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Optimization of Multi-objective Transportation Problem Using Evolutionary Algorithms

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Abstract

The Multi-objective transportation problem (MOTSP) is a of linear type optimization problem which contains all equality type constraints and the multi variable objectives. Here we have presented an application of Evolutionary Algorithms to the multi-objective transportation problem (MOTSP). We consider the multi-objective transportation problem as linear optimization problem and use a special type of Encoding method. The set of feasible solution of the multi-objective transportation problem (MOTSP) is encoding as Bipartite Graph and selected as the parent solution. The Evolutionary operates where applied to get an optimal compromise solution. This methodology is illustrated by a numerical example.

Keywords: Bipartite Graph, Chromosome, Evolutionary algorithms, feasible solutions.

1. Introduction

Optimization is a kind of the decision making, in which decision have to be taken to optimize one or more objectives under some prescribed set of circumstances. These problems may be a single or multi-objective and are to be optimized (maximized or minimized) under a specified set of constraints. The constraints usually are in the form of inequalities or equalities. Such problems which often arise as a result of mathematical modelling of many real life situations are called optimization problems.

Multi-objective optimization or multi-objective programming is the process of simultaneously optimizing more than one objective subject to certain constraints. Applications of Multi-objective optimization problems were found in the fields:

product and process design, finance, aircraft design, the oil and gas industry, automobile design and many more. Maximizing profit and minimizing the cost of a product; maximizing performance and minimizing fuel consumption of a vehicle; and minimizing weight while maximizing the strength of a particular component.

The Optimization technique can be fruitfully applied to transportation problem. The basic transportation problem involves the transportation or physical distribution of goods from several supply points to demand points and it also involves minimization of cost for distribution of product from factories to the number of cities. We should consider the requirement of goods at each demand points, variety of shipping routes and associated cost of distribution or transportation of goods or products from each origin to each destination. In conventional programming method requires the parameters to be known as constants. In practical situation, however, the parameters are known exactly and have to be estimated. The imprecision may follow from the inexact environment or may be a consequence of certain flexibility for the given enterprise associated with the cost of objective function.

Evolutionary algorithms suffer from the large size problem of the Pareto set e. g. [14]. Here for some methods have been proposed to reduce the Pareto set to a manageable size. However, the goal is not only to prune a given set, but rather to generate a representative subset, which maintains the characteristics of the generated set[10]. Also evolutionary algorithms such as, genetic algorithms (GAs) can be used as a global optimization tool for continuous and discrete functions problems. However, a simple GA may suffer from slow convergence, and instability of results [13, 24]. GAs' problem solution power can be increased by local searching.

2. Transportation Problem

The transportation problem minimizes the cost of transporting some commodity that is available at m sources (supply nodes) and required at n destinations (demand nodes). The source parameter ($\boldsymbol{s_{ij}}$) may be production facilities, warehouse, etc., whereas the destination parameter ($\boldsymbol{d_{ij}}$) may be warehouse, sales outlet, etc. The penalty ($\boldsymbol{a_{ij}}$) that is, the co-efficient of the objective functions, could represent transportation cost, delivery time, number of goods transposed, unfulfilled demand, and many others.

2.1 Multi-Objective Transportation Problem (MOTSP)

The transportation problem which involves multiple objective functions. This type of problem is called multi-objective transportation problem. The mathematical model of multi-objective transportation problem can be stated as follows.

$$(MOTSP)\ Z_K(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^{k}\, x_{ij}$$

$$Subject\ to\ \sum_{j=1}^{n} x_{ij} = s_i, i = 1, 2, 3, \dots m$$

$$\sum_{i=1}^{m} x_{ij} = d_j, j = 1, 2, 3, \dots n$$
$$x_{ij} \geq 0 \; \forall \; i, j$$

2.2 Evolutionary Algorithms (EA) Formulation

Evolutionary algorithms (EAs) are a natural selection method from biological world which deals the survival of the fittest. EAs is an optimization techniques in which involve a search from a "population" of solutions, not from a single point. Each iteration of an EA selection will rule out poor solutions. The fitted solutions where recombined with other solutions by swapping parts of a solution with another and mutated is down by a small change of an element. New generations of the solutions are got by recombination and mutation.

