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Intuitionistic fuzzification functions

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Abstract

Fuzzification is the process of converting crisp quantity into fuzzy. If uncertainty happens to arise because of imprecision, ambiguity or vagueness, then the variable is probably fuzzy and can be represented by a membership function. Fuzzification determines the degree of membership. In practice, due to insufficiency of the information available, the evaluation of membership and non-membership values upto decision maker's satisfaction is not always possible. Consequently, there remains an indeterministic part of which hesitation survives. While various methods for the fuzzification of fuzzy sets have been devised, no such attempt has been found in the case of intuitionistic fuzzy sets. The term *intuitionistic fuzzification functions* refers to formulating membership and non-membership functions of an intuitionistic fuzzy set. In this paper, an attempt has been made to introduce various types of intuitionistic fuzzification functions such as triangular, trapezoidal, Gaussian, bell-shaped, sigmoidal, S-shaped, Z-shaped functions which will be more useful in modeling real world situations in intuitionistic fuzzy environment.

AMS subject classification:
Keywords: Intuitionistic fuzzy sets, membership and non-membership functions, intuitionistic fuzzy index, intuitionistic fuzzification functions.

1. Introduction

Real world problems sometimes emerge to be complex owing to uncertainty either in parameters or in situations in which the problem occurs. Uncertainty may arise due to partial information about the problem, or due to unreliable information, or due to inherent imprecision in the language. For example, to categorize a person 'tall' or 'not tall' is problematic. As there is no distinct cut off point at which tallness begins, it is far more difficult to define the set of tall people. If threshold is selected, say 180 cm at which the set *tall* begins, the output of the reasoning system using this definition would not be smooth with respect to the height of a person. A person of height 179 cm would produce a different output than a person of 181 cm [9]. A more natural way would be to relax the strict separation between tall and not tall. This can be done by allowing not only the (crisp) decision yes/no, but more flexible rules like "Approximately tall". To deal these situations, in 1965, Lotfi A Zadeh introduced the concept, *fuzzy set* [10].

In crisp, all the elements are coded with 0 or 1. A straight way to generalize this concept is to allow more values between 0 and 1. These values are referred to as the membership grades of these elements in the set. Such a function is called a *membership function*, μ_A by which a fuzzy set A is defined [3, 6]. Fuzzy sets allow elements to be a partial membership in the set. Therefore a person with height 179 cm can be categorized both in tall and in not tall sets with a particular degree of membership. As with the increase in the height of the person, the membership grade within the tall set would increase whereas the membership grade within the not tall set would decrease simultaneously. The membership grade in a fuzzy set is not a matter of affirmation or denial, but rather a matter of degree. In fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one.

Fuzziness in the fuzzy set is characterized by its membership function [8, 9]. It classifies the element in the set, whether it is discrete or continuous. The membership functions can be formed by graphical representations of different shapes. In MATLAB, the Fuzzy Logic toolbox includes 11 built-in functions [11]. These functions are in turn built from several basic functions: piecewise linear functions, the Gaussian distribution function, the sigmoid curve and quadratic & cubic polynomial curves.

Due to the insufficiency in the availability of information, the evaluation of membership values is not possible upto our satisfaction. Therefore a generalization of fuzzy sets was introduced by K. T. Atanassov in 1983 as *intuitionistic fuzzy sets* (IFSs), which include both membership and non-membership of the element in the set, where the non-membership value = 1 - the membership value. However, in reality, it may not exist, because there remains a part indeterministic on which hesitation survives [4, 5]. This value is called *intuitionistic fuzzy index (hesitancy index)*.

In such situations, the Intuitionistic Fuzzy Set theory introduced by Atanassov seems to be applicable to address this issue of uncertainty. In the case, when the degree of rejection is defined simultaneously with the degree of acceptance and when both these degrees are not complementary to each other, then IFS can be used as a more general and full tool for describing uncertainty.

The steps involved in modeling real life problems via intuitionistic fuzzy sets are

1. Intuitionistic fuzzification:
   Fuzzification is the process of converting crisp to fuzzy. The term intuitionistic fuzzification refers to formulating membership and non-membership values of an IFS. As far as image is concerned fuzzification refers to conversion of gray levels [0, 255] of the pixels into real values in [0, 1].

2. Modification of membership and non-membership values (intuitionistic fuzzy operators/rules)

3. Intuitionistic defuzzification:
   Defuzzification is the reverse process of fuzzification, where the output obtained is a crisp quantity.

This paper concentrates on step 1 given above to define membership and non-membership functions by treating hesitancy index as a parameter. The membership, non-membership functions and hesitancy index of an IFS can be used to express three states namely support, opponent and neutral, with more delicate depiction and expression of fuzzy essence of objective world [7]. Further more, it appears more agile and applied when it comes to deal with uncertain problems. Various types of membership and non-membership functions of IFSs are defined with suitable illustrations and fuzzification methods are extended to intuitionistic fuzzy sets.

The remaining part of the paper is organized as follows. Section 2 gives basic definitions of IFSs. Section 3 deals with the various types of intuitionistic fuzzification functions. The process of intuitionistic fuzzification allows the system inputs and outpus to be expressed in linguistic terms so that the rules can be applied in a simple manner to express a complex system. Section 4 comprises the concluding part.

2. Preliminaries

In this section, some basic definitions, which are prerequisites for the study, are outlined.

Definition 2.1. [4] Let the universal set X be fixed. An *intuitionistic fuzzy set* A in X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where the functions $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ define the degrees of membership and non-membership of the element $x \in X$ respectively, and for every $x \in X$ in A, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ holds.

