Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 2 (2016), pp. 1201-1210

http://www.ripublication.com

Constructing Volatility Model of Portfolio Return by Using GARCH

Tarno
Department of Statistics, Universitas Diponegoro
Jl. Prof. Sudarto, SH, Semarang 50275; Indonesia.

Hasbi Yasin
Department of Statistics, Universitas Diponegoro
Jl. Prof. Sudarto, SH, Semarang 50275; Indonesia.

Budi Warsito
Department of Statistics, Universitas Diponegoro
Jl. Prof. Sudarto, SH, Semarang 50275; Indonesia.

Abstract

The aim of this research is forecasting volatility of portfolio return using GARCH model. Portfolio return is combination of several return assets. The mean model of stock return is constructed using Autoregressive Integrated Moving Average (ARIMA), and the variance is determined using GARCH model. Based on squares of residual that yielded from mean model, the variance model is constructed using GARCH. The optimal GARCH model is implemented for forecasting volatility of several stock return such as Bank Mandiri (BMRI), Bank BCA (BBCA), Unilever (UNVR) stock return and their portfolio as case studies. The weight(proportion) of each asset in the portfolio return is determined based on Lagrange Multiplier Method.

Keywords: Portfolio return, Volatility, ARIMA, GARCH.

1. Introduction

Financial time series data are usually characterized by volatility clustering, persistence autocorrelation and leptokurtic behavior [1, 2, 3, 4, 5]. The data are usually non-stationary and non-linear [3, 4, 5]. One of the most popular models which applied for time series modeling is ARIMA [6, 7, 8, 9, 10]. Whereas Autoregressive Conditional Heteroscedasticity (ARCH) model was proposed by Engle in 1982 [1] and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model that developed by Bollerslev in 1986 [2] are popular variance models. ARIMA-GARCH has been applied in a lot of research for forecasting financial time series data [11, 12, 13, 14, 15]. The aim of this research is forecasting volatility of portfolio return using GARCH. The procedure of volatility modeling consists of two global steps, firstly, steps of constructing mean model and secondly, steps of constructing variance model [1, 2, 3]. The remaining paper is organized as follows: section 2 discusses about basic concept of mean model (Box-Jenkins ARIMA) and variance model (ARCH/GARCH); section 3 discusses about application GARCH model for forecasting volatility of LQ-45 stock return; and the conclusion is discussed in section 4.

2. Basic Concept of Mean and Variance Models

Basic concept of time series analysis that discussed in this section covers general forms of mean model ARIMA and variance model ARCH/GARCH.

2.1 ARIMA Model

Autoregressive Integrated Moving Average (ARIMA) model is the method introduced by Box-Jenkins [2]. To date, ARIMA become the most popular model for forecasting univariate time series data. Generally, ARIMA(p, d, q) model can be written as (see [6, 8, 10])

$$\phi_p(B)(1-B)^d Z_t = \theta_q(B)a_t \tag{1}$$

where $\phi_p(B) = 1-\phi_1 B-\phi_2 B^2-\cdots-\phi_p B^p$,

$\theta_q(B) = 1-\theta_1 B-\theta_2 B^2-\cdots-\theta_q B^q$,

where B is backward shift operator, p and q denotes order of autoregressive and moving average respectively and d denotes order of differences.

2.2 Volatility Models

2.2.1 GARCH Model

Given stationary time series Z_t such as financial return, so Z_t can be expressed as summation of its mean and a white noise [1, 2], if there is no autocorrelation among Z_t itself, i.e

$$Z_t = \mu_t + a_t \quad \text{and} \quad a_t = \sigma_t \varepsilon_t \tag{2}$$

where μ_t is process mean of Z_t and $\varepsilon_t \sim N(0,1)$. To investigate the volatility clustering or conditional heteroscedasticity, it is assumed that $\mathrm{Var}_{t-1}(a_t) = \sigma_t^2$, where $\mathrm{Var}_{t-1}(\bullet)$ express conditional variance given information at time (t-1), and

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_p a_{t-p}^2 \tag{3}$$

Because mean of a_t is 0, $Var_{t-1}(a_t) = E_{t-1}(a_t^2) = \sigma_t^2$. Therefore, Eq.2 can be written as:

$$a_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_p a_{t-p}^2 + u_t \quad (4)$$

where $u_t = a_t^2 - E_{t-1}(a_t^2)$ is white noise with mean 0. Model (2) and (3) is called ARCH model [1].
In practice, the number of lags p are frequently large, then the number of parameters in the model that should be estimated are also very large. Bollerslev (1986) proposed more parsimonious model to substitute AR model (3) with equation below [2].