Population:

The simple genetic algorithm often uses a sequence representation. A string usually represents a unique member of the search space. Following the biological analogy, strings are sometimes called chromosomes.

Selection:

The selection operator is responsible for detecting better regions of the search space. The "fitness" of a member is its objective function value. Selection computes an ordering among all the members of the population and gives more copies to the better strings at the expense of less "fit" members.

Crossover:

Crossover works by swapping portions between two strings. Single-point crossover is often used in the simple genetic algorithm. It works by first randomly picking a point between and the participating strings are then split at that point, followed by a swapping of the split halves. In absence of any bias, uniform application of crossover simply correlates the individual distribution of genes in the population.

Mutation:

Mutation randomly changes the entries of a string. Mutation is usually treated as a low profile operator in a genetic algorithm because blind mutation cannot make an algorithm transcend the limits of random enumerative search.

2.3 Structure of the (MOTSP)

Consider a set of $Z_K(x) = \{z_1(x), z_2(x), z_3(x), \dots, z_k(x)\}$ is a vector of K objective functions. Where a_{ij}^k are the corresponding cost of objective functions (k = 1, 2,..., K), and m and n are the number of sources and destinations, respectively. The above problem implies that the total supply $\sum_{i=1}^{m} s_i$ must be equal to the total demand $\sum_{j=1}^{n} d_j$. When total supply is equals to the total demand the resulting formulation is called a balanced transportation problem

2.4 Encoding Problem

The chromosome is represented by a Bipartite Graph. The Destination and Source are represented by two set of Vertices. The relation between the two set of vertices is obtained from the basic feasible solutions. This is as following bipartite graph as follows.

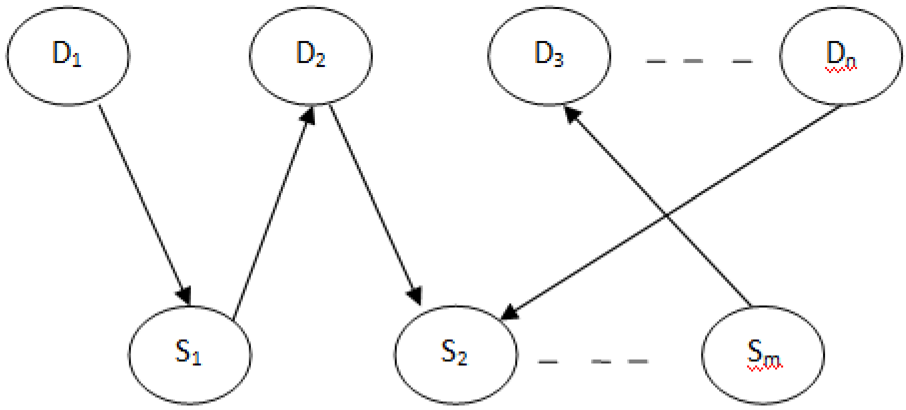

Figure 1: Encoded solution

2. Numerical Calculation

The transportation problem which involves two objectives is taken as a multiple objective problem whose supply and demand is as follows.

Table 1: Transportation allocations

a_{ij}^{k}	D_1	D_2	D_3	D_4	Supply
S_1	1 4	2 4	7 3	7 4	8
S_2	1 5	9 8	3 9	4 10	19
S_3	8 6	9 2	4 5	6 1	17
Demand	11	3	14	16	44

3.1 Initial Population

A Bipartite Graph representation of the chromosome, and in order to create and to permit our set of solutions to evolve in a very large domain, we shall use a combination of some methods. **NWCR**: North West corner rule. **LCM**: Least cost method. **VAM Method**: Vogele's approximation method.

