Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 2 (2016), pp. 1481-1490

http://www.ripublication.com

Patterns obtained from digit and iterative digit sums of Palindromic, Repdigit and Repunit numbers, its variants and subsets

[1]Sheila A. Bishop, *[1]Hilary I. Okagbue, [2]Muminu O. Adamu, [1]Abiodun A. Opanuga

[1]Department of Mathematics, Covenant University, Canaanland, Ota, Nigeria.
[2]Department of Mathematics, University of Lagos, Akoka, Lagos, Nigeria.

Abstract

The digit and iterative digit sums of Palindromic numbers, their primes and squares, repdigit, repunit, their squares and cubes produced different patterns and sequences. The digit and iterative digits sum of the Palindromic, repdigits and repunit numbers are the same but with different patterns.

Keywords: Palindromic numbers, repdigit, repunit, digit sum, iterative digit sum

Introduction

Palindrome derived from the Greek word "palindromus" which literally mean "to run back again "can be in word or numerical. A Palindrome is a word, sentence, phrase, and letters and so on, that reads the same from left or right. Examples are Abba, Bob, nun, noon, refer, dad, mum, level, rear, did, civic, dud, eke, deed, and so on.

Numeral palindrome or Palindromic numbers are numbers that remains unchanged when their respective digit are reversed. This palindromic numbers can be computed in several number bases. The Palindromic numbers (in decimal) can be found in OEIS-A002113 which forms the sequence:

0,1,2,3,4,5,6,7,8,9,11,22, ... (A)

Repdigit and repunit are the subsets of the palindromic numbers. Repdigit are made up of repeated digit of the same number and it is the multiple of repunit. The unique properties [1][2][3] of Palindromic numbers have been a subject of research and application. Palindromic numbers can be represented in unique ways such as the one proposed by [4], its complexity [5]. Palindromic numbers have been studied in different forms such as prime pyramids [6], different powers [7], and squares [8].

Palindromic numbers have been studied and related to different number sequences such as; Smith numbers [9], composite numbers [10], and polygonal numbers [11]. Consequently, repdigit which are related to palindromic have also been studied extensively to determine their properties such as; no Keith numbers are repdigit[12], all base decimal repdigit are sums of three Fibonacci numbers [13], perfect repdigit [14], and an algorithm of determining which polygonal numbers are repdigit [15].
The aim of this paper is introduce the concept of digit and iterative digit sums to deepen our understanding the pattern of behavior of the palindromic numbers and their related sequence of numbers. See [16-21] for details on the concept and application of sum of digit of integers and integer sequences. The integers sequences usedin this paper can be accessed in [22].
The results are summarized in different sections in the following order.

a). The digits and iterative digit sum of Palindromic numbers.
b). The digits and iterative digit sums of Palindromic primes.
c). The digits and iterative digit sums of Palindromic squares.
d). The digits and iterative digit sums of Palindromic cubes.
e). The digits and iterative digit sums of Repdigit.
f). The digits and iterative digit sums of Repunit.
g). The digits and iterative digit sums of squared repunit numbers.
h). The digits and iterative digit sums of cubed repunit numbers

The digit and iterative digit sum of Palindromic numbers.
The sequence generated by the digit of Palindromic numbers is the positive integer sequence but graphically a pattern is observed. This is shown in **figure 1.** The graph is cyclic in nature.