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i a_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (5)$$

where $\alpha_i > 0$ $(i = 0,1,2,\ldots,p)$; $\beta_j > 0$ $(j = 1,2,\ldots,q)$ to guarantee that conditional variance σ_t^2 is always positive. Eq.5 together with Eq.2 is called as generalized ARCH or GARCH(p, q). If q=0 the GARCH model become ARCH model [1].

2.2.2 EGARCH Model

Nelson proposed Exponential GARCH (EGARCH) model with leverage effect that written as follow [7].

$$h_t = \alpha_0 + \sum_{i=0}^{p} \alpha_i \frac{|a_{t-i}| + \gamma_i a_{t-i}}{\sigma_{t-i}} + \sum_{j=0}^{q} \beta_j h_{t-j} \quad (6)$$

where $h_t = \ln \sigma_t^2$. The conditional variance of EGRACH σ_t is guaranteed to be positive regardless the coefficients in model (6), because $\ln \sigma_t^2$ has substituted to σ_t^2 itself in the model [5].

2.3 Portfolio Return

Portfolio return is summation of single asset stock return multiplied by its weight (proportion). The weight of each stock to be determined based on Lagrange Multiplier method. The optimal weight can be solved by minimizing portfolio variance function with constraint $\mathbf{w}^T \mathbf{1}_N = 1$ [13]. Define the portfolio variance as: $\sigma_p^2 = \frac{1}{2}\mathbf{w}^T \Sigma \mathbf{w}$. Minimizing function $\frac{1}{2}\mathbf{w}^T \Sigma \mathbf{w}$ with respect to $\mathbf{w}$ is equivalent to minimizing function $\mathbf{w}^T \Sigma \mathbf{w}$. The aim of minimizing function $\frac{1}{2}\mathbf{w}^T \Sigma \mathbf{w}$ is minimizing risk based on the mean of portfolio return. Mathematically, it can be written as: $\min(\sigma_p^2) = \min_{\mathbf{w}} \left(\frac{1}{2}\mathbf{w}^T \Sigma \mathbf{w} \right)$ with constraint $\mathbf{w}^T \mathbf{1}_N = 1$. The optimization problem can be solved by using Lagrange function.

$$L = \frac{1}{2}\mathbf{w}^T \Sigma \mathbf{w} - \lambda(\mathbf{w}^T \mathbf{1}_N - 1) \quad (7)$$

where L: Lagrange function and λ: Lagrange multiplier. The optimal weight **w** is obtained by minimizing Eq.7. Based on theory of calculus, we obtain

$$w = \frac{\Sigma^{-1} 1_N}{1_N^T \Sigma^{-1} 1_N}. \tag{8}$$

Therefore, the portfolio return of N assets can be determined using formula:

$$r_p = \sum_{i=1}^{N} w_i r_{t,i} \tag{9}$$

where w_i : weight of $i-th$ asset and $r_{t,i}$: return of $i-th$ asset.

3. Application

As an implementation of GARCH modeling for forecasting volatility of portfolio return, GARCH models to be constructed for forecasting volatility of Bank Mandiri (BMRI) stock return, Bank BCA (BBCA) stock return, Unilever (UNVR) stock return and their portfolio. The daily stock return of BMRI, BBCA and UNVR from 2 January 2013 until 16 April 2014 are used for constructing models (see www.finance.yahoo.com).

Procedure of GARCH modeling can be divided into two main steps, the first one are mean modeling steps and the second one are variance modeling steps. The steps of constructing ARIMA model consists of model identification, parameter estimation, and verification model. The estimated model that satisfied all of the assumptions can be used for forecasting, but if the estimated model didn't satisfy the assumption especially homoscedasticity assumption (there is GARCH effect) then the variance model should be constructed. The model should be constructed based on the squares of residual. Results of constructing GARCH models for forecasting volatility of single asset and portfolio can be described as follows.