Definition 2.2. [4] For every common intuitionistic fuzzy subset A on X, we have $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ called the *intuitionistic fuzzy index or hesitancy index* of x in A. $\pi_A(x)$ is the degree of indeterminacy of $x \in X$ to the IFS A. $\pi_A(x)$ expresses the degree of lack of knowledge of every $x \in X$ belongs to IFS or not. Obviously, for every $x \in X$ and $0 \leq \pi_A(x) \leq 1$.

Definition 2.3. [9] *Membership function* for an intuitionistic fuzzy set A on the universe of discourse X is defined as $\mu_A : X \to [0, 1]$, where each element X is mapped to a value between 0 and 1. The value $\mu_A(x), x \in X$ is called the membership value or degree of membership.

Definition 2.4. [9] *Non-membership function* for an intuitionistic fuzzy set A on the universe of discourse X is defined as $\nu_A : X \to [0, 1]$, where each element X is mapped to a value between 0 and 1. The value $\nu_A(x)$, $x \in X$is called the non-membership value or degree of non-membership.

3. Intuitionistic fuzzification functions

In this paper, the term *intuitionistic fuzzification functions* refers to formulation of membership and non-membership functions of an IFS. As there are infinite number of ways to characterize fuzziness to depict the membership functions graphically which describe fuzziness. The choice of which of the methods to be used depends entirely on the problem under consideration. The graphical representations may include different shapes formed using straight lines and simple curves. The formulated membership and non-membership functions themselves can take any form the system requires like triangles, trapezoids, bell curves or any other shape as long as those shapes accurately represent the distribution of information within the system.

The simplest membership and non-membership functions are formed using straight lines. Among these, *intuitionistic fuzzy triangular functions* are formed by the collection of three points forming a triangle and *intuitionistic fuzzy trapezoidal functions* are just a truncated triangle curve with a flat top.

The *intuitionistic fuzzy Gaussian* and *bell-shaped functions* are formed by smooth curves and *intuitionistic fuzzy function* are also simple curves which is either open left or right. *Intuitionistic fuzzy S-shaped* and *Z-shaped functions* are formed by polynomial based curves.

This section discusses the formulation and the features of the above-mentioned intuitionistic fuzzy functions. Suitable illustrations are also dealtwith. Throughout this paper, A represents an *intuitionistic fuzzy set.*

3.1. Intuitionistic fuzzy triangular function (*iftrif*)

The *iftrif*, is specified by three parameters, a lower limit a, an upper limit c, and a value b, where $a \leq b \leq c$. The precise appearance of the function is determined by the choice of the parameters a, b, c which in turn forms a triangle. In this a and c locates the *feet* of the triangle and the parameter b locates the *peak.*

Intuitionistic fuzzy triangular membership function of A takes the form

$$\mu_A(x) = \begin{cases} 0 & ; \quad x \leq a \\ \left(\dfrac{x-a}{b-a}\right) - \epsilon & ; \quad a < x \leq b \\ \left(\dfrac{c-x}{c-b}\right) - \epsilon & ; \quad b \leq x < c \\ 0 & ; \quad x \geq c \end{cases} \tag{3.1}$$

The corresponding, intuitionistic fuzzy triangular non-membership function is of the form

$$\nu_A(x) = \begin{cases} 1-\epsilon & ; \quad x \le a \\ 1-\left(\dfrac{x-a}{b-a}\right) & ; \quad a < x \le b \\ 1-\left(\dfrac{c-x}{c-b}\right) & ; \quad b \le x < c \\ 1-\epsilon & ; \quad x \ge c \end{cases} \tag{3.2}$$

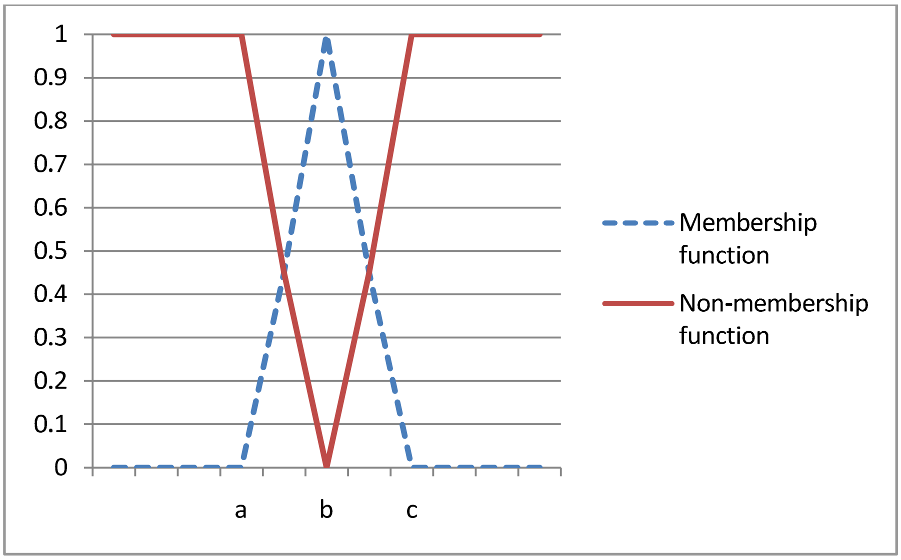

Figure 1: (a) Intuitionistic fuzzy triangular function.

The diagrammatic representation of membership and non-membership functions are shown in Fig. 1(a).

Note 1
When $\epsilon = 0$, *iftrif* tends to *trif* in fuzzy.