3.1.1 Bipartite Graphical Representation
Two solutions of the table 1 in figure 2 are taken as parent

Parent 1: Z_1=156, Z_2=200

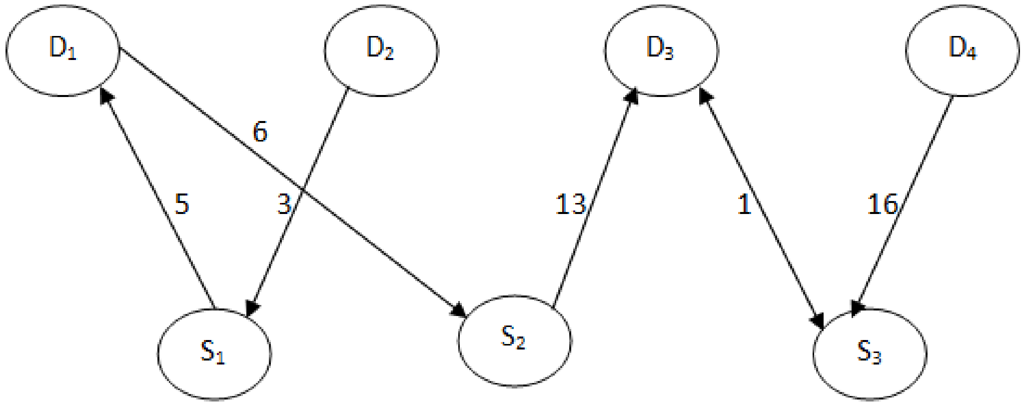

Figure 2: Parent 1

Parent 2: Z_1=208, Z_2=167

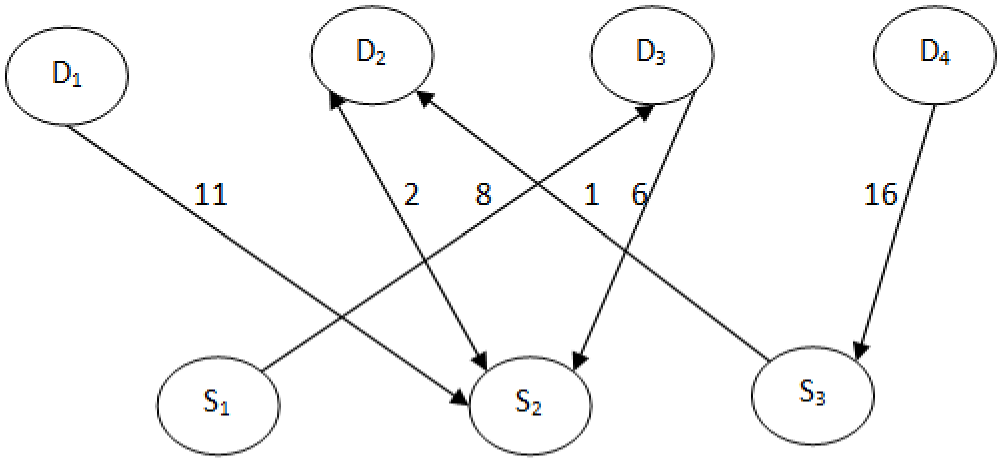

Figure 3: Parent 2

3.2 Crossover Operator
Crossover involves combining elements from two parent chromosomes into one or more child chromosomes The role of the crossover is to generate a better solution by exchanging information contained in the current good ones.
Here we apply single point crossover operation.

