

Risk Estimation and Validation for Unconditional Stock Market Returns

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Abstract

Market risk analysis using Value-at-Risk (VaR) and conditional Value-at-Risk (CVaR) have become a popular concept in financial risk management nowadays. At the same time, statistical distributions also play a vital role in market risk analysis. This paper presents the concepts, methods and tools with the use of statistical distribution in risk estimation and validation. In this paper, we estimate the unconditional stock market returns distribution, Value-at-Risk (VaR) and conditional Value-at-Risk (CVaR) for monthly rates of returns for FTSE Bursa Malaysia Hijrah Shariah Index, FTSE Bursa Malaysia EMAS Shariah Index and FTSE Bursa Malaysia ACE Index. First, we present the application in empirical finance where we fit our real data based on its best-fitting distribution. Next, we present the application of risk analysis where we apply the best-fitting distribution to estimate the Value-at-Risk (VaR) and conditional Value-at-Risk (CVaR). Lastly, we evaluate the model validation for both risk measures.

Keywords: Market behavior; Market risk; Risk management; Statistical distribution

1. Introduction

Due to rapid globalization, financial risk management analysis has become a relatively important discipline and an increasingly vital aspect to both financial institutions and practitioners. The increased focus on financial risk management analysis thus led to the development of various techniques, methods and tools to measure the risks. There are several types of risk in financial markets, for example, market risk. Market risk are risks due to changing markets, market prices and the uncertainty of future returns due to fluctuations of financial asset quantities such as stock prices.

One of the fundamental issues in the financial risk management analysis is to characterize the returns distribution. A good approximation for the unconditional returns distribution is very significant for risk construction. Therefore, the first step in financial risk management analysis is to find a suitable model for asset returns. Once the returns are successfully modeled, hence, the risk measures can be constructed. Thus, specification of the distribution function is very crucial and plays a vital role to measure risk accurately.

Note that risk assessment tools at financial institutions strongly rely on the shape of a return distribution. A serious estimation bias could happen when one ally with a wrongly distributional assumptions. Imagine the hazardous if we apply it to risk measurement and risk management. For example, it might lead to a hefty loss in capital management. Also, one tends to seriously underestimate or perhaps overestimate the actual risk.

Thus, in this paper, first, we estimate the unconditional stock market returns distribution for three Bursa Malaysia monthly series, where we fit our real data based on its best-fitting distribution. We then provide empirical computation of Value-at-Risk (VaR) and conditional Value-at-Risk (CVaR) using the best-fitting distribution for Malaysia stock returns. Next, we perform model validation for both risk measures.

2. Stylized Facts of Malaysia Stock Market Returns

In this section, we present the stylized facts of three FTSE Bursa Malaysia Index Series. The index description, index identifier and starting date of each index are summarized in Table 1. The data are obtained from DATASTREAM database.

Table 1. List of FTSE Bursa Malaysia Index

Index description	Index identifier	Period
A. Tradable Index		
1. FTSE Bursa Malaysia Hijrah Shariah Index	FBMHS	Feb 2007 – July 2012
B. Benchmark Index		
1. FTSE Bursa Malaysia EMAS Shariah Index	FBMS	Oct 2006 – July 2012
2. FTSE Bursa Malaysia ACE Index	FBMMES	July 2007 – July 2012

Notes:

- FTSE Bursa Malaysia Hijrah Shariah Index: This index comprises the 30 largest Shariah-compliant companies in FTSE Bursa Malaysia Emas Index

screened by Yasaar Ltd and the Securities Commission's Shariah Advisory Council (SAC).

- FTSE Bursa Malaysia EMAS Shariah Index: This index comprises the Shariah-compliant constituents of the FTSE Bursa Malaysia Emas Index that meet the screening requirements of the Securities Commission's Shariah Advisory Council (SAC).
- FTSE Bursa Malaysia ACE Index: This index comprises all the companies listed on the ACE Market.

The FTSE Bursa Malaysia Index Series was officially launched on 26 June 2006 (<http://www.bursamalaysia.com>). We use one tradable index namely the FTSE Bursa Malaysia Hijrah Shariah Index; and two benchmark indices which are FTSE Bursa Malaysia EMAS Shariah Index and FTSE Bursa Malaysia ACE Index. All indices are based on the Main Market. The series are denominated in Malaysian Ringgit (MYR). Monthly series are used in this study. Prior to analysis, the series are analyzed in returns, which is the first difference of natural logarithms multiplied by 100 over the whole period. This is done to express things in percentage terms.

