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Weyl and Weyl type theorems for m-quasi-n-class $A(k)$ and algebraically m-quasi-n-class $A(k)$ operators

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Abstract

In this paper, we introduced m-quasi-N-class $A(k)$ operators, where k and m are positive integers for a fixed $N > 0$. We prove that, if T is m-quasi-N-class $A(k)$ operator, then T is an isoloid, T is of finite ascent, T is reguloid, Weyl's theorems holds T and also for $f(T)$ for every $f \in H(\sigma(T))$. We define algebraically m-quasi-N-class $A(k)$ operators and prove if T is algebraically m-quasi-N-class $A(k)$, then Weyl's theorem hold for T and $f(T)$ for every $f \in H(\sigma(T))$, T is polaroid. We discussed H property, (β) property, SVEP, Generalized Weyl's and Weyl type theorems.

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1. Introduction

Let $T \in B(H)$ be the Banach algebra of all bounded linear operator on a non-zero complex Hilbert space H. By an operator T, we mean an element from $B(H)$. If T lies in $B(H)$, then T^* denoted as adjoint of $T \in B(H)$. An operator T is called paranormal if $\|T^2x\| \geq \|Tx\|^2$, for every unit vector $x \in H$. An operator T belongs to class A, if $|T^2| \geq |T|^2$. An operator T is said to be p-hyponormal if $(T^*T)^p \geq (TT^*)^p$. If p=1 then T is called hyponormal operator. An operator T is said to be class $A(k)$ for $k > 0$, if $(T^* |T|^{2k} T)^{\frac{1}{k+1}} \geq |T|^2$. An operator T is called normaloid if $r(T) = \|T\|$, where $r(T) = sup \{|\lambda| : \lambda \in \sigma(T)\}$ and isoloid if every isolated point of $\sigma(T)$ is an eigen value of T. The ascent of T denoted by $p\,(T)$, is the least non-negative integer n such that $kerT^n = kerT^{n+1}$. The descent of T denoted by $q\,(T)$, is the least non-negative integer n such that $ran\left(T^n\right) = ran\left(T^{n+1}\right)$. T is said to be of finite ascent if $p\,(T - \lambda) < \infty$, for all $\lambda \in C$. If $p\,(T)$ and $q\,(T)$ are both finite, then $p\,(T) = q\,(T)$ [18]. Moreover, $0 < p\,(\lambda I - T) = q\,(\lambda I - T) < \infty$ precisely when λ is a pole of the resolvent of T.

For $T \in B(H)$, we write $kerT$ and $ranT$ for the null space and range of T, respectively. An operator $T \in B(H)$ is called upper semi-Fredholm if it has closed range and finite dimensional null space $(i.e., \alpha(T) := dimkerT < \infty)$, and $T \in B(H)$ is called lowe semi-Fredholm if it has closed range and finite co-dimensional null space $(i.e., \beta(T) := dimkerT^* < \infty)$. If $T \in B(H)$ is both upper semi-Fredholm and lower semi-Fredholm, we call it is Fredholm. If $T \in B(H)$ is semi-Fredholm, then the index of T, is denote by $ind(T)$, is given by $ind(T) = \alpha(T) - \beta(T)$.

We denote the spectrum of $T \in B(H)$ by $\sigma(T)$, and the sets of isolated points and accumulation points of spectrum of $\sigma(T)$ denoted by $iso\sigma(T)$ and $acc\sigma(T)$, respectively.

$$\sigma\,(T) = \{\lambda \in C : T - \lambda I is\ not\ invertible\},$$

The essential spectrum of T is defined as

$$\sigma_e(T) = \{\lambda \in \mathbb{C} : T - \lambda\ is\ not\ Fredholm\},$$

The appropriate point spectrum of T is

$$\sigma_a\,(T) = \{\lambda \in C : T - \lambda I is\ not\ bounded\ below\},$$

The essential appropriate point spectrum of T is defined as

$$\sigma_{ea}\,(T) = \{\lambda \in C : T - \lambda I is\ not\ semi - Weyl\ operator\},$$

