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A New Algorithm for Developing Block Methods for Solving Fourth Order Ordinary Differential Equations

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Abstract

Block methods as an approach for solving higher order ordinary differential equations (ODEs) have been seen to be very useful in recent literature. However, the development of the block methods for higher order, such as fourth order ODEs is seen to include a lot of steps and transformations. This is irrespective of the approach adopted; be it interpolation, numerical integration or Taylor series. Hence, this study investigates into producing an algorithm that can produce the desired block method directly for any value of the stepnumber k, whose computational complexity is also shown.

Keywords: Generalized Algorithm, Computational Complexity, Block Method, Fourth Order, Initial Value Problems, Boundary Value Problems.

Introduction

The numerical solutions of fourth order ordinary differential equations have been vastly explored in literature. Numerous approaches have been adopted such as numerical integration (Waeleh et al, 2011; Yap & Ismail, 2015) and interpolation approaches (Mohammed, 2010; Olabode & Alabi, 2013). Likewise, Li (2008) spoke of splines being considered to construct finite difference schemes for solving higher order boundary value problems of ordinary differential equations and this area has been researched well into over time (Ramadan, Lashien & Zahra, 2009; Akram, & Amin, 2012; Pervaiz & Ahmad, 2015).

However, one challenge faced when developing block methods of the form

$$A^0 Y_{n+k} = A^i Y_{n-k} + B^i Y'_{n-k} + D^i Y''_{n-k} + E^i Y'''_{n-k} + h^4 \left(C^0 Y^{iv}_{n+k} + C^i Y^{iv}_{n-k} \right) \quad (1)$$

for solving fourth order ordinary differential equations is the computational burden associated with the step by step process of developing the schemes and corresponding derivatives of the desired k-step block method as seen in the work of authors such as Adesanya et al (2012) and Omar and Kuboye (2015) who have adopted block methods of this form to solve fourth order ordinary differential equations. Hence, the introduction of an algorithm that will directly produce the required coefficients of any k-step block method under consideration for solving

$$y^{iv} = f\left(x, y, y', y'', y'''\right) \qquad (2)$$

is very expedient and timely.

Hence, this article as described in the sections following will state the generalized algorithm for developing block methods of the form (1) above and this algorithm will be verified by recovering one of such block methods previously existing in literature.

Generalized Algorithm for k-step Block Methods for $y^{iv} = f\left(x, y, y', y'', y'''\right)$

In a bid to develop block methods of the form (1) above where $Y_{n+k} = \left(y_{n+1}, y_{n+2}, \cdots, y_{n+k}\right)$ and $Y_{n+k}^{(i)} = \left(y_{n+1}^{(i)}, y_{n+2}^{(i)}, \cdots, y_{n+k}^{(i)}\right)$, the following generalized algorithm is given

$$y_{n+\xi} = \sum_{i=0}^{3} \frac{(\xi h)^i}{i!} y_n^{(i)} + \sum_{i=0}^{k} \phi_i f_{n+i}, \quad \xi = 1, 2, \ldots, k \qquad (3)$$

with derivatives

$$y_{n+\xi}^{(a)} = \sum_{i=0}^{4-(a+1)} \frac{(\xi h)^i}{i!} y_n^{(i+a)} + \sum_{i=0}^{k} \omega_{ia} f_{n+i}, \quad a = 1_{(\xi=1,2,\ldots,k)}, 2_{(\xi=1,2,\ldots,k)}, 3_{(\xi=1,2,\ldots,k)} \qquad (4)$$