3.1 Forecasting volatility of single asset

3.1.1 Forecasting volatility of BMRI

Estimated model of BMRI return is: ARIMA([2], 0, [2])-EGARCH(1, 1) that can be written as: $r_t = 0.7133 r_{t-2} - 0.8350 a_{t-2} + a_t$,

where $a_t \sim N(0, \sigma_t^2)$ and

$$\ln(\sigma_t^2) = -0.1237 - 0.0880 \left| \frac{a_{t-1}}{\sigma_{t-1}} \right| - 0.1076 \frac{a_{t-1}}{\sigma_{t-1}} + 0.9741 \ln(\sigma_{t-1}^2).$$

3.1.2 Forecasting volatility of BBCA

The estimated model of BBCA stock return is: ARIMA(2, 0, 2)-GARCH(1, 1) that can be written as:

$$r_t = -0.5899 r_{t-1} - 0.9820 r_{t-2} + 0.5657 a_{t-1} + 1.0302 a_{t-2} + a_t$$

where $a_t \sim N(0, \sigma_t^2)$ and $\sigma_t^2 = 0.000044 + 0.050571 a_{t-1}^2 + 0,83505 \sigma_{t-1}^2$.

3.1.3 Forecasting volatility of unilever (UNVR)

Estimated model of UNVR stock return is: ARIMA([2], 0, [2])-IGARCH(1, 1) that can be written as:

$r_t = 0.9303 r_{t-2} - 0.9941 a_{t-2} + a_t$,

where $a_t \sim N(0, \sigma_t^2)$ and $\sigma_t^2 = 0.0263 a_{t-1}^2 + 0.9737 \sigma_{t-1}^2$.

The result of predicted volatility of single asset return BMRI, BBCA and UNVR are respectively shown as Figure 1, Figure 2 and Figure 3.