The Weyl spectrum of T is defined as

$$w(T) = \{\lambda \in \mathbb{C} : T - \lambda\ is\ not\ Weyl\},$$

The Browder spectrum of T is defined as

$$\sigma_b\,(T) = \{\lambda \in C : T - \lambda I\ is\ not\ Weyl\}.$$

It is well known that $\sigma_e(T) \subseteq w(T)$. The set of isolated eigen values of finite multiplicity denoted by $\pi_{00}(T) := \{\lambda \in \mathbb{C} : \lambda \in iso\sigma(T) and 0 < \alpha(T-\lambda) < \infty\}$. Let $T \in B(H)$ and let λ_0 be an isolated point of $\sigma(T)$. Then there exists a small enough positive number $r > 0$ such that $\{\lambda \in C : |\lambda - \lambda_0| \leq r\} \cap \sigma(T) = \lambda_0$. We say that a-Weyl's theorem holds for T [33], if T satisfies the equality

$$\sigma_a(T) - \sigma_{ea} = \pi_{00}^a(T).$$

We say that T satisfies property (w) if

$$\sigma_a(T) - \sigma_{ea}(T) = \pi_{00}(T)$$

and satisfies property (b) if

$$\sigma_a(T) - \sigma_{ea}(T) = p_{00}(T).$$

For an operator T and a non-negative integer n, define $T_{[n]}$ to be the restriction of T to $R(T^n)$ viewed as a map from $R(T^n)$ into $R(T^n)$. In particular, $T_{[0]} = T$. If for some integer n, $R(T^n)$ is closed and $T_{[n]}$ is an upper $(resp.alower)$ semi-Fredholm operator, then T is called upper $(resp.lower)$ semi-B-Fredholm operator. Moreover, if $T_{[n]}$ is a Fredholm operator, then T is called B-Fredholm operator. A semi-B-Fredholm operator is an upper or lower semi-B-Fredholm operator. The index of a semi-B-Fredholm operator T is the index of semi-Fredholm operator $T_{[d]}$, where d is the degree of the stable iteration of T and defined as

$$d = inf\left\{n \in N; \forall m \in N, m \geq n \Rightarrow (R(T^n) \cap N(T)) \subset (R(T^m) \cap N(T))\right\}.$$

Then B-Weyl spectrum $\sigma_{BW}(T) = \{\lambda \in C : T - \lambda I is\ not\ a\ B - Weyloperator\}$. We say that T satisfies generalized Weyl's theorem [6], if $\sigma(T) - \sigma_{BW}(T) = E(T)$, by [8], if Generalized Weyl's theorem holds for T, then Weyl's theorem holds for T.

2. weyl's theorem for m-quasi-N-class $A(k)$ operators

In this section, we introduced m-quasi-N-class $A(k)$ operator and prove that Weyl's theorem holds for them. We prove that T is m-quasi-N-class $A(k)$ operator, then T is isoloid, finite ascent, property H, property (β) and the Riesz idempotent operator E_λ with respect to λ is self-adjoint and satisfies

$$E_\lambda H = ker(T - \lambda I) = ker(T - \lambda I)^*.$$

Definition 2.1. An operator $T \in B(H)$ is defined to be m-quasi-N-class $A(k)$, if $T^{*m}\left(N\left(T^*|T|^{2k}T\right)^{\frac{1}{k+1}} - |T|^2\right)T^m \geq 0$, where k and m are positive integers for a fixed $N > 0$.

Theorem 2.2. [35] If T is p-hyponormal or log-hyponormal operator, then T is N-class $A(k)$ operator for each positive integer k.

Theorem 2.3. [35] Let T be an invertible and class A operator. Then

1. T is of N-class $A(k)$ operator for every positive integer k
2. N-class $A\ (1) \subseteq N - classA\ (2) \subseteq N - classA\ (3)$.....
3. For all positive integer n, T^n is of N-class $A(k)$ operator for every positive integer k
4. T^{-1} is of N-class $A(k)$ operator for every positive integer k.

Theorem 2.4. [35] If T is of N-class $A(k)$ for some positive integer k, then T is k-paranormal.

Theorem 2.5. If T belongs to N-class $A(k)$ for some $k \geq 1$, then T belongs to m-quasi-N-class $A(k)$.