$\phi_i = A^{-1}B$ and $\omega_{ia} = A^{-1}D$, where

$$A = \begin{pmatrix} 1 & 1 & 1 & \ldots & 1 \\ 0 & h & 2h & \ldots & kh \\ 0 & \frac{(h)^2}{2!} & \frac{(2h)^2}{2!} & \ldots & \frac{(kh)^2}{2!} \\ . & . & . & \ldots & . \\ . & . & . & \ldots & . \\ . & . & . & \ldots & . \\ 0 & \frac{h^k}{k!} & \frac{(2h)^k}{k!} & \ldots & \frac{(kh)^k}{k!} \end{pmatrix}, \quad B = \begin{pmatrix} \frac{(\xi h)^4}{4!} \\ \frac{(\xi h)^5}{5!} \\ \frac{(\xi h)^6}{6!} \\ . \\ . \\ . \\ \frac{(\xi h)^{(4+k)}}{(4+k)!} \end{pmatrix}, \quad D = \begin{pmatrix} \frac{(\xi h)^{(4-a)}}{(4-a)!} \\ \frac{(\xi h)^{(4-a)+1}}{((4-a)+1)!} \\ \frac{(\xi h)^{(4-a)+2}}{((4-a)+2)!} \\ . \\ . \\ . \\ \frac{(\xi h)^{(4-a)+k}}{((4-a)+k))!} \end{pmatrix}$$

Note the following proposition

Proposition 2.1:

There exists only one block form for every k-step block method.

This proposition is seen evident in the next section as the generalized algorithm will be adopted to develop a sample 5-step block method for solving fourth order ordinary differential equations and the coefficients obtained correspond with the coefficients of the block method in the similar 5-step block method of Adesanya et al (2012).

5-Step Block Method Derived From the Generalized Algorithm

To verify this algorithm, we develop the 5-step block method using the generalized algorithm and then compare the output to the k = 5 block method derived in literature.

$$A^0 Y_{n+k} = A^i Y_{n-k} + B^i Y'_{n-k} + D^i Y''_{n-k} + E^i Y'''_{n-k} + h^4 \left(C^0 Y^{iv}_{n+k} + C^i Y^{iv}_{n-k} \right), \quad k = 5.$$

First, the generalized algorithm (3) is expanded together with the expression for the derivatives (4)

$$\begin{aligned}
y_{n+1} &= y_n + hy'_n + \tfrac{h^2}{2!}y''_n + \tfrac{h^3}{3!}y'''_n + \left(\phi_0 f_n + \phi_1 f_{n+1} + \phi_2 f_{n+2} + \phi_3 f_{n+3} + \phi_4 f_{n+4} + \phi_5 f_{n+5}\right),\\
y_{n+2} &= y_n + 2hy'_n + \tfrac{(2h)^2}{2!}y''_n + \tfrac{(2h)^3}{3!}y'''_n + \left(\phi_0 f_n + \phi_1 f_{n+1} + \phi_2 f_{n+2} + \phi_3 f_{n+3} + \phi_4 f_{n+4} + \phi_5 f_{n+5}\right),\\
y_{n+3} &= y_n + 3hy'_n + \tfrac{(3h)^2}{2!}y''_n + \tfrac{(3h)^3}{3!}y'''_n + \left(\phi_0 f_n + \phi_1 f_{n+1} + \phi_2 f_{n+2} + \phi_3 f_{n+3} + \phi_4 f_{n+4} + \phi_5 f_{n+5}\right),\\
y_{n+4} &= y_n + 4hy'_n + \tfrac{(4h)^2}{2!}y''_n + \tfrac{(4h)^3}{3!}y'''_n + \left(\phi_0 f_n + \phi_1 f_{n+1} + \phi_2 f_{n+2} + \phi_3 f_{n+3} + \phi_4 f_{n+4} + \phi_5 f_{n+5}\right),\\
y_{n+5} &= y_n + 5hy'_n + \tfrac{(5h)^2}{2!}y''_n + \tfrac{(5h)^3}{3!}y'''_n + \left(\phi_0 f_n + \phi_1 f_{n+1} + \phi_2 f_{n+2} + \phi_3 f_{n+3} + \phi_4 f_{n+4} + \phi_5 f_{n+5}\right).
\end{aligned} \quad (5)$$