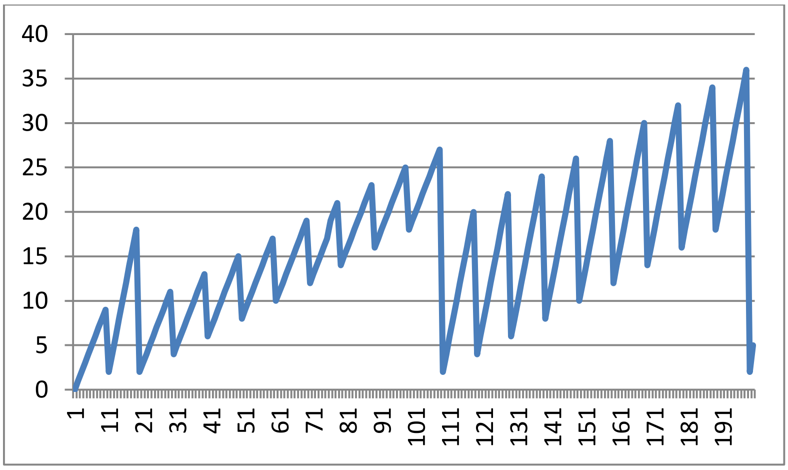

Figure 1: The sum of digit of the first 200 Palindromic numbers

All the single digit of positive integers are obtained from the iterative digit sum of the Palindromic numbers. A patterned graph is obtained shown in **figure 2**.

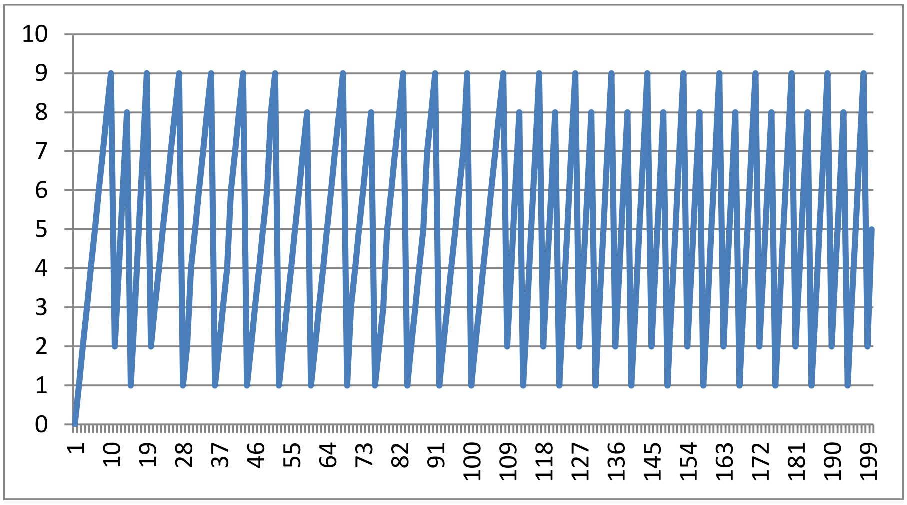

Figure 2: The iterative digit sum of the first 200 Palindromic numbers.

The numbers obtained from the iterative digit sum of Palindromic numbers are uniformly distributed. The probability of the occurrence of each number is the same. This is depicted in **table 1**. This is the result for the first 200 Palindromic numbers excluding the number zero.

Table 1: The frequency and probability of occurrence of numbers obtained from the iterative sum of the first 200 Palindromic numbers

Number	Frequncy	Probability
1	22	0. 110
2	23	0. 115
3	22	0. 110
4	22	0. 110
5	23	0. 115
6	22	0. 110
7	22	0. 110
8	23	0. 115
9	21	0. 105
Total	200	1. 000

The digit and iterative digit sums of Palindromic primes

The sequence of palindromic prime (OEIS-A002385) is given by:

2, 3, 5, 7, 11, 101, 131, ... (B)

Sequence (C) is the outcome of the digit sum of the Palindromic primes.

2, 3, 4, 5, 7, 8, 10, 11, 13, ... (C)

Excluding the second term, sequence (C) is the sequence that excludes all the multiples of 3. This is because multiples of 3 imply divisibility by 9 and as such they are not primes. The sum of the digit increases as the primes increases. This is shown in **figure 3** for the first 75 consecutive Palindromic primes.