Theorem 2.6. Let T be an invertible and class A operator. Then for each positive integer m,

1. T is of m-quasi-N-class $A(k)$ for every positive integer k.
2. m-quasi-N-class $A(1) \subseteq$ m-quasi-N-class $A(2) \subseteq$ m-quasi-N-class $A(3) \subseteq$...
3. For all positive integers n, T^n is of m-quasi-N-class $A(k)$.
4. T^{-1} is of m-quasi-N-class $A(k)$ operator.

Matrix representation of an operator is used to study various properties of an operator. N-class $A(k)$ operator have the matrix representation [35], $T = \begin{bmatrix} A & S \\ 0 & 0 \end{bmatrix}$ with respect to direct sum of closure of range of T and ker T^*.

Proposition 2.7. $[HansenInequality]$ **[15]** If $A, B \in B(H)$ satisfy $A \geq 0$ and $\|B\| \leq 1$, then $\left(B^* AB\right)^{\delta} \geq B^* A^{\delta} B$ for all $\delta \in (0, 1]$.

Theorem 2.8. Assume that $T \in B(H)$ is m-quasi-N-class $A(k)$ operator for some positive integer k and m, T has no dense and T has the following representation $T = \begin{bmatrix} T_1 & T_2 \\ 0 & T_3 \end{bmatrix}$ on $H = \overline{ran\ (T^m)} \oplus ker\ \left(T^{m*}\right)$. Then T_1 is N-class $A(k)$ operator on $\overline{ran\ (T^m)}$ and T_3 is nilpotent. Furthermore, $\sigma\ (T) = \sigma\ (T_1) \cup \{0\}$.

Proof. Let P be the orthogonal projection onto $\overline{ran\ (T^m)}$. Then $\begin{bmatrix} T_1 & 0 \\ 0 & 0 \end{bmatrix} = TP = PTP$. Since T is m-quasi-N-class $A(k)$ operator, $P\left(N\left(T^* |T|^{2k} T\right)^{\frac{1}{k+1}} - |T|^2\right) P \geq 0$. By

Hansen's inequality,

$$\begin{aligned} PN\left(T^{*}\left|T\right|^{2k}T\right)^{\frac{1}{k+1}}P &= PN\left(T^{*k+1}T^{k+1}\right)^{\frac{1}{k+1}}P \\ &\leq \left(PNT^{*k+1}T^{k+1}P\right)^{\frac{1}{k+1}} \\ &= \begin{bmatrix} N\left(T_1^{*}\left|T_1\right|^{2k}T_1\right)^{\frac{1}{k+1}} & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

and

$$P\left|T\right|^{2}P = PT^{*}TP = \begin{bmatrix} \left|T_1\right|^{2} & 0 \\ 0 & 0 \end{bmatrix}.$$

Therefore,

$$\begin{aligned} \begin{bmatrix} N\left(T_1^{*}\left|T_1\right|^{2k}T_1\right)^{\frac{1}{k+1}} & 0 \\ 0 & 0 \end{bmatrix} &\geq PN\left(T^{*}\left|T\right|^{2k}T\right)^{\frac{1}{k+1}}P \\ &\geq P\left|T\right|^{2}P = \begin{bmatrix} \left|T_1\right|^{2} & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned}$$

Hence

$$N\left(T_1^{*}\left|T_1\right|^{2k}T_1\right)^{\frac{1}{k+1}} \geq \left|T_1\right|^{2}.$$

Hence T_1 is N-class $A(k)$ operatoor on $\overline{ran\left(T^{m}\right)}$. For any $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in H$,

$$\begin{aligned} \left\langle T_3^{m}x_2, x_2\right\rangle &= \left\langle T^{m}\left(I-P\right)x, \left(I-P\right)x\right\rangle \\ &= \left\langle \left(I-P\right)x, T^{m*}\left(I-P\right)x\right\rangle = 0 \end{aligned}$$

Hence $T_3^{m} = 0$. By [16], Corollary 2.9,

$$\sigma\left(T_1\right)\cup\sigma\left(T_3\right) = \sigma\left(T\right)\cup\tau,$$

where τ is the union of certain of the holes in $\sigma\left(T\right)$ which happens to be a subset of $\sigma\left(T_1\right)\cap\sigma\left(T_3\right)$, and $\sigma\left(T_1\right)\cap\sigma\left(T_3\right)$ has no interior points. Therefore,

$$\sigma\left(T\right) = \sigma\left(T_1\right)\cup\sigma\left(T_3\right) = \sigma\left(T_1\right)\cup\{0\}.$$

■

Since N-class $A(k)$ operators are isoloid [35], we immediately have the following corollary.