Note 2
Hereafter, ϵ is an arbitrary parameter chosen in such a way that $\mu_A(x) + \nu_A(x) + \epsilon = 1$ and $0 < \epsilon < 1$.

Example 3.1. Suppose the room temperature varies from $-5°C$ to $+5°C$, then the corresponding membership and non-membership triangular functions for *approximately zero degree celsius* temperature specified by the three parameters $a = -5$, $b = 0$ and

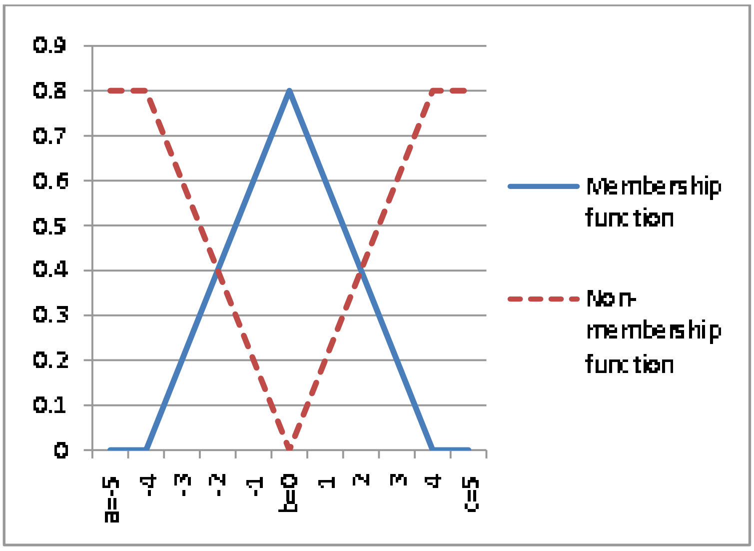

Figure 1: (b) Intuitionistic fuzzy triangular function.

$c = +5$ are as follows: $(\epsilon = 0.2)$

$$\mu_A(x) = \begin{cases} 0 & ; \quad x \leq -5 \\ \left(\dfrac{x+5}{5}\right) - 0.2 & ; \quad -5 < x \leq 0 \\ \left(\dfrac{5-x}{5}\right) - 0.2 & ; \quad 0 \leq x < 5 \\ 0 & ; \quad x \geq 5 \end{cases} \tag{3.3}$$

$$\nu_A(x) = \begin{cases} 0.8 & ; \quad x \leq -5 \\ 1 - \left(\dfrac{x+5}{5}\right) & ; \quad -5 < x \leq 0 \\ 1 - \left(\dfrac{5-x}{5}\right) & ; \quad 0 < x \leq 5 \\ 0.8 & ; \quad x \geq 5 \end{cases} \tag{3.4}$$

The graph of the intuitionistic fuzzy triangular function is displayed in Fig. 1(a). The $iftrif$ for the intuitionistic fuzzy set *approximately zero degree celsius* is shown in Fig. 1(b).

3.2. Intuitionistic fuzzy trapezoidal function (*iftraf*)

The *iftraf*, has a flat top and is a truncated triangle. The *iftraf* function is defined by four parameters, a lower limit a, an upper limit d, a lower support limit b and an upper support limit c, where $a \leq b \leq c \leq d$. Here, a and d locate the feet of the trapezium and b and c locate the shoulder point.

The intuitionistic fuzzy trapezoidal membership function is defined as follows:

$$\mu_A(x) = \begin{cases} 0 & ; \quad x \le a \\ \left(\dfrac{x-a}{b-a}\right) - \epsilon & ; \quad a < x < b \\ 1-\epsilon & ; \quad b \le x \le c \\ \left(\dfrac{d-x}{d-c}\right) - \epsilon & ; \quad c < x < d \\ 0 & ; \quad x \ge d \end{cases} \tag{3.5}$$

The corresponding intuitionistic fuzzy trapezoidal non-membership function is given by

$$\nu_A(x) = \begin{cases} 1-\epsilon & ; \quad x \le a \\ 1-\left(\dfrac{x-a}{b-a}\right) & ; \quad a < x < b \\ 0 & ; \quad b \le x \le c \\ 1-\left(\dfrac{d-x}{d-c}\right) & ; \quad c < x < d \\ 1-\epsilon & ; \quad x \ge d \end{cases} \tag{3.6}$$