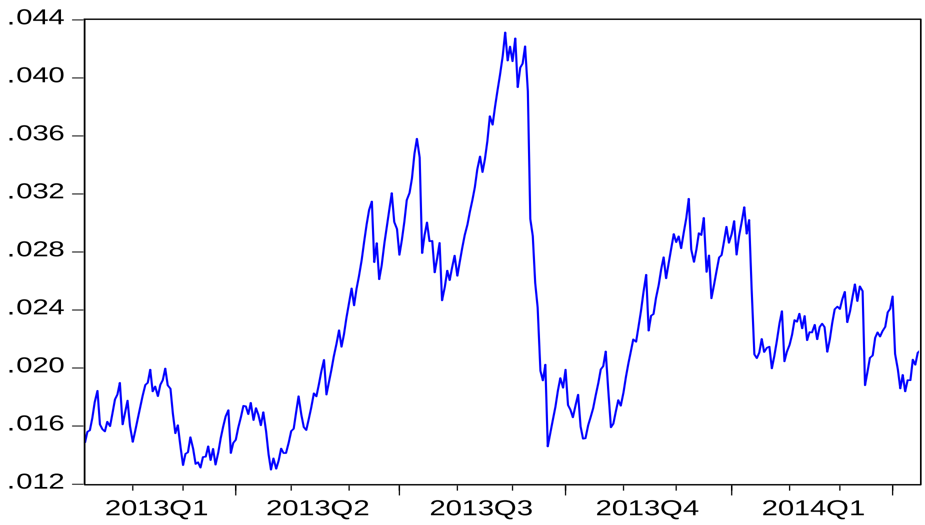

Figure 1: The estimated volatility of BMRI

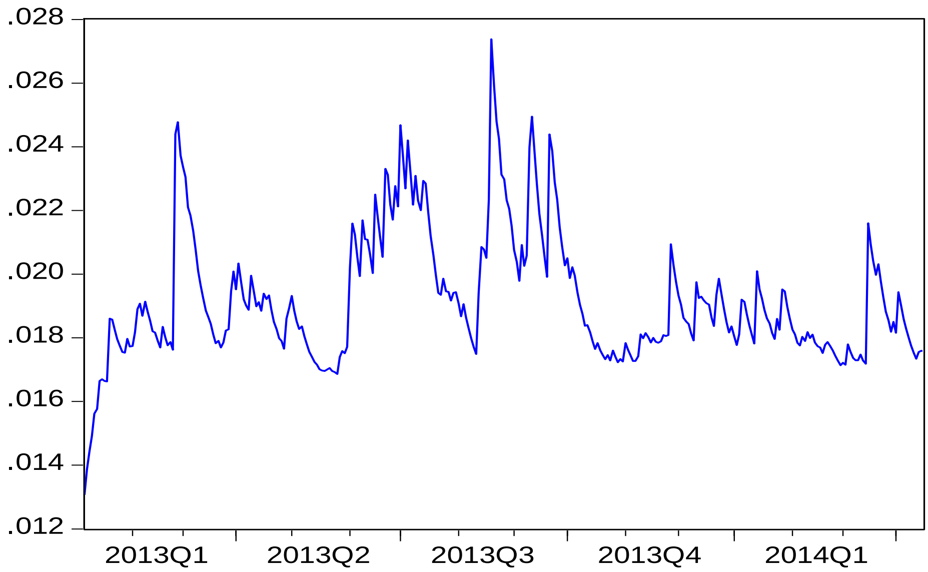

Figure 2: The estimated volatility of BBCA

Figure 3: The estimated volatility of UNVR

3.2 Forecasting volatility of portfolio asset

3.2.1 Forecasting volatility of portfolio: BMRI and BBCA

The weight of each asset is calculated by minimizing Lagrange multiplier function. The weight of BMRI and BBCA is 23% and 77% respectively. Portfolio return is determined using Eq.9. The constructed model of portfolio return is: ARIMA([2], 0, [2])-GARCH(1, 1) that can be written as:

$$r_t = -0.8121 r_{t-2} + 0{,}8337 a_{t-2} + a_t,$$

where $a_t \sim N(0, \sigma_t^2)$ and $\sigma_t^2 = 0.000037 + 0.0598 a_{t-1}^2 + 0.8387 \sigma_{t-1}^2$.

3.2.2 Forecasting volatility of portfolio: BMRI and UNVR

The optimal weight of BMRI and UNVR is 48.5% and 51.5%. By using Eq.9, the estimated model of portfolio return is: ARIMA(2, 0, 2)-GARCH(1, 1) that can be written as:

$$r_t = 1.5051 r_{t-1} - 0.9187 r_{t-2} - 1{,}5541 a_{t-1} + 0.9509 a_{t-2} + a_t$$

where $a_t \sim N(0, \sigma_t^2)$ and $\sigma_t^2 = 0.0000097 + 0.05114 a_{t-1}^2 + 0.92679 \sigma_{t-1}^2$.

3.2.3 Forecasting volatility of portfolio: BBCA and UNVR

The optimal weight of BMRI and UNVR is 48.5% and 51.5% respectively. Return portfolio is determined using Eq.13. The estimated model of portfolio stock return is: ARIMA([3], 0, 0)-EGARCH(1, 1) that can be written as:

$r_t = -0.1076 r_{t-3} + a_t$, where $a_t \sim N(0, \sigma_t^2)$, $\ln(\sigma_t^2) = -0.3811 - 0.1078 \left| \frac{a_{t-1}}{\sigma_{t-1}} \right| + 0.9619 \ln(\sigma_{t-1}^2)$.

3.2.4 Forecasting volatility of portfolio: BMRI, BBCA and UNVR

The optimal weight of BMRI, BBCA and UNVR is 15.8%, 59.2% and 25.0% respectively. Portfolio return is determined using Eq.9. The estimated model of portfolio return is: ARIMA([2], 0, [2])-IGARCH(1, 1) that can be written as: $r_t = 0.9018 r_{t-2} - 0.9950 a_{t-2} + a_t$,

where $a_t \sim N(0, \sigma_t^2)$ and $\sigma_t^2 = 0.0305 a_{t-1}^2 + 0.9695 \sigma_{t-1}^2$.

The result of predicted volatility of portfolio return are shown as Figure 4, Figure 5, Figure 6 and Figure 7 below.