Corollary 2.9. Assume that $T \in B(H)$ is m-quasi-N-class $A(k)$ operator for some positive integer k and m, T has no dense range and T has the following representation

$T = \begin{bmatrix} T_1 & T_2 \\ 0 & T_3 \end{bmatrix}$ on $H = \overline{ran\,(T^m)} \oplus ker\,\left(T^{m*}\right)$. Then T_1 is isoloid and T_3 is nilpotent. Further, $\sigma\,(T) = \sigma\,(T_1) \cup \{0\}$.

Theorem 2.10. If $T \in B(H)$ is m-quasi-N-class $A(k)$ operator for some positive integers k and m, then T is isoloid.

Theorem 2.11. If $T \in B(H)$ is m-quasi-N-class $A(k)$ operator for some positive integers k and m, $0 \neq \lambda \in \sigma_p\,(T)$ and T is of the form $T = \begin{bmatrix} \lambda & T_2 \\ 0 & T_3 \end{bmatrix}$ on ker $(T - \lambda I) \oplus ker\,(T - \lambda I)^{\perp}$, then

1. $T_2 = 0$ and
2. T_3 is m-quasi-N-class $A(k)$.

Corollary 2.12. If $T \in B(H)$ is m-quasi-N-class $A(k)$ operator for some positive integers k and m and $(T - \lambda I)\,x = 0$ for $\lambda \neq 0$ and $x \in H$, then $(T - \lambda I)^*\,x = 0$.

Theorem 2.13. Let $T \in B(H)$ be a m-quasi-N-class $A(k)$ operator for some positive integers k and m, then T satisfies

$$N\left\|T^{k+m+1}x\right\|^{\frac{2}{k+1}}\left\|T^m x\right\|^{\frac{2}{k+1}} \geq \left\|T^{m+1}x\right\|^2.$$

Theorem 2.14. If $T \in B(H)$ is m-quasi-N-class $A(k)$ operator for some positive integers k and m, then T is of finite ascent.

Proposition 2.15. $[Theorem\ 2.13]$ **[27]** For given operators $A, B, C \in B(H)$, there is an equality $w\,(A) \cup w\,(B) = w\,(M_C) \cup \tau$, where $M_C = \begin{bmatrix} A & C \\ 0 & B \end{bmatrix}$ and τ is the union of certain holes in $w\,(M_C)$ which happens to be a subset of $w\,(A) \cap w\,(B)$.

Proposition 2.16. $[Corollary\ 2.12]$ **[27]** Suppose $A \in B(H)$ and $B \in B(K)$ are isoloids. If Weyl's theorem holds for A and B, and if $w\,(A) \cap w\,(B)$ has no interior points, then Weyl's theorem holds for $\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$.

Proposition 2.17. **[28]** If either $SP\,(A)$ or $SP\,(B)$ has no pseudoholes and if A is an isoloid operator for which Weyl's theorem holds, then for every $C \in B\,(K, H)$, Weyl's theorem holds for $\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} \Rightarrow$ Weyl's theorem holds for $\begin{bmatrix} A & C \\ 0 & B \end{bmatrix}$.

Proposition 2.18. **[35]** If T is N-class $A(k)$ operator for some positive integer k, then $f\,(w\,(T)) = w\,(f\,(T))$ for every $f \in H\,(\sigma\,(T))$.

Proposition 2.19. $[Theorem 5]$ **[17]** If $T \in B(H)$ then the following are equivalent

1. $ind\,(T - \lambda I)\, ind\,(T - \mu I) \geq 0$ for each pair $\lambda, \mu \in C - \sigma_e(T)$
2. $f(w(T)) = w(f(T))$ for every $f \in H(\sigma(T))$.