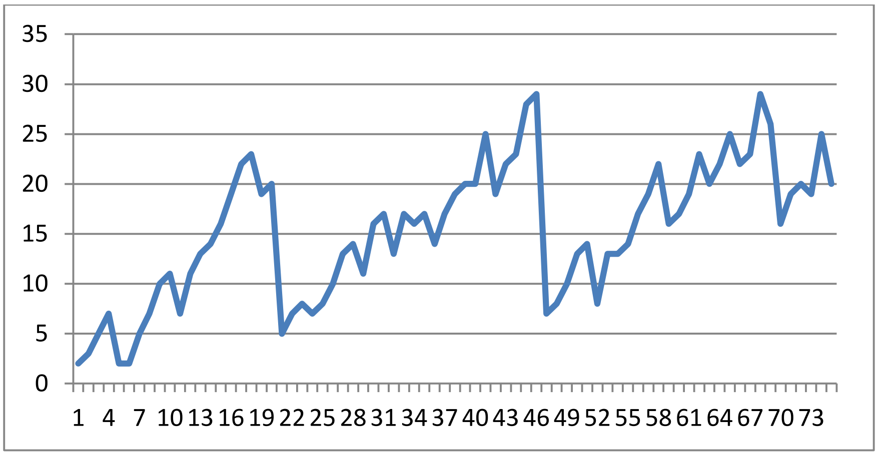

Figure 3: Sum of digit of the first 75 Palindromic primes

The iterative sum of digit of Palindromic primes produced the following numbers; 1, 2, 3, 4, 5, 7 and 8. The number 3 appear only once.
The digit and iterative digit sum results are in agreement with primality tests. Multiples of 3 imply divisibility and hence the numbers are not primes.

The digit and iterative digit sums of Palindromic squares
The sequence of Palindromic squares (OEIS-A002779) is given by:

0, 1, 4, 9, 121, 484, 676, ... (D)

The digit sum of the Palindromic squares will most likely summed to squares, even though some non-squares can be obtained. Sequence (E) is the outcome of the digit sum of the Palindromic squares.

0, 1, 4, 9, 16, 25, 28, 34, ... (E)

The sequence is closed related to the sequence obtained by the sum of digits of squared positive integers or simply the numbers with digits root 1, 4, 7 or 9. The sequence approximates to the sequence of numbers generated by the sum of digit of squared positive numbers is subject to further research.
Excluding the number zero, the iterative sum of the digit of Palindromic numbers yielded 4 numbers namely: 1, 4, 7 and 9.

The digit and iterative digit sums of Palindromic cubes
The sequence of Palindromic cubes (OEIS-A002781) is given by:

0, 1, 8, 343, 1331, 1030301, ... (F)

The digit sum of the Palindromic cubes will most likely summed to cubes, even though some non-cubes can be obtained. Attempt on generation of sequence was inconclusive since few terms were obtained but a pattern was observed from the line graph shown in **figure 4.**

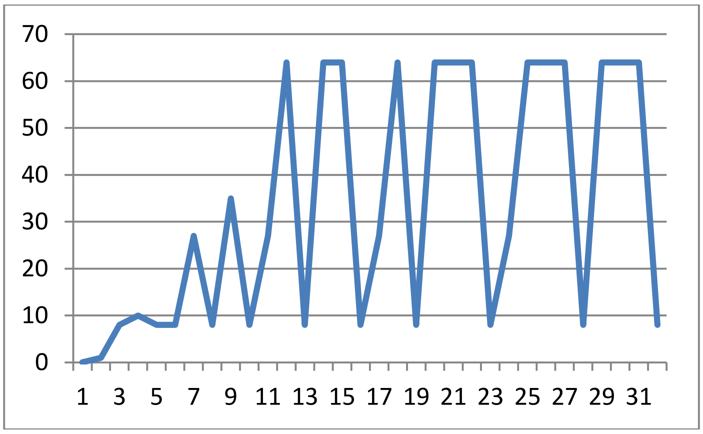

Figure 4: Sum of digit of the first 35 Palindromic cubes

Excluding the number zero, the iterative sum of the digitof Palindromic numbers yielded 3 numbers namely: 1, 8 and 9.