with derivatives

$$\begin{aligned}
y'_{n+1} &= y'_n + hy''_n + \tfrac{h^2}{2!}y'''_n + \left(\omega_{01} f_n + \omega_{11} f_{n+1} + \omega_{21} f_{n+2} + \omega_{31} f_{n+3} + \omega_{41} f_{n+4} + \omega_{51} f_{n+5}\right),\\
y'_{n+2} &= y'_n + 2hy''_n + \tfrac{(2h)^2}{2!}y'''_n + \left(\omega_{01} f_n + \omega_{11} f_{n+1} + \omega_{21} f_{n+2} + \omega_{31} f_{n+3} + \omega_{41} f_{n+4} + \omega_{51} f_{n+5}\right),\\
y'_{n+3} &= y'_n + 3hy''_n + \tfrac{(3h)^2}{2!}y'''_n + \left(\omega_{01} f_n + \omega_{11} f_{n+1} + \omega_{21} f_{n+2} + \omega_{31} f_{n+3} + \omega_{41} f_{n+4} + \omega_{51} f_{n+5}\right),\\
y'_{n+4} &= y'_n + 4hy''_n + \tfrac{(4h)^2}{2!}y'''_n + \left(\omega_{01} f_n + \omega_{11} f_{n+1} + \omega_{21} f_{n+2} + \omega_{31} f_{n+3} + \omega_{41} f_{n+4} + \omega_{51} f_{n+5}\right),\\
y'_{n+5} &= y'_n + 5hy''_n + \tfrac{(5h)^2}{2!}y'''_n + \left(\omega_{01} f_n + \omega_{11} f_{n+1} + \omega_{21} f_{n+2} + \omega_{31} f_{n+3} + \omega_{41} f_{n+4} + \omega_{51} f_{n+5}\right),\\
y''_{n+1} &= y''_n + hy'''_n + \left(\omega_{02} f_n + \omega_{12} f_{n+1} + \omega_{22} f_{n+2} + \omega_{32} f_{n+3} + \omega_{42} f_{n+4} + \omega_{52} f_{n+5}\right),\\
y''_{n+2} &= y''_n + 2hy'''_n + \left(\omega_{02} f_n + \omega_{12} f_{n+1} + \omega_{22} f_{n+2} + \omega_{32} f_{n+3} + \omega_{42} f_{n+4} + \omega_{52} f_{n+5}\right),\\
y''_{n+3} &= y''_n + 3hy'''_n + \left(\omega_{02} f_n + \omega_{12} f_{n+1} + \omega_{22} f_{n+2} + \omega_{32} f_{n+3} + \omega_{42} f_{n+4} + \omega_{52} f_{n+5}\right),\\
y''_{n+4} &= y''_n + 4hy'''_n + \left(\omega_{02} f_n + \omega_{12} f_{n+1} + \omega_{22} f_{n+2} + \omega_{32} f_{n+3} + \omega_{42} f_{n+4} + \omega_{52} f_{n+5}\right),\\
y''_{n+5} &= y''_n + 5hy'''_n + \left(\omega_{02} f_n + \omega_{12} f_{n+1} + \omega_{22} f_{n+2} + \omega_{32} f_{n+3} + \omega_{42} f_{n+4} + \omega_{52} f_{n+5}\right),\\
y'''_{n+1} &= y'''_n + \left(\omega_{03} f_n + \omega_{13} f_{n+1} + \omega_{23} f_{n+2} + \omega_{33} f_{n+3} + \omega_{43} f_{n+4} + \omega_{53} f_{n+5}\right),\\
y'''_{n+2} &= y'''_n + \left(\omega_{03} f_n + \omega_{13} f_{n+1} + \omega_{23} f_{n+2} + \omega_{33} f_{n+3} + \omega_{43} f_{n+4} + \omega_{53} f_{n+5}\right),\\
y'''_{n+3} &= y'''_n + \left(\omega_{03} f_n + \omega_{13} f_{n+1} + \omega_{23} f_{n+2} + \omega_{33} f_{n+3} + \omega_{43} f_{n+4} + \omega_{53} f_{n+5}\right),\\
y'''_{n+4} &= y'''_n + \left(\omega_{03} f_n + \omega_{13} f_{n+1} + \omega_{23} f_{n+2} + \omega_{33} f_{n+3} + \omega_{43} f_{n+4} + \omega_{53} f_{n+5}\right),\\
y'''_{n+5} &= y'''_n + \left(\omega_{03} f_n + \omega_{13} f_{n+1} + \omega_{23} f_{n+2} + \omega_{33} f_{n+3} + \omega_{43} f_{n+4} + \omega_{53} f_{n+5}\right).
\end{aligned} \quad (6)$$