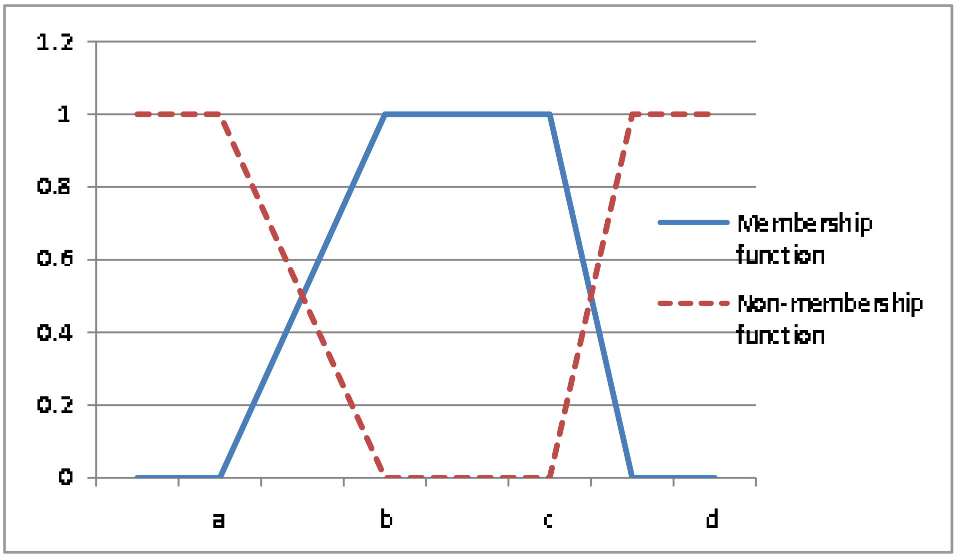

Figure 2: Intuitionistic fuzzy trapezoidal function

The graph of the intuitionistic fuzzy trapezoidal functions is shown in Fig. 2. The intuitionistic fuzzy trapezoidal functions may be symmetric or asymmetric in shape. The symmetric *iftraf* function is shown in Fig. 2. Obviously, the *intuitionistic fuzzy triangular function* is a special case of *intuitionistic fuzzy trapezoidal function*.

Example 3.2. In problems like testing the youthness of the people according to age of the person, the trapezoidal membership function may be used. Suppose A be the set of ages of *old men* which vary *around 55*. Assuming that men whose ages above 65 is treated as *very old*. In this example, the trapezoidal membership function is specified by the parameters $\{a = 50,\ b = 55,\ c = 60 \text{ and } d = 65\}$ and the corresponding membership and non-membership functions are defined as follows: ($\epsilon = 0.1$)

$$\mu_A(x) = \begin{cases} 0 & ; \quad x \leq 50 \\ \left(\dfrac{x-50}{5}\right) - 0.1 & ; \quad 50 < x < 55 \\ 0.9 & ; \quad 55 \leq x \leq 60 \\ \left(\dfrac{65-x}{5}\right) - 0.1 & ; \quad 60 < x < 65 \\ 0 & ; \quad x \geq 65 \end{cases} \tag{3.7}$$

and

$$\nu_A(x) = \begin{cases} 0.9 & ; \quad x \leq 50 \\ 1 - \left(\dfrac{x-50}{5}\right) & ; \quad 50 < x < 55 \\ 0 & ; \quad 55 \leq x \leq 60 \\ 1 - \left(\dfrac{65-x}{5}\right) & ; \quad 60 < x < 65 \\ 0.9 & ; \quad x \geq 65 \end{cases} \tag{3.8}$$

The intuitionistic fuzzy trapezoidal functions, are categorized into two, namely, *intuitionistic fuzzy R-functions* and *intuitionistic fuzzy L-functions*.

3.2.1 Intuitionistic fuzzy R-functions

An *intuitionistic fuzzy R-function* is the right intuitionistic fuzzy *trapezoidal* function. *Intuitionistic fuzzy R-function* is specified by two parameters c and d with $a = b = -\infty$, whose membership functions is defined as follows:

$$\mu_A(x) = \begin{cases} 0 & ; \quad x \geq d \\ \left(\dfrac{d-x}{d-c}\right) - \epsilon & ; \quad c < x < d \\ 1-\epsilon & ; \quad x \leq c \end{cases} \tag{3.9}$$

The corresponding, non-membership function takes the form

$$\nu_A(x) = \begin{cases} 1-\epsilon & ; \quad x \geq d \\ 1 - \left(\dfrac{d-x}{d-c}\right) & ; \quad c < x < d \\ 0 & ; \quad x \geq c \end{cases} \tag{3.10}$$

The diagrammatic representation of intuitionistic fuzzy R-function is shown in Fig. 3.

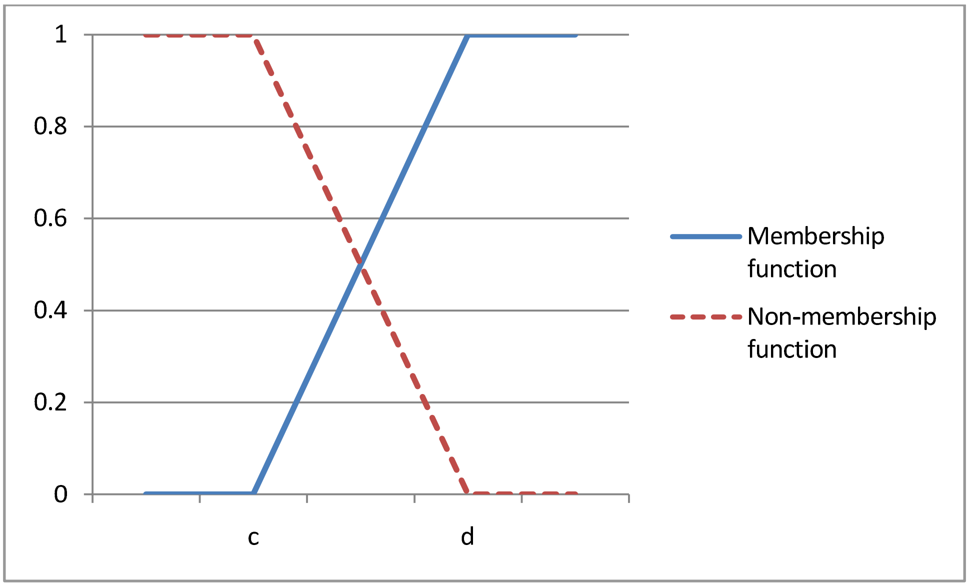

Figure 3: Intuitionistic fuzzy R-function

Example 3.3. If the parameters of the *intuitionistic fuzzy R -function* are specified by the parameters $c = 5.6$, $d = 5.8$, then the corresponding membership and non-membership functions are as follows: $(\epsilon = 0.2)$

$$\mu_A(x) = \begin{cases} 0 & ; \quad x \geq 5.8 \\ \left(\dfrac{5.8 - x}{0.2}\right) - 0.2 & ; \quad 5.6 < x < 5.8 \\ 0.8 & ; \quad x \leq 5.6 \end{cases} \tag{3.11}$$

$$\nu_A(x) = \begin{cases} 0.8 & ; \quad x \geq 5.8 \\ 1 - \left(\dfrac{5.8 - x}{0.2}\right) & ; \quad 5.6 < x < 5.8 \\ 0 & ; \quad x \geq 5.6 \end{cases} \tag{3.12}$$