3.2.1 Graphical Representation of Crossover Operator

Then the offspring's after crossover is as

Child 1: Z_1=162, Z_2=201

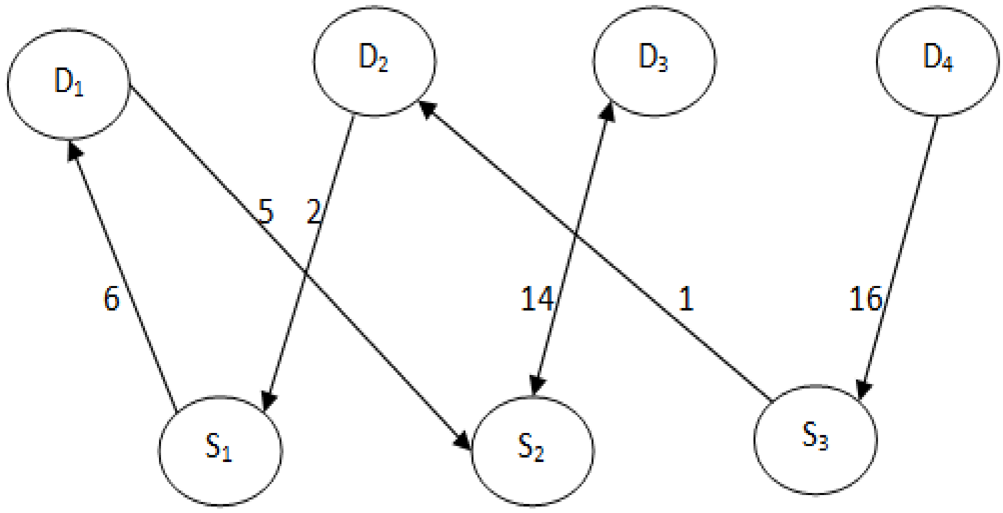

Figure 4: Child 1

Child 2: Z_1=209, Z_2=169

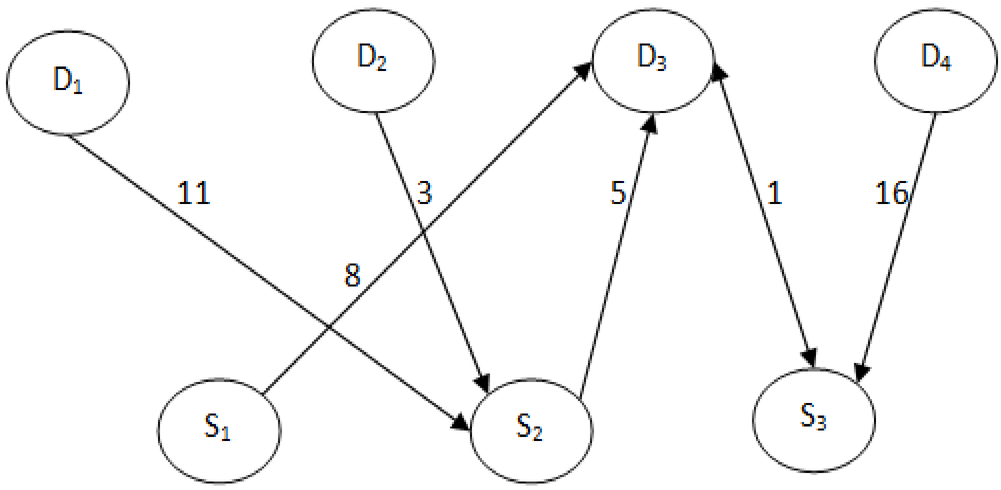

Figure 5: Child 2

3.3 Mutation

After crossover, each child produced by the crossover undergoes mutation with a low probability. Hear the mutation is down as changing the direction of the arrows