The digit and iterative digit sums of Repdigit

Repdigit were considered since they are subsets of the general Palindromic numbers. A repdigit or simply called a monodigit is a positive integer composed of repetition of a single digit. This can be generalized into different number base system. This research is for the case of decimal or base 10. The sequence of repdigit(OEIS-A010785) is given by;

0,1,2,3,4,5,6,7,8,9,11,22,33, ... (G)

The sum of digit of repdigit yield the sequence of positive integers with a pattern shown in **figure 5.** The sum of digits increases with the repdigit.

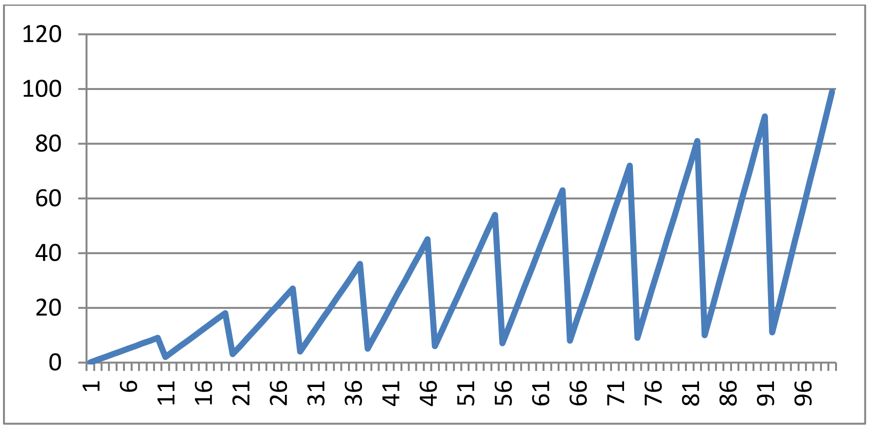

Figure 5: The sum of digits of the first 100 repdigit numbers

The iterative digits sum of repdigit yield the following numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. A wing-like pattern is obtained as shown in **figure 6**.

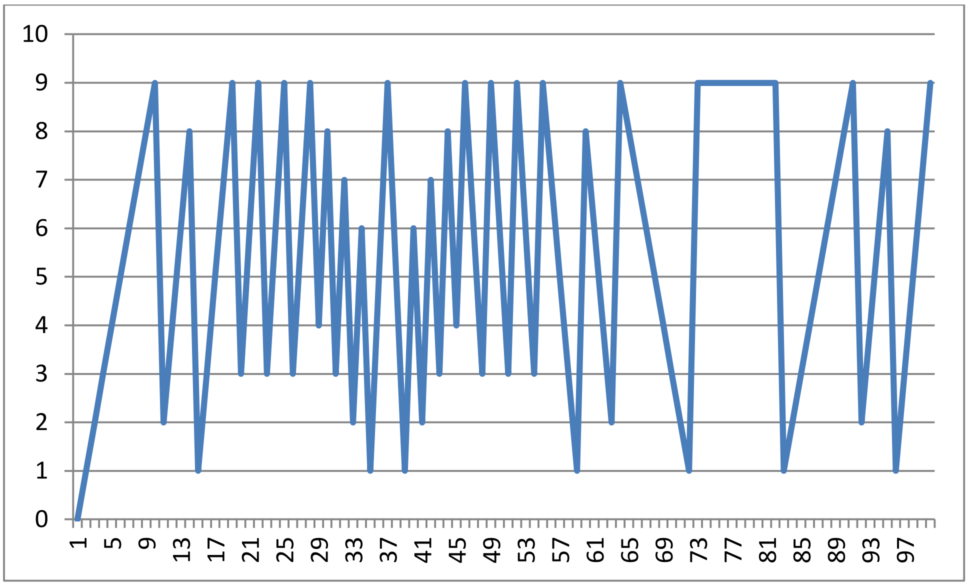

Figure 6: Iterative sum of digit of the first 100 repdigits.

The iterative sum of digit of repdigit is cyclic and recursive as seen in figure 6. There is also a gap between each cycle.