3.2.2 Intuitionistic fuzzy L-functions

Intuitionistic fuzzy L-function is the left intuitionistic fuzzy trapezoidal function. *Intuitionistic fuzzy L*-function is specified by two parameters a and b with $c = d = +\infty$, whose membership takes the form

$$\mu_A(x) = \begin{cases} 0 & ; \quad x \leq a \\ \left(\dfrac{x - a}{b - a}\right) - \epsilon & ; \quad a < x < b \\ 1 - \epsilon & ; \quad x \geq b \end{cases} \tag{3.13}$$

The corresponding, non-membership function is given as

$$\nu_A(x) = \begin{cases} 1-\epsilon & ; \quad x \le a \\ 1-\left(\dfrac{x-a}{b-a}\right) & ; \quad a < x < b \\ 0 & ; \quad x \ge b \end{cases} \tag{3.14}$$

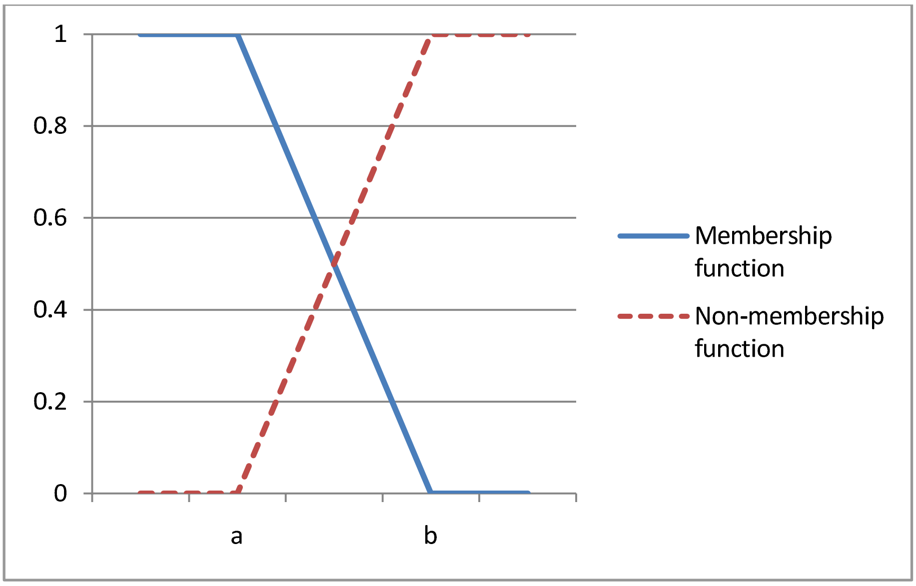

Figure 4: Intuitionistic fuzzy L-function.

Example 3.4. If the parameters of the intuitionistic fuzzy L-function are specified by the parameters $a = 5.2$, $b = 5.4$, then the corresponding membership and non-membership functions are as follows: $(\epsilon = 0.2)$

$$\mu_A(x) = \begin{cases} 0 & ; \quad x \le 5.2 \\ \left(\dfrac{x-5.4}{0.2}\right) - 0.2 & ; \quad 5.2 < x < 5.4 \\ 0.8 & ; \quad x \ge 5.4 \end{cases} \tag{3.15}$$

$$\nu_A(x) = \begin{cases} 0.8 & ; \quad x \le 5.2 \\ 1-\left(\dfrac{x-5.4}{0.2}\right) & ; \quad 5.2 < x < 5.4 \\ 0 & ; \quad x \ge 5.4 \end{cases} \tag{3.16}$$

3.3. Intuitionistic fuzzy Gaussian function (*ifgaussf*)

Ifgaussf is specified by two parameters. The Gaussian function is defined by a central value m and width $k > 0$. The smaller the k, the narrower the curve is.

Intuitionistic fuzzy Gaussian membership and non-membership functions are defined as