3.3.1 Graphical Representation of Mutation

Child 3: Z_1=162, Z_2=192

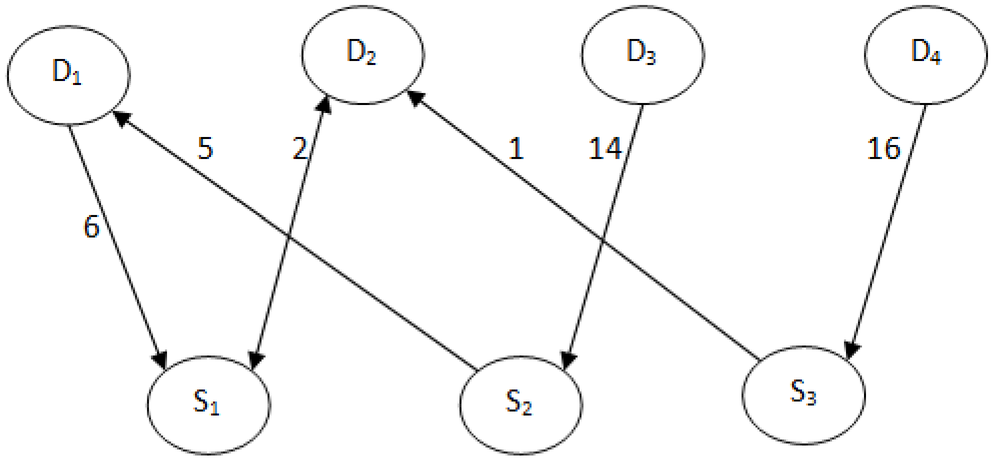

Figure 6: Child 3

Child 4: Z_1=174, Z_2=169

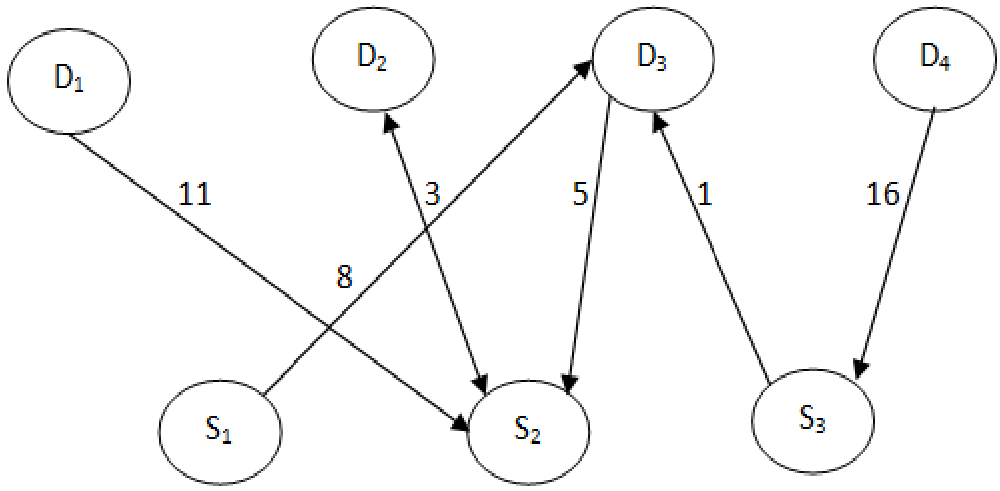

Figure 7: Child 4

After applying the Evolutionary variation operators like crossover and mutation we have got the solution as Z_1=162, Z_2=169

To evaluate the performance of the approach, let us consider the solution of the problem by using different methods. The interactive approach in [20] provides the following results: Z_1 =186 and Z_2 =174, the fuzzy approach in [2] gave the following results: $Z_1 = 170$ and $Z_2 = 190$, the fuzzy approach in [4] gave the following results: $Z_1 = 160$ and $Z_2 = 195$, the IFGP approach in [1] gave the following results: $Z_1 =168$ and $Z_2=185$.

Conclusion

The transportation problem is the central nerve system to keep the balance in economical world from ancient day until today. In earlier days, transportation problem developed with the assumption that the supply, demand, and cost parameters are exactly known. But in real-life applications, all the parameters of the transportation problem are not generally defined precisely. Here we have presented an application of Evolutionary Algorithms to the multi-objective transportation problem (MOTSP). We consider the multi-objective transportation problem as linear optimization problem and use a special type of Encoding method. The set of feasible solution of the multi-objective transportation problem (MOTSP) is encoding as Bipartite Graph and selected as the parent solution. The Evolutionary operates where applied to get an optimal compromise solution. This methodology is illustrated by a numerical example.

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