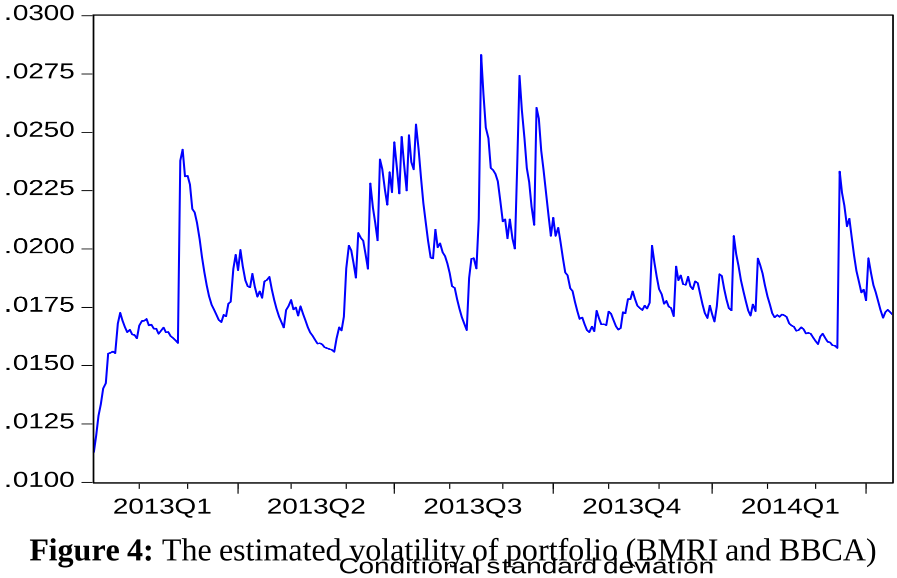

Figure 4: The estimated volatility of portfolio (BMRI and BBCA)

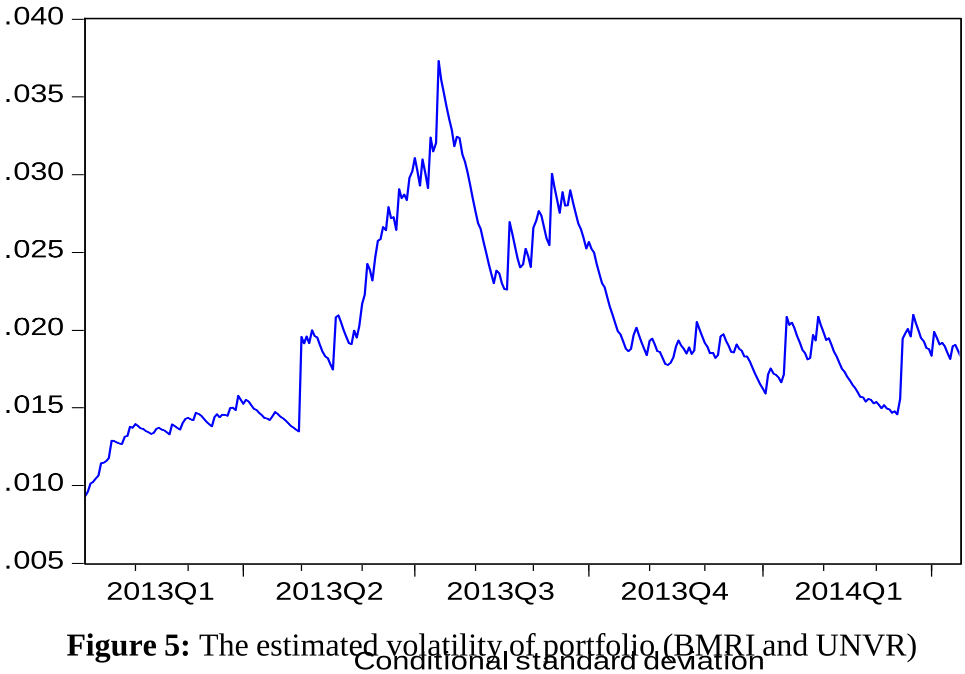

Figure 5: The estimated volatility of portfolio (BMRI and UNVR)