$$\mu_A(x) = \exp\left(-\frac{(x-m)^2}{2(k)^2}\right) - \epsilon$$

and

$$\nu_A(x) = 1 - \left(\exp\left(-\frac{(x-m)^2}{2(k)^2}\right)\right)$$

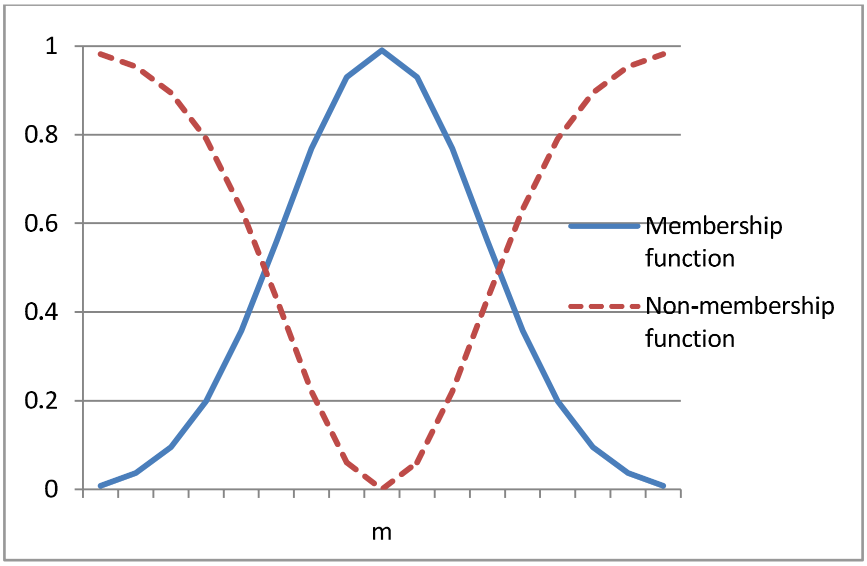

Figure 5: Intuitionistic fuzzy Gaussian function.

The diagrammatic representation of intuitionistic fuzzy Gaussian function is shown in Fig. 5.

Example 3.5. The exponential growth of the bacteria can be expressed by the intuitionistic fuzzy Gaussian function. If the Gaussian membership function is determined by the parameters $m = 5$ and $k = 1$, then the Gaussian membership and non-membership functions are as follows

$$\mu_A(x) = \exp\left(-\frac{(x-5)^2}{2}\right) - \epsilon \tag{3.17}$$

and

$$\nu_A(x) = 1 - \left(\exp\left(-\frac{(x-5)^2}{2}\right)\right) \tag{3.18}$$

3.4. Intuitionistic fuzzy bell-shaped function (*ifbellf*)

Intuitionistic fuzzy bell-shaped function is specified by three parameters a, b, c and usally the parameter b is positive. The parameter c locates the center of the curve and b control the slopes at the crossover points. The intuitionistic fuzzy bell-shaped membership and non-membership functions are defined as

$$\mu_A(x) = 1 - \epsilon - \left(\frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}} \right) \tag{3.19}$$

and

$$\nu_A(x) = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}} \tag{3.20}$$

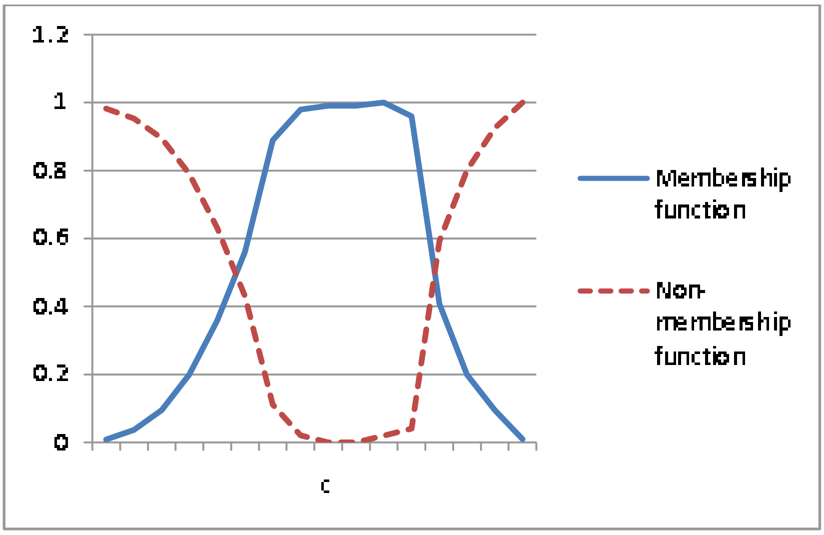

Figure 6: Intuitionistic fuzzy bell-shaped function.

As the shape of the membership resembles the bell and non-membership resembles the inverted bell in Fig. 6, it is called intuitionistic fuzzy bell-shaped function.

3.5. Intuitionistic fuzzy sigmoidal function (*ifsigf*)

Intuitionistic fuzzy sigmoidal function depends on two parameters a and c, where c locates the distance from the origin and a determines the steepness of the function. Depending on the sign of the parameter a, the intuitionistic fuzzy sigmoidal membership function is inherently open to the right or to the left. If a is positive, the function is open to the right, whereas if it is negative it is open to the left. As the parameter increases, the transition from 0 to 1 becomes sharper.

The intuitionistic fuzzy sigmoidal membership and non-membership functions are defined as