The digit and iterative sums of digit of repunit

Repunit or repeated unit is a number that contains only the repetition or copies of the solitary digit 1. Repunit are the subsets of repdigit which are the subsets of the palindromic numbers. The digit and iterative digit sums of repunit are trivial but emphasis is placed on the roots of positive integer powers of the repunit.

The digit and iterative sums of digit of squared repunit numbers

The repunit sequence (OEIS-A002275) is given by;

0, 1, 11, 111, 1111, 11111, ... (H)

The squared repunitor Denlo numbers (OEIS-A002477) sequence is given by;

$0^2, 1^2, 11^2, 111^2, 1111^2, 11111^2, \ldots$ (I)

Different patterns of sequences were observed from the sum of digit of squared repunits. It is already in scientific literature that the square of the first 10 repunits produces Palindromic numbers. It is observed that the sum of digits of the first 10 squared repunit numbers follow a particular pattern as shown in **table 2.**

Table 2: The pattern of the sum of digit of the first 10 squared repunit

Repunit	Sum of digit	Pattern
0	0	0^2
1	1	1^2
11	4	2^2
111	9	3^2
1111	16	4^2
11111	25	5^2
111111	36	6^2
1111111	49	7^2
11111111	64	8^2
111111111	81	9^2

The sum of digit of the first 10 squared repunit numbers is the number of digit raised to power two. From the 11th term onwards, a different pattern was observed. The numbers obtained from the sum of digit formed a sequence and the difference between two consecutive terms is the sequence of odd numbers. This is shown in **table 3.**

Table 3: The sum of digit of squared repunit from 11th to 19th terms

Term	Sum of digit	Difference between terms
11	82	3
12	85	5
13	90	7
14	97	9
15	106	11
16	117	13
17	130	15
18	145	17
19	162	

This can be helpful in predicting the sum of digit of squared repunit numbers without going through the tedious process of computation. The sum of digit from the 20th terms upwards is 181, 202, 225, 250, 277, 306, 337, 370 and so on.

The iterative sum of digit of squared repunit numbers produces a single pattern for all the terms of the sequence. Excluding the first term which is zero, the pattern observed was recursive and the pattern is repeated after every nine consecutive terms. This is shown in **table 4.**

Table 4: Iterative digit sum of the first 19^{th} terms of the squared repunit.

Term	Iterative sum of digit	Term	Iterative sum of digit
2	1	11	1
3	4	12	4
4	9	13	9
5	7	14	7
6	7	15	7
7	9	16	9
8	4	17	4
9	1	18	1
10	9	19	9

This can be helpful in predicting the iterative sum of digit of squared repunit number s without going through the tedious process of computation. This is shown in **table 5**.

Table 5: The iterative digit sum of squared repunit of different terms

Iterative sum of digit	Terms
1	2, 11, 20, 29, …
4	3, 12, 21, 30, …
9	4, 13, 22, 31, …
7	5, 14, 23, 32, …
7	6, 15, 24, 33, …
9	7, 16, 25, 34, …
4	8, 17, 26, 35, …
1	9, 18, 27, 36, …
9	10, 19, 28, 37, …

The digit and iterative sums of digit of cubed repunit numbers

The cubed repunit sequence is given by;

$0^3, 1^3, 11^3, 111^3, 1111^3, 11111^3, ...$ (J)

A sequence (K) is obtained by the digit sum of cubed repunit. The sequence is given by;

0, 1, 8, 27, 28, 44, 72, 64, 71, ... (K)

However, a pattern was observed from the iterative digit sum of cubed repunit numbers. Excluding the first term which is zero, the iterative digit sum produces only 3 numbers 1, 8 and 9. The pattern is shown in **table 6.**

Table 6: Iterative digit sum of the cubed repunit of different terms

Iterative sum of digit	Terms
1	2, 5, 8, 11, …
8	3, 6, 9, 12, …
9	4, 7, 10, 13, …

This can be helpful in predicting the iterative sum of digits of cubed repunit numbers without going through the tedious process of computation.