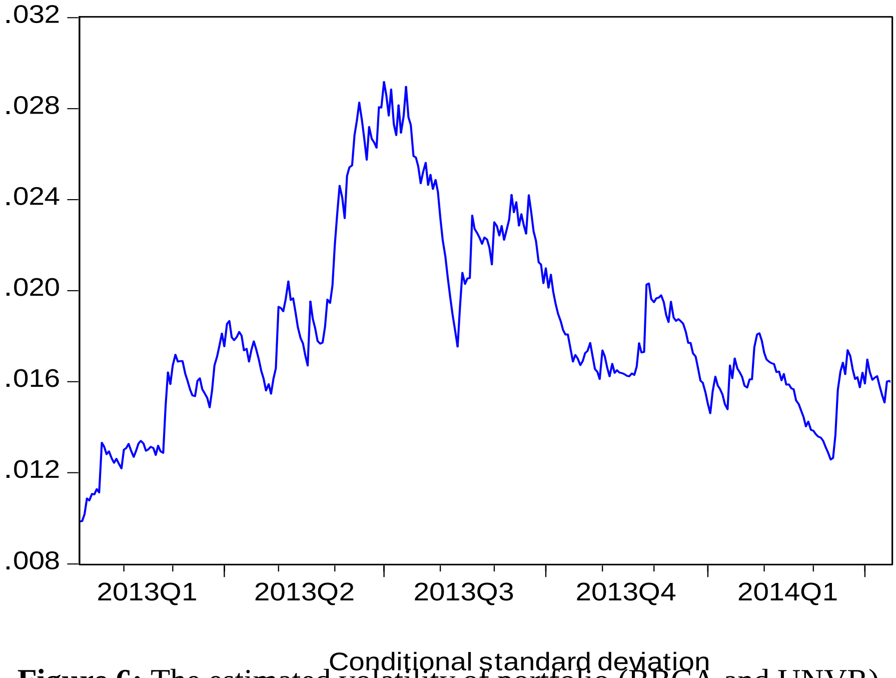

Figure 6: The estimated volatility of portfolio (BBCA and UNVR)

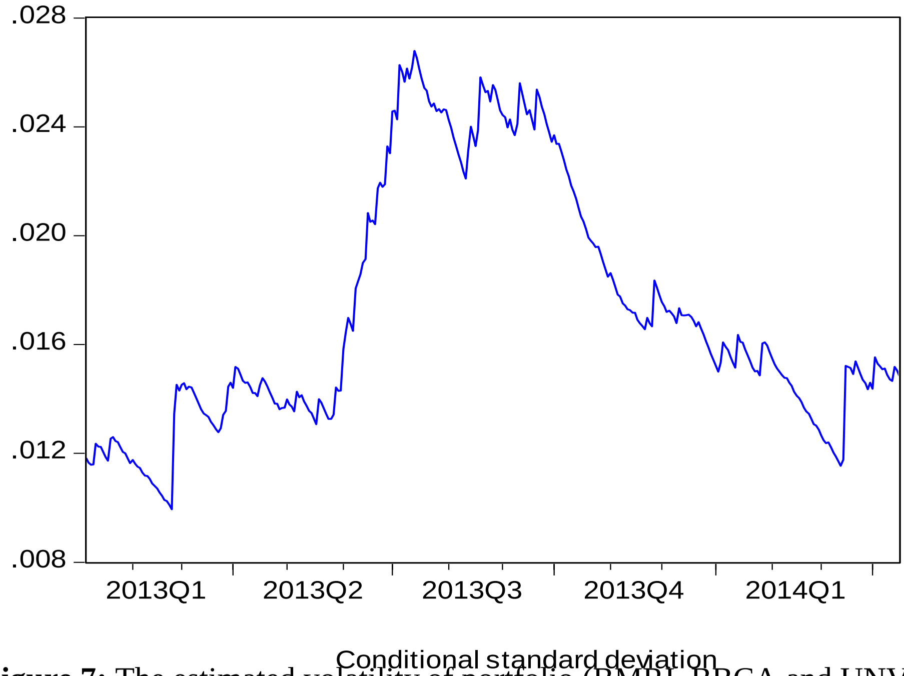

Figure 7: The estimated volatility of portfolio (BMRI, BBCA and UNVR)

The examples of predictied volatility for single asset return of BMRI, BBCA and UNVR and their portfolio return from 3 April 2014 until 15 April 2014 are given on Table 1.