$$\mu_A(x) = \left(\frac{1}{1 + \exp(-a(x-c))} \right) - \epsilon \tag{3.21}$$

and

$$\nu_A(x) = 1 - \frac{1}{1 + \exp(-a(x-c))} \tag{3.22}$$

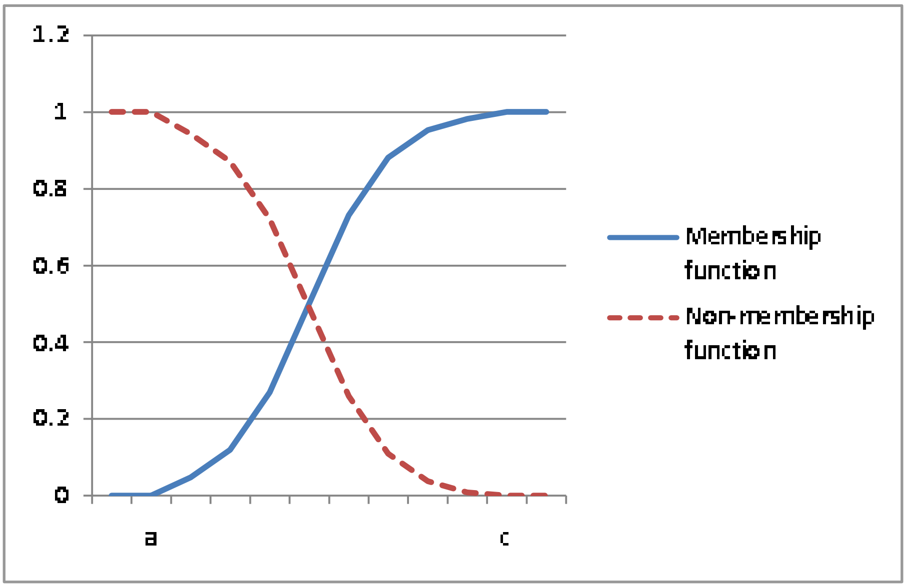

Figure 7: Intuitionistic fuzzy Sigmoidal function.

In Fig. 7, the intuitionistic fuzzy sigmoidal function is open to the right. Intuitionistic fuzzy sigmoidal function is commonly used as an activation function in Neural networks.

Example 3.6. Suppose the two parameters of the intuitionistic fuzzy sigmoidal membership function is given to be $a = 1$ and $c = 0$, then the corresponding membership and non-membership functions are as follows ($\epsilon = 0.01$)

$$\mu_A(x) = \left(\frac{1}{1 + \exp(-1(x - 0))}\right) - 0.01 \tag{3.23}$$

$$\nu_A(x) = 1 - \left(\frac{1}{1 + \exp(-1(x - 0))}\right) \tag{3.24}$$

3.6. Intuitionistic fuzzy S-shaped function (*ifSf*)

The precise appearance of *ifSf* is determined by the choice of the parameters a, b and the parameters locate the extremes of the sloped portion of the curve.

Intuitionistic fuzzy S-shaped membership function takes the form

$$\mu_A(x) = \begin{cases} 0 & ; \quad x \leq a \\ 2\left(\dfrac{x-a}{b-a}\right)^2 - \epsilon & ; \quad a < x \leq \dfrac{a+b}{2} \\ 1 - 2\left(\dfrac{x-b}{b-a}\right)^2 - \epsilon & ; \quad \dfrac{a+b}{2} \leq x < b \\ 1 - \epsilon & ; \quad x \geq b \end{cases} \tag{3.25}$$

Similarly, intuitionistic fuzzy S-shaped non-membership function is given by

$$\nu_A(x) = \begin{cases} 1-\epsilon & ; \quad x \leq a \\ 1-2\left(\dfrac{x-a}{b-a}\right)^2 & ; \quad a < x \leq \dfrac{a+b}{2} \\ 2\left(\dfrac{x-b}{b-a}\right)^2 & ; \quad \dfrac{a+b}{2} \leq x < b \\ 0 & ; \quad x \geq b \end{cases} \tag{3.26}$$

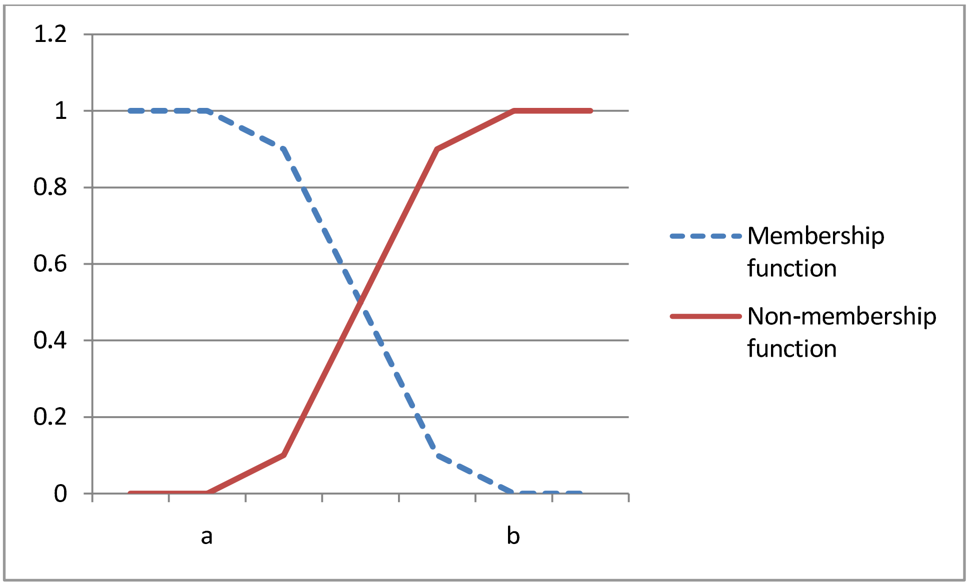

Figure 8: Intuitionistic fuzzy S-shaped function.

The graphical representation of intuitionistic fuzzy S-shaped function is shown in Fig. 8.

Example 3.7. If the two parameters of the intuitionistic fuzzy S-shaped function are given to be $a = 5.1$ and $b = 5.5$, then the corresponding membership and non-membership functions are as follows: $(\epsilon = 0.1)$

$$\mu_A(x) = \begin{cases} 0 & ; \quad x \leq 5.1 \\ 2\left(\dfrac{x-5.1}{0.4}\right)^2 - 0.1 & ; \quad 5.1 < x \leq 5.3 \\ 1-2\left(\dfrac{x-5.5}{0.4}\right)^2 - 0.1 & ; \quad 5.3 \leq x < 5.5 \\ 0.9 & ; \quad x \geq 5.5 \end{cases} \tag{3.27}$$

and

$$\nu_A(x) = \begin{cases} 0.9 & ; \quad x \leq 5.1 \\ 1 - 2\left(\dfrac{x - 5.1}{0.4}\right)^2 & ; \quad 5.1 < x \leq 5.3 \\ 2\left(\dfrac{x - 5.5}{0.4}\right)^2 & ; \quad 5.3 \leq x < 5.5 \\ 0 & ; \quad x \geq 5.5 \end{cases} \tag{3.28}$$

3.7. Intuitionistic fuzzy Z-shaped function (*ifZf*)

The *ifZf*, is given by two parameters, a and b which locate the extremes of the sloped portion of the curve.