Table 2 reports the descriptive statistics for the monthly stock price indices over the sample period. All three indices are negatively skewed and have positive excess kurtosis except for the FTSE Bursa Malaysia ACE Index. Only FTSE Bursa Malaysia ACE Index confirms normality assumption.

Table 2. Statistical properties of Malaysia stock price indices

Statistics	FBMHS	FBMS	FBMMES
N	64	68	59
Mean	0.7081	0.7434	-0.5512
Median	1.5734	1.4448	-0.3938
Maximum	16.8037	15.2228	16.0750
Minimum	-19.6753	-20.0504	-20.5718
Std. Dev.	5.6573	5.7194	6.4789
Skewness	-0.7471	-0.8794	-0.1454
Kurtosis	5.7956	5.4062	3.5938
Jarque-Bera	26.7939	25.1692	1.0749
Probability	0.0000	0.0000	0.5842

Notes:

FBMHS: FTSE Bursa Malaysia Hijrah Shariah Index

FBMS: FTSE Bursa Malaysia EMAS Shariah Index

FBMMES: FTSE Bursa Malaysia ACE Index

3. Distributional Fitting

In this section, we present the distributional fitting for all three indices. The statistical ranking method used in this routine is the Kolmogorov-Smirnov test.

Table 3 reports the distributional fitting results. The results also show the

Kolmogorov-Smirnov test statistics and p-values of the best-fitting distribution of three FTSE Bursa Malaysia indices where the higher the p-value, the better the distribution fits the data. From Table 3, Normal mixture distribution is the best-fitting distribution for FTSE Bursa Malaysia Hijrah Shariah Index and FTSE Bursa Malaysia EMAS Shariah Index. Meanwhile, Normal distribution is the best-fitting distribution for FTSE Bursa Malaysia ACE Index.

Table 3. Distributional fitting results

Indices	Distribution	KS test statistic	P-value
FBMHS	Normal mixture	0.0516	0.9923
FBMS	Normal mixture	0.0556	0.9770
FBMMES	Normal	0.0500	0.9980

Notes:

FBMHS: FTSE Bursa Malaysia Hijrah Shariah Index

FBMS: FTSE Bursa Malaysia EMAS Shariah Index

FBMMES: FTSE Bursa Malaysia ACE Index

Table 4 reports number of components, k , in the mixture. We perform cross-validation to confirm the number of components for the Normal mixture distribution. What we do is a simple data-set splitting, where a randomly-selected half of the data is used to fit the model, and another half to test. The basic idea is to split a data set into train and test. We fit the model using the training points, and then calculate the log-likelihood of the test points under the model. We pick the number of component which maximizes the likelihood of the data. The boldface entry in Table 4 confirms the two-component in the mixture for FTSE Bursa Malaysia Hijrah Shariah Index and FTSE Bursa Malaysia EMAS Shariah Index.

Table 4. Log-likelihoods of the 10 fold cross-validation for Normal mixture distribution

k	FBMHS	FBMS
1	-98.2017	-114.0325
2	-94.9708	-111.3669
3	-97.7560	-113.8726
4	-98.7322	-125.7174
5	-95.0923	-118.4525
6	-98.7117	-135.2373
7	-100.3097	-134.3250
8	-100.5387	-144.9777
9	-99.5476	-144.2491
10	-106.3629	-123.1648

Notes:

k is the number of components

FBMHS: FTSE Bursa Malaysia Hijrah Shariah Index

FBMS: FTSE Bursa Malaysia EMAS Shariah Index

Meanwhile, for parameter estimation of Normal mixture distribution, we use the maximum likelihood method via the EM algorithm as introduced by [1]. The results are summarized in Table 5. The low-variance component has a higher probability to occur for FTSE Bursa Malaysia EMAS Shariah Index. However, it is vice versa for FTSE Bursa Malaysia Hijrah Shariah Index. The FTSE Bursa Malaysia EMAS Shariah Index indicates that the first normal is a low mean high variance regime while the second normal is a high mean low variance regime. The FTSE Hijrah Shariah Index indicates that the first normal is a low mean low variance regime. Meanwhile, the weights indicate that the second regime is more prevalent regime for both indices.