Proposition 2.20. [29] If $T \in B(H)$ is isoloid, then

$$f(\sigma(T) - \pi_{00}(T)) = \sigma(f(T)) - \pi_{00}(f(T)),$$

for ever $f \in H(\sigma(T))$.

Lemma 2.21. If T is m-quasi-N-class $A(k)$ operator then $ind\,(T - \lambda I) \leq 0$ for all $\lambda \in C$.

Lemma 2.22. If T is m-quasi-N-class $A(k)$ operator then

$$f(\sigma(T) - \pi_{00}(T)) = \sigma(f(T)) - \pi_{00}(f(T))$$

for ever $f \in H(\sigma(T))$.

By Lemma 2.21 and Proposition 2.19, the following result is trivial.

Lemma 2.23. If T is m-quasi-N-class $A(k)$ operator then $f(w(T)) = w(f(T))$ for ever $f \in H(\sigma(T))$.

Theorem 2.24. If $T \in B(H)$ is m-quasi-N-class $A(k)$ operator for some positive integers k and m, then Weyl's theorem holds for T.

Proof. By Theorem 2.8, if $T = \begin{bmatrix} T_1 & T_2 \\ 0 & T_3 \end{bmatrix}$ on $H = \overline{ran\,(T^m)} \oplus ker\left(T^{m*}\right)$, then T_1 is N-class $A(k)$ operator on $\overline{ran\,(T^m)}$ and T_3 is nilpotent. Also b [35], T_1 is isoloid and Weyl's theorem holds for T_1, since $0 \notin w(T_1)$. Hence by Proposition 2.16, Weyl's theorem holds for $\begin{bmatrix} T_1 & 0 \\ 0 & T_3 \end{bmatrix}$. Therefore by Proposition 2.17, Weyl's theorem holds for $T = \begin{bmatrix} T_1 & T_2 \\ 0 & T_3 \end{bmatrix}$. ■

Theorem 2.25. If $T \in B(H)$ is m-quasi-N-class $A(k)$ operator for some positive integers k and m, then Weyl's theorem holds for $f(T)$ for every $f \in H(\sigma(T))$.

Proof. By Lemma 2.22, Theorem 2.24 and Lemma 2.23, for every $f \in H(\sigma(T))$,

$$\begin{aligned} &\sigma(f(T)) - \pi_{00}(f(T)) \\ &\quad = f(\sigma(T) - \pi_{00}(T)) \\ &\quad = f(w(T)) = w(f(T)). \end{aligned}$$

Hence Weyl's theorem holds for $f(T)$, for all $f \in H(\sigma(T))$. ■

In Theorem 2.5, the converse is true, if T is invertible. Duggal [25], has shown that k-paranormal operators are heriditarily normaloid. Since N-class $A(k)$ operators are k-paranormal, it follows that N-class $A(k)$ are normaloid.

Theorem 2.26. If T is N-class $A(k)$ operator for some positive integer k and for $\lambda \in C$, $\sigma(T) = \lambda$ then $T = \lambda$.

Theorem 2.27. If T is m-quasi-N-class $A(k)$ operator for positive integer k and for all $\lambda \in C$, $\sigma(T) = \lambda$ then $T = \lambda$, if $\lambda \neq 0$ and $T - \lambda I$ is nilpotent.

Proof. If $\lambda = 0$, then by Theorem 2.8, $T = \begin{bmatrix} T_1 & T_2 \\ 0 & 0 \end{bmatrix}$ on on $H = \overline{ran\,(T^m)} \oplus ker\left(T^{m*}\right)$, where T_1 is N-class $A(k)$ operator and $\sigma(T) = \sigma(T_1) \cup \{0\}$. Hence $\sigma(T_1) = 0$. Hence by Theorem 2.26, $T_1 = 0$. Hence $T^m = 0$. Therefore $T - \lambda I$ is nilpotent. Assume that $\lambda \neq 0$. Then T is an invertible quasi-N-class $A(k)$ operator and hence N-class $A(k)$ with $\sigma(T) = \lambda$. Then again by Theorem 2.26, $T = \lambda$. ∎

Theorem 2.28. If T is m-quasi-N-class $A(k)$ operator for positive integers m and k and M is an invariant subspace of T, then the restriction $T_{|M}$ is N-class $A(k)$.