For y_{n+1}

$$\begin{pmatrix} \phi_0 \\ \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & h & 2h & 3h & 4h & 5h \\ 0 & \frac{h^2}{2!} & \frac{(2h)^2}{2!} & \frac{(3h)^2}{2!} & \frac{(4h)^2}{2!} & \frac{(5h)^2}{2!} \\ 0 & \frac{h^3}{3!} & \frac{(2h)^3}{3!} & \frac{(3h)^3}{3!} & \frac{(4h)^3}{3!} & \frac{(5h)^3}{3!} \\ 0 & \frac{h^4}{4!} & \frac{(2h)^4}{4!} & \frac{(3h)^4}{4!} & \frac{(4h)^4}{4!} & \frac{(5h)^4}{4!} \\ 0 & \frac{h^5}{5!} & \frac{(2h)^5}{5!} & \frac{(3h)^5}{5!} & \frac{(4h)^5}{5!} & \frac{(5h)^5}{5!} \end{pmatrix}^{-1} \begin{pmatrix} \frac{h^4}{4!} \\ \frac{h^5}{5!} \\ \frac{h^6}{6!} \\ \frac{h^7}{7!} \\ \frac{h^8}{8!} \\ \frac{h^9}{9!} \end{pmatrix} = \begin{pmatrix} \frac{49126h^4}{1814400} \\ \frac{49045h^4}{1814400} \\ -\frac{40160h^4}{1814400} \\ \frac{25430h^4}{1814400} \\ -\frac{9310h^4}{1814400} \\ \frac{1469h^4}{1814400} \end{pmatrix}$$

Likewise for y_{n+2}, $\begin{pmatrix} \phi_0 \\ \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{pmatrix} = \begin{pmatrix} \frac{4264h^4}{14175} \\ \frac{7960h^4}{14175} \\ -\frac{4910h^4}{14175} \\ \frac{3080h^4}{14175} \\ -\frac{1120h^4}{14175} \\ \frac{176h^4}{14175} \end{pmatrix}$; y_{n+3}, $\begin{pmatrix} \phi_0 \\ \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{pmatrix} = \begin{pmatrix} \frac{25488h^4}{22400} \\ \frac{63315h^4}{22400} \\ -\frac{26460h^4}{22400} \\ \frac{19170h^4}{22400} \\ -\frac{7020h^4}{22400} \\ \frac{1107h^4}{22400} \end{pmatrix}$;

$$y_{n+4},\ \begin{pmatrix} \phi_0 \\ \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{pmatrix} = \begin{pmatrix} \frac{40448h^4}{14175} \\ \frac{116480h^4}{14175} \\ -\frac{29440h^4}{14175} \\ \frac{33280h^4}{14175} \\ -\frac{11360h^4}{14175} \\ \frac{1792h^4}{14175} \end{pmatrix};\ y_{n+5},\ \begin{pmatrix} \phi_0 \\ \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{pmatrix} = \begin{pmatrix} \frac{418250h^4}{72576} \\ \frac{1315625h^4}{72576} \\ -\frac{175000h^4}{72576} \\ \frac{418750h^4}{72576} \\ -\frac{106250h^4}{72576} \\ \frac{18625h^4}{72576} \end{pmatrix};\ y'_{n+1},\ \begin{pmatrix} \omega_{01} \\ \omega_{11} \\ \omega_{21} \\ \omega_{31} \\ \omega_{41} \\ \omega_{51} \end{pmatrix} = \begin{pmatrix} \frac{3929h^3}{40320} \\ \frac{4975h^3}{40320} \\ -\frac{3862h^3}{40320} \\ \frac{2422h^3}{40320} \\ -\frac{883h^3}{40320} \\ \frac{139h^3}{40320} \end{pmatrix};$$

