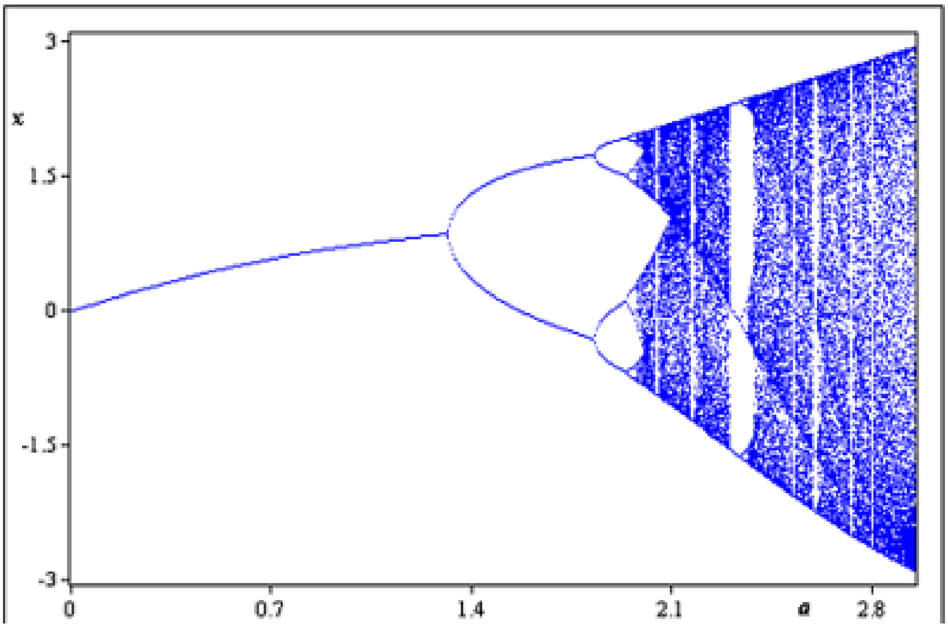

Figure 1: The bifurcation diagram for the map (1) in the cases (2.1) till (2.7) obtained for $b = -3.66$ and $0 \leq a \leq 2.95$.

investigate a new simplest 3-D sinusoid discrete map with two nonlinearities sine and cosine maps topologically different from any other know 3-D maps, that realizes a new physical phenomenon in which the new chaotic attractors for our proposed map (1) in the cases (2.1) till (2.7) are obtained from identical period doubling bifurcation route to chaos as shown in Fig. 1.

2. The proposed 3-D map

Here we consider a new simple 3-D sinusoid discrete map described by:

$$\begin{pmatrix} \overline{x} \\ \overline{y} \\ \overline{z} \end{pmatrix} = \begin{pmatrix} a \cos x \\ b \sin y \\ f(x, y, z) \end{pmatrix} \tag{2.1}$$

where

$$f(x, y, z) = a_0 + a_1 x + a_2 y + a_3 z + a_4 x^2 + a_5 y^2 + a_6 z^2 + a_7 xy + a_8 xz + a_9 yz$$

and $(a, b, a_i)_{0 \leq i \leq 9} \in \mathbb{R}^{12}$ are bifurcation parameters and $(x, y, z) \in \mathbb{R}^3$ are the state variables. On the other hand the associated map of the new 3-D sinusoid discrete map (1) is continuous and differentiable on $\mathbb{R}^3$. For the case (2.1) the map (1) is of class $C^\infty(\mathbb{R}^3)$ and is one-to one.

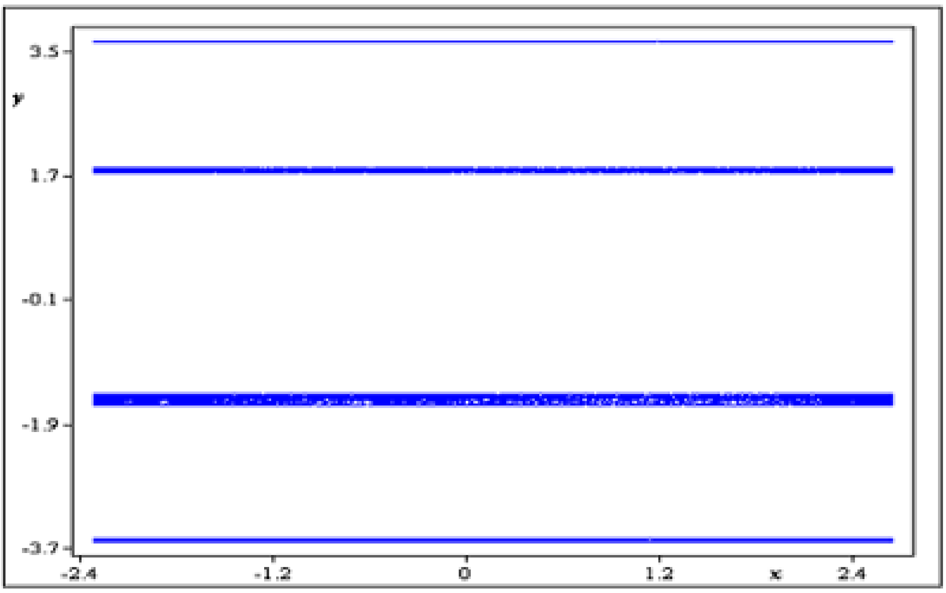

Figure 2: Phase portraits of the map (1) in the cases (2.1) till (2.7) obtained for $a = 2.64$ and $b = -3.66$.

3. Numerical investigation

There are several possible methods for a discrete dynamical map to make a transition from regular behavior to chaotic behavior. The numerical computing of bifurcation diagrams display these methods (routes) and let one to distinguish the chaotic regions from which the chaotic attractors can be determined. In general there are three most popular routes to chaos. For example the chaotic attractor for the famous 2-D Hénon map [1] is obtained from a period doubling bifurcation route to chaos, and the Lozi chaotic attractor [2] is obtained from a border-collision bifurcation route to chaos.

With the same initial conditions $x_0 = 0.25$, $y_0 = 0.2$, $z_0 = 0.02$ and bifurcation parameters $a = 2.64$ and $b = -3.66$ the dynamical behaviors of the map (1) in the cases (2.1) till (2.7) are investigated numerically. Fig.1 shows the bifurcation diagram of the proposed map (1) in the cases (2.1) till (2.7), the bifurcation diagram in Fig. 1 exhibit a period doubling bifurcation scenario route to chaos for the selected values of the bifurcation parameters a and b, i.e., the chaotic attractors of the seven cases of the map (1) are obtained through a period doubling bifurcation route to chaos. Fig. 2 shwos the phase portraits of the map (1) in the cases (2.1) till (2.7).

$$f(x, y, z) = z \tag{2.1}$$

$$f(x, y, z) = y + z \tag{2.2}$$

$$f(x, y, z) = x + y + z \tag{2.3}$$

$$f(x, y, z) = x + y + z + x^2 \tag{2.4}$$

$$f(x, y, z) = x + y + z + y^2 \tag{2.5}$$

$$f(x, y, z) = x + y + z + x^2 + y^2 \tag{2.6}$$

$$f(x, y, z) = x + y + z + xy + x^2 + y^2 \tag{2.7}$$

4. Conclusion

This short paper investigate a new simplest 3-D sinusoid discrete map with two nonlinearities sine and cosine maps that realizes a new physical phenomenon in which the new chaotic attractors for our proposed map (1) in the cases (2.1) till (2.7) are obtained from identical period doubling bifurcation route to chaos this phenomenon is justified by numerical investigation.

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