Table 1. Predicted volatility of BMRI, BBCA, UNVR and their portfolio return

Date	Predicted Volatility						
	BMRI	BBCA	UNVR	BMRI, BBCA	BMRI, UNVR	BBCA, UNVR	BMRI, BBCA, UNVR
4/3/2014	0.0199	0.0190	0.0169	0.0189	0.0195	0.0145	0.0153
4/4/2014	0.0186	0.0186	0.0167	0.0184	0.0191	0.0142	0.0152
4/7/2014	0.0195	0.0183	0.0168	0.0181	0.0192	0.0142	0.0151
4/8/2014	0.0184	0.0180	0.0170	0.0178	0.0190	0.0141	0.0151
4/9/2014	0.0192	0.0177	0.0169	0.0174	0.0186	0.0139	0.0149
4/10/2014	0.0192	0.0175	0.0167	0.0171	0.0181	0.0136	0.0147
4/11/2014	0.0206	0.0173	0.0165	0.0173	0.0190	0.0133	0.0147
4/14/2014	0.0202	0.0176	0.0175	0.0174	0.0191	0.0141	0.0152
4/15/2014	0.0210	0.0176	0.0173	0.0173	0.0187	0.0140	0.0150

4. Conclusion

Based on the in sample data of return BMRI, BBCA and UNVR as case studies, the optimal weight of each asset can be determined using Lagrange Multiplier method for constructing portfolio return. The volatility of portfolio return can be predicted. The GARCH model can work well for forecasting volatility of BMRI, BBCA, UNVR and portfolio return.

Acknowledgement

We would like to give thank to Directorate of Research and Public Services, The Ministry of Research, Technology and Higher Education Republic of Indonesia for their support. This research was funded by "Fundamental" Research Grant 2014-2015.

References

[1] Engle, R.F., 1982, Autoregressive Conditional Heteroscedasticity with Estimates of Variance United Kingdom Inflation, Econometrica, Vol. 50, No.4., pp. 987-1007.

[2] Bollerslev, T., 1986, Generalized Autoregressive Conditional Heteroscedasticity, Journal of Econometrics, 31, 307-327.

[3] Morgan, J.P., 1996, Risk Metrics – Technical Document, J. P. Morgan Global Research Fourth Edition, Reuters.

[4] Danielsson, J., 2011, Financial Risk Forecasting, John Wiley and Sons, Ltd. United Kingdom.

[5] Tsay, R. S., 2005, Analysis of Financial Time Series, 2nd Edition, Wiley-Interscience, A John Wiley & Sons, Inc., Publication.

[6] Wei, W.W.S., 2006, Time Series Analysis: Univariate and Multivariate Methods, Second Edition, Pearson Education Inc. Boston.
[7] Zivot, E. and Wang, J., 2002, Modeling Financial Time Series with S-PLUS.
[8] Box, G.E.P. and Jenkins, G.M., 1978, Time Series Analysis: Forecasting and Control, 3rd Edition, Holden Day, San Francisco.
[9] Brockwell, P.J. and Davis, R.A., 1991, Time Series: theory and Methods, 2nd edition, New York: Springer Verlag.
[10] Makridakis, S., Wheelwright, S. C. and Hyndman, R.J., 1998, Forecasting: Methods and Applications, John Wiley & Sons Inc., New York.
[11] Engle, R.F. and Manganeli, S., 2001, Value at Risk Models in Finance, Working Paper Series No. 75 August 2001 European Central Bank. Germany.
[12] Holton, G., 2003, Value at Risk, Theory and Practice, Academic Press, Boston.
[13] Jorion, P., 2002, Value at Risk: The New Benchmarking for Managing Financial Risk, Mc Graw Hill.
[14] Ledenyov, D.O. and Ledenyov, V. O., 2012, On the Risk Management with Application of Econophysics Analysis in Central Banks and Financial Institutions, The Financial Times, The Bodley Head and The Random House first annual essay competitions, London, UK.
[15] Lopez, J.A., 1999, Evaluating the Predictive Accuracy of Volatility Models, Economics Research Department, Federal Reserve Bank of San Fransisco.