$$y'_{n+2},\ \begin{pmatrix} \omega_{01} \\ \omega_{11} \\ \omega_{21} \\ \omega_{31} \\ \omega_{41} \\ \omega_{51} \end{pmatrix} = \begin{pmatrix} \frac{317h^3}{630} \\ \frac{734h^3}{630} \\ -\frac{380h^3}{630} \\ \frac{244h^3}{630} \\ -\frac{89h^3}{630} \\ \frac{14h^3}{630} \end{pmatrix};\ y'_{n+3},\ \begin{pmatrix} \omega_{01} \\ \omega_{11} \\ \omega_{21} \\ \omega_{31} \\ \omega_{41} \\ \omega_{51} \end{pmatrix} = \begin{pmatrix} \frac{5481h^3}{4480} \\ \frac{16119h^3}{4480} \\ -\frac{4374h^3}{4480} \\ \frac{4230h^3}{4480} \\ -\frac{1539h^3}{4480} \\ \frac{243h^3}{4480} \end{pmatrix};\ y'_{n+4},\ \begin{pmatrix} \omega_{01} \\ \omega_{11} \\ \omega_{21} \\ \omega_{31} \\ \omega_{41} \\ \omega_{51} \end{pmatrix} = \begin{pmatrix} \frac{712h^3}{315} \\ \frac{2336h^3}{315} \\ -\frac{224h^3}{315} \\ \frac{704h^3}{315} \\ -\frac{200h^3}{315} \\ \frac{32h^3}{315} \end{pmatrix};$$

$$y'_{n+5},\ \begin{pmatrix} \omega_{01} \\ \omega_{11} \\ \omega_{21} \\ \omega_{31} \\ \omega_{41} \\ \omega_{51} \end{pmatrix} = \begin{pmatrix} \frac{29125h^3}{8064} \\ \frac{101875h^3}{8064} \\ -\frac{1250h^3}{8064} \\ \frac{38750h^3}{8064} \\ -\frac{4375h^3}{8064} \\ \frac{1375h^3}{8064} \end{pmatrix};\ y''_{n+1},\ \begin{pmatrix} \omega_{02} \\ \omega_{12} \\ \omega_{22} \\ \omega_{32} \\ \omega_{42} \\ \omega_{52} \end{pmatrix} = \begin{pmatrix} \frac{2462h^2}{10080} \\ \frac{4315h^2}{10080} \\ -\frac{3044h^2}{10080} \\ \frac{1882h^2}{10080} \\ -\frac{682h^2}{10080} \\ \frac{107h^2}{10080} \end{pmatrix};\ y''_{n+2},\ \begin{pmatrix} \omega_{02} \\ \omega_{12} \\ \omega_{22} \\ \omega_{32} \\ \omega_{42} \\ \omega_{52} \end{pmatrix} = \begin{pmatrix} \frac{355h^2}{630} \\ \frac{1088h^2}{630} \\ -\frac{370h^2}{630} \\ \frac{272h^2}{630} \\ -\frac{101h^2}{630} \\ \frac{16h^2}{630} \end{pmatrix};$$

$$y''_{n+3},\begin{pmatrix}\omega_{02}\\ \omega_{12}\\ \omega_{22}\\ \omega_{32}\\ \omega_{42}\\ \omega_{52}\end{pmatrix}=\begin{pmatrix}\frac{984h^2}{1120}\\ \frac{3501h^2}{1120}\\ -\frac{72h^2}{1120}\\ \frac{870h^2}{1120}\\ -\frac{288h^2}{1120}\\ \frac{45h^2}{1120}\end{pmatrix};\ y''_{n+4},\begin{pmatrix}\omega_{02}\\ \omega_{12}\\ \omega_{22}\\ \omega_{32}\\ \omega_{42}\\ \omega_{52}\end{pmatrix}=\begin{pmatrix}\frac{376h^2}{315}\\ \frac{1464h^2}{315}\\ \frac{176h^2}{315}\\ \frac{608h^2}{315}\\ -\frac{80h^2}{315}\\ \frac{16h^2}{315}\end{pmatrix};\ y''_{n+5},\begin{pmatrix}\omega_{02}\\ \omega_{12}\\ \omega_{22}\\ \omega_{32}\\ \omega_{42}\\ \omega_{52}\end{pmatrix}=\begin{pmatrix}\frac{3050h^2}{2016}\\ \frac{11875h^2}{2016}\\ \frac{2500h^2}{2016}\\ \frac{6250h^2}{2016}\\ \frac{1250h^2}{2016}\\ \frac{275h^2}{2016}\end{pmatrix};$$