Corollary 2.29. If T is m-quasi-N-class $A(k)$ operator for positive integers m and k, and $0 \neq \lambda \in \sigma_p(T)$, then T is of the form $T = \begin{bmatrix} \lambda & 0 \\ 0 & T_3 \end{bmatrix}$ on $ker\,(T - \lambda) \oplus \overline{(T - \lambda)^*}$, where T_3 is m-quasi-N-class $A(k)$ operator and $ker\,(T_3 - \lambda) = \{0\}$.

Theorem 2.30. If T is m-quasi-N-class $A(k)$ operator for positive integers m and k and $\lambda \in \sigma(T)$ is an isolated point, then the Riesz idempotent operator E_λ with respect to λ satisfies $E_\lambda H = ker\,(T - \lambda I)$, hence λ is an eigenvalue of T.

Theorem 2.31. If T is m-quasi-N-class $A(k)$ operator for positive integers m and k, then T has SVEP and $p(\lambda I - T) \leq 1$ for all $\lambda \in C$. Furthermore, both T and T^* are reguloid.

Theorem 2.32. If T is m-quasi-N-class $A(k)$ operator for positive integers m and k, then Weyl's theorem holds for T and T^*. If in addition, T^* has SVEP, then a-Weyl's theorem holds for both T and T^*.

Theorem 2.33. If T is m-quasi-N-class $A(k)$ operator for positive integers m and k and T^* has SVEP, then a-Weyl's theorem holds for $f(T)$ for every $f \in H(\sigma(T))$.

Theorem 2.34. Let T be m-quasi-N-class $A(k)$ operator for positive integers m and k and $\lambda \neq 0$ be an isolated point in $\sigma(T)$. Then the Riesz idempotent operator E_λ with respect to λ is self-adjoint and satisfies $E_\lambda H = ker\,(T - \lambda I) = ker\,(T - \lambda I)^*$.

Theorem 2.35. If T is m-quasi-N-class $A(k)$ operator for positive integers m and k, then T satisfies property (β).

3. weyl's theorem for algebraically m-quasi-N-class $A(k)$ operators

In this section, we define algebraically m-quasi-N-class $A(k)$ operators and we prove that Weyl's theorem hold for them. We also prove that T is algebraically m-quasi-N-class $A(k)$ operators, then T is polaroid and other Weyl type theorems are discussed.

Definition 3.1. An operator T is defined to be of algebraically m-quasi-N-class $A(k)$ for positive integers m and k, if there exists a non-constant complex ploynomial $p(t)$ such that $p(T)$ is of m-quasi-N-class $A(k)$ operator.

Theorem 3.2. If T is algebraically m-quasi-N-class $A(k)$ operators for positive integers k and m and $\sigma(T) = \mu_0$, then $T - \mu_0$ is nilpotent.

Theorem 3.3. If T is algebraically m-quasi-N-class $A(k)$ operators for positive integer k, then Weyl's theorem holds for T.

Proof. Assume that $\lambda \in \sigma(T) - w(T)$, then $T - \lambda$ is Weyl and not invertible. Claim: $\lambda \in \partial_\sigma(T)$. Assume the contrary that λ is an interior point of $\sigma(T)$. Then there exists a neighborhood $\cup$ of λ such that $dimN(T - \mu) > 0$ for all $\mu \in \cup$. Hence by ([13], $Theorem$ 10) T does not have SVEP which is a contradiction. Hence $\lambda \in \partial_\sigma(T) - w(T)$. Therefore by punctured neighborhood theorem, $\lambda \in \pi_{00}(T)$. Conversely suppose that $\lambda \in \pi_{00}(T)$. Using the Riesz idempotent E_λ with respect to λ, we can represent T as the direct sum $T = \begin{bmatrix} T_1 & 0 \\ 0 & T_2 \end{bmatrix}$ where $\sigma(T_1) = \{\lambda\}$ and $\sigma(T_2) = \sigma(T) - \{\lambda\}$. Then by Theorem 3.2, $T_1 - \lambda$ is nilpotent. Since $\lambda \in \pi_{00}(T)$, $T_1 - \lambda$ is a finite dimensional operator, so $T_1 - \lambda$ is Weyl. But since $T_2 - \lambda$ is invertible, $T - \lambda$ is Weyl. Hence $\lambda \in \sigma(T) - w(T)$, therefore $\sigma(T) - w(T) = \pi_{00}(T)$. ■

By [2], Theorem 2.16, we get the following result.