Conclusion

The digit and iterative digit sum of the Palindromic, repdigit and repunit numbers are the same with different patterns. The numbers obtained by the iterative sum of digit of Palindromic numbers are uniformly distributed. The digit and iterative digit sums of Palindromic primes produced numbers without the multiples of 3, but with the exception of 3, which is the 2nd term. The results showed that the numbers investigated are strictly primes, as multiples of 3 would have implied divisibility. The results of the Palindromic squares and squared repunit numbers are the same and in agreement with the results of [18]. Also the results of Palindromic cubes and cubed repunit numbers are the same and in agreement with the results of [19].

References

[1] H. Kresova, T. Salat. On Palindromic numbers. Acta Math, Univ. Comenian. 42, 43, 293-298 (1984).

[2] M. Kirch. Elementary number theory: a computer approach. IntextEdu. Publ. (1974).

[3] W. D. Banks, I. E. Shparlinski. Prime divisors of Palindromes. PeriodicaMathematicaHungarica. 51, 1, 1-10 (2005).

[4] J. Taneja. Representations of Palindromic, prime, and Fibonacci Sequence Patterns. RGMIA Research Report Collection. 18, 1-24 (2015).

[5] J. –P. Allouche, M. Baake, J. Cassaigne, D. Damanik. Palindrome complexity. Theoretical Computer Science. 292, 1, 9-31 (2003).

[6] G. L. Honaker, C. K. Caldwell. Palindromic prime pyramids. Journal of Recreational Mathematics. 30, 3, 169-176 (2000).

[7] S. H. Hernandez, F. Luca. Palindromic powers. Rev. Colonbiana Mat. 40, 2, 81-86 (2006).

[8] I.Korec. Palindromic Squares for various number system bases. MathematicaSlovaca. 41, 3, 261-276 (1991).

[9] W. L. McDaniel. Palindromic Smith numbers. Journal of Recreational Mathematics. 19, 34-37 (1987). 10. W. D. Banks, D. N. Hart, M. Sakata.

Almost all palindromes are composite. Mathematical Research Letters. 11, 5-6, 853-868 (2004).
[10] M. F. Ezerman, B. Meyer, P. Solé. On polynomial pairs of integers. Journal of Integer Sequences. 18, 3, 1-4 (2015).
[11] J. J. Bravo, S. Guzman, F. Luca. Repdigit Keith numbers. Lithuanian Mathematical Journal. 53, 2, 143-148 (2013).
[12] F. Luca. Repdigits as sums of three Fibonacci numbers. Mathematical Communications. 17, 1, 1-11 (2012).
[13] K. Broughan, S. G. Sanchez, F. Luca. Perfect repdigits. Mathematics of Computation. 82, 284, 2439-2459 (2013).
[14] M. Keith. On repdigit polygonal numbers. Journal of Integer Sequence. 1, Article 98:1. 6 (1998).
[15] J. C. Saunders. Sum of digits in q-aryexpansions. Int. Journal of Number Theory. 11, 2, 593-611 (2015).
[16] F. T. Adams-Watters, F. Ruskey. Generating functions for the digital sum and other digit counting sequences. Journal of Integer Sequence. 12, Article 09:5. 6 (2009).
[17] H. I. Okagbue, M. O. Adamu, S. A. Iyase, A. A. Opanuga. Indian J. Sci& Tech, 8, 15, 69912 (2015)
[18] H. I. Okagbue, M. O. Adamu, S. A. Bishop, A. A. Opanuga. Indian J. Nat. Sci, 6, 32, 10190 (2015)
[19] J. P. Allouche, J. O. Shallit. Discrete Math. Theo. Comp. Sci. 4, 1 (2000).
[20] L. Ericksen. Divisibility, iterated digit sums, primality tests. MathematicaSlovaca. 59, 3, 261-274.
[21] N. J. A. Sloane. The online encyclopedia of integer sequences. Published electronically at http://oeis. org. Sequences: A002113, A002385, A002779, A002781, A010785, A002275, A002477.