$$y'''_{n+1},\begin{pmatrix}\omega_{03}\\ \omega_{13}\\ \omega_{23}\\ \omega_{33}\\ \omega_{43}\\ \omega_{53}\end{pmatrix}=\begin{pmatrix}\frac{475h}{1440}\\ \frac{1427h}{1440}\\ -\frac{798h}{1440}\\ \frac{482h}{1440}\\ -\frac{173h}{1440}\\ \frac{27h}{1440}\end{pmatrix};\ y'''_{n+2},\begin{pmatrix}\omega_{03}\\ \omega_{13}\\ \omega_{23}\\ \omega_{33}\\ \omega_{43}\\ \omega_{53}\end{pmatrix}=\begin{pmatrix}\frac{28h}{90}\\ \frac{129h}{90}\\ \frac{14h}{90}\\ \frac{14h}{90}\\ -\frac{6h}{90}\\ \frac{h}{90}\end{pmatrix};\ y'''_{n+3},\begin{pmatrix}\omega_{03}\\ \omega_{13}\\ \omega_{23}\\ \omega_{33}\\ \omega_{43}\\ \omega_{53}\end{pmatrix}=\begin{pmatrix}\frac{51h}{160}\\ \frac{219h}{160}\\ \frac{114h}{160}\\ \frac{114h}{160}\\ -\frac{21h}{160}\\ \frac{3h}{160}\end{pmatrix};$$

$$y'''_{n+4},\begin{pmatrix}\omega_{03}\\ \omega_{13}\\ \omega_{23}\\ \omega_{33}\\ \omega_{43}\\ \omega_{53}\end{pmatrix}=\begin{pmatrix}\frac{14h}{45}\\ \frac{64h}{45}\\ \frac{24h}{45}\\ \frac{64h}{45}\\ \frac{14h}{45}\\ 0\end{pmatrix};\ y'''_{n+5},\begin{pmatrix}\omega_{03}\\ \omega_{13}\\ \omega_{23}\\ \omega_{33}\\ \omega_{43}\\ \omega_{53}\end{pmatrix}=\begin{pmatrix}\frac{95h}{288}\\ \frac{375h}{288}\\ \frac{250h}{288}\\ \frac{250h}{288}\\ \frac{375h}{288}\\ \frac{95h}{288}\end{pmatrix}$$

On substituting these coefficients back in (5) and (6) above gives the same block method derived by Adesanya et al (2012). Hence, the validity of the algorithm is verified.

Computational Complexity of Generalized Algorithm for Developing 5-Step Block Method

The development of the 5-step block method using the new generalized algorithm involved obtaining the coefficients for the block corrector schemes and its corresponding derivatives at grid points $x_{n+1}, x_{n+2}, x_{n+3}, x_{n+4}$ and x_{n+5}.

Algorithm

Step 1: Evaluate

$$y_{n+\xi}=\sum_{i=0}^{3}\frac{(\xi h)^i}{i!}y_n^{(i)}+\sum_{i=0}^{k}\phi_i f_{n+i},\quad \xi=1,2,\ldots,k$$

with derivatives

$$y_{n+\xi}^{(a)}=\sum_{i=0}^{4-(a+1)}\frac{(\xi h)^i}{i!}y_n^{(i+a)}+\sum_{i=0}^{k}\omega_{ia}f_{n+i},\quad a=1_{(\xi=1,2,\ldots,k)},2_{(\xi=1,2,\ldots,k)},3_{(\xi=1,2,\ldots,k)}$$