Corollary 3.4. If T is algebraically N-class $A(k)$ for some positive integer k, and T^* has SVEP then a-Weyl's theorem and propeerty (w) hold for T.

Thorem 3.5. If T is algebraically m-quasi-N-class $A(k)$ for some positive integers k and m for a fixed $N > 0$, then $w(f(T)) = f(w(T))$ for every $f \in H(\sigma(T))$.

Theorem 3.6. If T is algebraically m-quasi-N-class $A(k)$ for some positive integers k and m for a fixed $N > 0$, then Weyl's theorem holds for $f(T)$, for every $f \in H(\sigma(T))$.

Theorem 3.7. If T or T^* is algebraically m-quasi-N-class $A(k)$ for some positive integers k and m for a fixed $N > 0$, then $\sigma_{ea}(f(T)) = f(\sigma_{ea}(T))$.

Theorem 3.8. If T is algebraically m-quasi-N-class $A(k)$ for some positive integers k and m for a fixed $N > 0$, then T is polaroid.

Corollary 3.9. If T is algebraically m-quasi-N-class $A(k)$ for some positive integers k and m for a fixed $N > 0$, then T is reguloid.

Corollary 3.10. If T is algebraically m-quasi-N-class $A(k)$ for some positive integers k and m for a fixed $N > 0$, then T is isoloid.

Corollary 3.11. If T is algebraically m-quasi-N-class $A(k)$ for some positive integers k and m for a fixed $N > 0$ and if in addition T^* has SVEP, then a-Weyl's theorem holds for $f(T)$ for every $f \in H(\sigma(T))$.

Corollary 3.12. If T^* is algebraically m-quasi-N-class $A(k)$ for some positive integers k and m, then $w(f(T)) = f(w(T))$.

By [3], Theorem 2.17, we get the following results.

Corollary 3.13. If T is algebraically m-quasi-N-class $A(k)$ for some positive integers k and m, and T^* has SVEP then property (b) hold for T.

Corollary 3.14. If T is algebraically m-quasi-N-class $A(k)$ for some positive integers k and m, Weyl's theorem, a-Weyl's theorem, property (w) and property (b) hold for T^*.

In the following theorem, we prove generalized Weyl's theorem for algebraically m-quasi-N-class $A(k)$ operators.

Theorem 3.15. If T is algebraically m-quasi-N-class $A(k)$ for some positive integers k and m, then generalized Weyl's theorem holds for T.

Proof. Assume that $\lambda \in \sigma(T) - \sigma_{BW}(T)$ then $T - \lambda$ is B-Weyl and not invertible. Then as in the necessary part of the proof of Theorem 3.3, we get $\lambda \in E(T)$. Conversely suppose that $\lambda \in E(T)$. Then λ is isolated in $\sigma(T)$. Using the Riesz idempotent E_λ with respect to λ, we can represent $T = \begin{bmatrix} T_1 & 0 \\ 0 & T_2 \end{bmatrix}$ where $\sigma(T_1) = \{\lambda\}$ and $\sigma(T_2) = \sigma(T) - \{\lambda\}$. Then by Theorem 3.2, $T_1 - \lambda$ is nilpotent. Since $\lambda \notin \sigma(T_2)$, $T_2 - \lambda$ is invertible. Hence both $T_1 - \lambda$ and $T_2 - \lambda$ have both finite ascent and descent. Hence $T - \lambda$ has both finite ascent and descent. Hence $T - \lambda$ is Drazin invertible. Therefore by [7], Lemma 4.1, $T - \lambda$ is B-Fredholm of index 0. Hence $\lambda \in \sigma(T) - \sigma_{BW}(T)$. Therefore $\sigma(T) - \sigma_{BW}(T) = E(T)$.

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