Intuitionistic fuzzy Z-shaped membership function is defined as

$$\mu_A(x) = \begin{cases} 1 - \epsilon & ; \quad x \leq a \\ 1 - 2\left(\dfrac{x - a}{b - a}\right)^2 - \epsilon & ; \quad a < x \leq \dfrac{a+b}{2} \\ 2\left(\dfrac{x - b}{b - a}\right)^2 - \epsilon & ; \quad \dfrac{a+b}{2} \leq x < b \\ 0 & ; \quad x \geq b \end{cases} \tag{3.29}$$

The corresponding, intuitionistic fuzzy Z-shaped non-membership function takes the form

$$\nu_A(x) = \begin{cases} 0 & ; \quad x \leq a \\ 2\left(\dfrac{x - a}{b - a}\right)^2 & ; \quad a < x < \dfrac{a+b}{2} \\ 1 - 2\left(\dfrac{x - a}{b - a}\right)^2 & ; \quad \dfrac{a+b}{2} \leq x < b \\ 1 - \epsilon & ; \quad x \geq b \end{cases} \tag{3.30}$$

The diagrammatic representation of intuitionistic fuzzy Z-shaped function is shown in Fig. 9.

Example 3.8. If the two parameters of the intuitionistic fuzzy Z-shaped function are given to be $a = 5.1$ and $b = 5.5$, then the corresponding membership and non-membership functions are as follows: $(\epsilon = 0.1)$

$$\mu_A(x) = \begin{cases} 0.9 & ; \quad x \leq 5.1 \\ 1 - 2\left(\dfrac{x - 5.1}{0.4}\right)^2 - 0.1 & ; \quad 5.1 < x \leq 5.5 \\ 2\left(\dfrac{x - 5.5}{0.4}\right)^2 - 0.1 & ; \quad 5.3 \leq x < 5.5 \\ 0 & ; \quad x \geq 5.5 \end{cases} \tag{3.31}$$

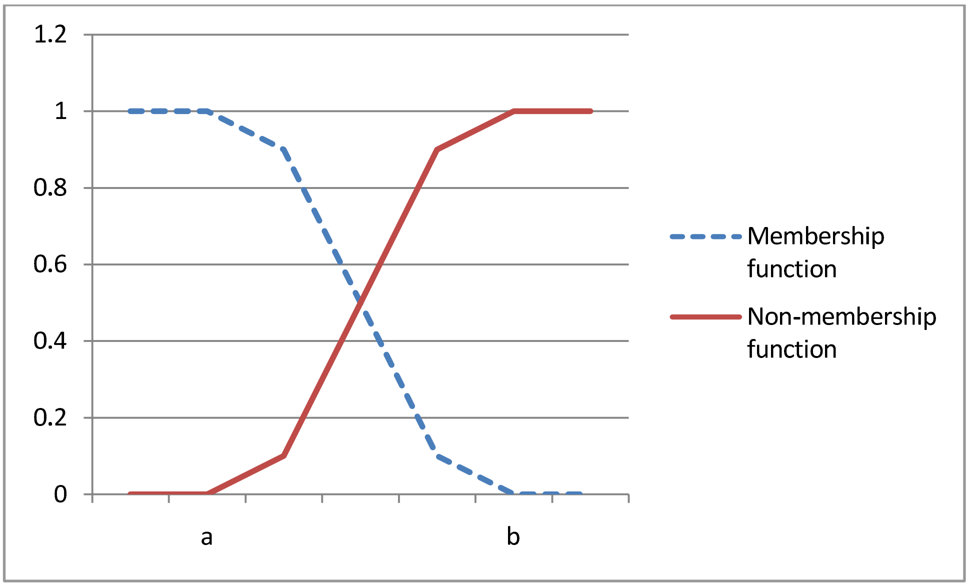

Figure 9: Intuitionistic fuzzy Z-shaped function

and

$$\nu_A(x) = \begin{cases} 0 & ; \quad x \leq 5.1 \\ 2\left(\dfrac{x-5.1}{0.4}\right)^2 & ; \quad 5.1 < x \leq 5.3 \\ 1-2\left(\dfrac{x-5.5}{0.4}\right)^2 & ; \quad 5.3 \leq x < 5.5 \\ 0.9 & ; \quad x \geq 5.5 \end{cases} \tag{3.32}$$

4. Conclusion

Intuitionistic fuzzification functions provide a flexible model to elaborate uncertainty and vagueness involved in real world problems. In this paper, several types of membership and non-membership functions with hesitancy index as an arbitrary parameter for triangular, trapezoidal, Gaussian, bell-shaped, sigmoidal, S-shaped, Z-shaped functions characterizing intuitionistic fuzzy sets are reviewed. An attempt has been made to formulate fuzzification functions for IFS. The authors further proposed to develop the membership and non-membership functions gallery in MATLAB, and also to concentrate on intuitionistic defuzzification.

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