$\phi_i = A^{-1}B$ and $\omega_{ia} = A^{-1}D$, where

$$A = \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & h & 2h & \dots & kh \\ 0 & \frac{(h)^2}{2!} & \frac{(2h)^2}{2!} & \dots & \frac{(kh)^2}{2!} \\ . & . & . & \dots & . \\ . & . & . & \dots & . \\ . & . & . & \dots & . \\ 0 & \frac{h^k}{k!} & \frac{(2h)^k}{k!} & \dots & \frac{(kh)^k}{k!} \end{pmatrix}, \quad B = \begin{pmatrix} \frac{(\xi h)^4}{4!} \\ \frac{(\xi h)^5}{5!} \\ \frac{(\xi h)^6}{6!} \\ . \\ . \\ . \\ \frac{(\xi h)^{(4+k)}}{(4+k)!} \end{pmatrix}, \quad D = \begin{pmatrix} \frac{(\xi h)^{(4-a)}}{(4-a)!} \\ \frac{(\xi h)^{(4-a)+1}}{((4-a)+1)!} \\ \frac{(\xi h)^{(4-a)+2}}{((4-a)+2)!} \\ . \\ . \\ . \\ \frac{(\xi h)^{(4-a)+k}}{((4-a)+k))!} \end{pmatrix}$$

and $k = 5$

Step 2: STOP

The computational complexity of taking the inverse of an $n \times n$ matrix is $O(n^3)$, while the computational complexity of the matrix multiplication of one $n \times s$ matrix with one $n \times p$ matrix is $O(nsp)$. Hence, the computational complexity of developing the 5-step block method using the generalized algorithm is obtained from,

$$4k\left\{\left[O\left((k+1)^3\right) + O\left((k+1)^2\right)\right]\right\}$$

as $O\left((k+1)^3\right)$.

Conclusion

A new generalized algorithm for developing any k-step block method for solving fourth order ordinary differential equations is presented in this paper. The algorithm has been validated by verifying the resulting coefficients with previously developed block methods (k = 5) in literature. Hence, the suitability of this algorithm when developing block methods of this kind is grounded. New improvements on the work can be compared to this present algorithm in terms of its computational complexity. A major advantage of this new algorithm is that it bypasses the rigour attached to the step by step approach used by previous studies. Hence, future research will look into developing an algorithm that will be suitable for developing block methods of any order and any steplength.

References

[1] Adesanya, A. O., Momoh, A. A., Alkali, M. A. and Tahir, A. (2012). Five Steps Block Method For The Solution Of Fourth Order Ordinary Differential Equations. *International Journal of Engineering Research and Applications, 2(5), 991-998*

[2] Akram, G., and Amin, N. (2012). Solution of a fourth order singularly perturbed boundary value problem using quintic spline. *In International Mathematical Forum 7(44), 2179-2190.*

[3] Li, C. J. (2008). A kind of multistep finite difference methods for arbitrary order linear boundary value problems. *Applied Mathematics and Computation, 196(2), 858-865.*

[4] Mohammed, U. (2010). A Six Step Block Method for Solution of Fourth Order Ordinary Differential Equations. *The Pacific Journal of Science and Technology, 11(1), 259-265*

[5] Olabode, B. T., and Alabi, T. J. (2013). Direct block predictor-corrector method for the solution of general fourth order ODEs. *Journal of Mathematics Research, 5(1), 26-33.*

[6] Kuboye, J. O., and Omar, Z. (2015). New Zero-stable Block Method for Direct Solution of Fourth Order Ordinary Differential Equations. *Indian Journal of Science and Technology, 8(12), 1-8*

[7] Pervaiz, A., and Ahmad, M. O. (2015). Polynomial Cubic Spline Method For Solving Fourth-Order Parabolic Two Point Boundary Value Problems. *Pakistan Journal of Science, 67(1), 64-67*

[8] Ramadan, M. A., Lashien, I. F., and Zahra, W. K. (2009). Quintic nonpolynomial spline solutions for fourth order two-point boundary value problem. *Communications in Nonlinear Science and Numerical Simulation, 14(4), 1105-1114.*

[9] Waeleh, N., Majid, Z. A., Ismail, F., and Suleiman, M. (2011). Numerical solution of higher order ordinary differential equations by direct block code. *Journal of Mathematics and Statistics, 8(1), 77-81.*

[10] Yap, L. K., & Ismail, F. (2015). Block Hybrid Collocation Method with Application to Fourth Order Differential Equations. *Mathematical Problems in Engineering, 2015, Article ID 561489, 1-6*