Table 5. Summary of five parameters two-component Normal mixture distributions model using EM algorithm

Indices	Estimation Method	Statistics	π_1	π_2	μ_1	μ_2	σ_1^2	σ_2^2
FBMHS	Maximum Likelihood Estimation	Estimate	0.0542	0.9458	-14.1710	1.5609	16.3816	18.9557
	Estimate log-likelihood: -196.1613	Std Err.	32.1024		0.3702	1.7367	0.0644	0.2673
		Ratio	0.0017		-38.2793	0.8988	254.3727	70.9155
		95% LCL	-0.0231		-21.7681	0.3363	-23.9713	10.7323
		95% UCL	0.1315		-6.5740	2.7856	56.7345	27.1790
FBMS	Maximum Likelihood Estimation	Estimate	0.0694	0.9306	-12.8641	1.7576	21.7568	18.1808
	Estimate log-likelihood: -209.0633	Std Err.	28.6035		0.3682	1.7912	0.0544	0.2773
		Ratio	0.0024		-34.9378	0.9812	399.9412	65.5636
		95% LCL	-0.0264		-21.2779	0.5218	-28.2487	10.0970
		95% UCL	0.1652		-4.4504	2.9935	71.7622	26.2646

Notes:

FBMHS: FTSE Bursa Malaysia Hijrah Shariah Index

FBMS: FTSE Bursa Malaysia EMAS Shariah Index

4. Risk Analysis

In this section, we present the application in risk analysis where we provide empirical computation of Value-at-Risk (VaR) and conditional Value-at-Risk (CVaR) using the best-fitting distribution for Malaysia stock returns.

Value-at-Risk (VaR) is the anticipated loss from an adverse market movement with a specified probability over a particular period of time. It is a loss that we are fairly sure will not be exceeded if the current portfolio is held over some period of time [2]. Meanwhile, conditional Value-at-Risk (CVaR) can be interpreted as the expected loss (in present value terms) given that the loss exceeds the value at risk [2].

As a risk measure, Value-at-Risk (VaR) has its drawbacks [2]. Value-at-Risk (VaR) does not measure the extent of exceptional loses where it states a level of loss that we are reasonably sure will not be exceeded. It tells us nothing about how much could be lost if Value-at-Risk (VaR) is exceeded. The conditional Value-at-Risk (CVaR) risk metric is more informative than Value-at-Risk (VaR). It tells us how much we expect to lose, given that the Value-at-Risk (VaR) is exceeded. Conditional Value-at-Risk (CVaR) gives a fuller description of the risks of a portfolio than just reporting the Value-at-Risk (VaR) alone.

For Value-at-Risk (VaR), the formula for the $100\alpha\%$ h -month VaR, as a percentage of the portfolio value, when the portfolio's returns are i.i.d. normally distributed with expectation μ_h and standard deviation σ_h is as follows

$$VaR_{h,\alpha} = \Phi^{-1}(1-\alpha)\sigma_h - \mu_h \quad (4.1)$$

where Φ is the standard Normal distribution function.

Normal mixture distribution can be used to estimate Value-at-Risk (VaR), capturing both leptokurtosis and skewness in returns distribution. To estimate the Normal mixture Value-at-Risk (VaR), we can apply a numerical algorithm to solve the following expression

$$\pi P(Y_1 < (x_\alpha - \mu_1)\sigma_1^{-1}) + (1-\pi) P(Y_2 < (x_\alpha - \mu_2)\sigma_2^{-1}) = \alpha. \quad (4.2)$$

Since Y_i are Normal variables, we know its quantiles. Specifically, we know everything in the expression (probabilities, means and volatilities) except the mixture quantile, x_α . Hence, the mixture quantile can be backed out from (4.2) using an iterative approximation method [2]. Finally, we find the Normal mixture VaR by setting $VaR_\alpha = -x_\alpha$.

For conditional Value-at-Risk (CVaR), let the random variable X denote a portfolio's h -month return. If $X \sim N(\mu_h, \sigma_h^2)$ then

$$CVaR_{h,\alpha}(X) = \alpha^{-1} \varphi(\Phi^{-1}(\alpha)) \sigma_h - \mu_h \quad (4.3)$$

where φ and Φ denote the standard Normal density and distribution functions.

Hence, $\Phi^{-1}(\alpha)$ is the α quantile of the standard Normal distribution and $\varphi(\Phi^{-1}(\alpha))$ is the height of the standard Normal density at this point.

Meanwhile, the Normal mixture conditional Value-at-Risk (CVaR) is as follows

$$CVaR_\alpha = \alpha^{-1} \sum_{i=1}^2 (\pi_i \sigma_i \varphi(\sigma_i^{-1} x_\alpha)) - \sum_{i=1}^2 \pi_i \mu_i. \quad (4.4)$$

Table 6 reports the Value-at-Risk (VaR) and conditional Value-at-Risk (CVaR) for all three indices. These two risk measures are calculated using parametric linear model over a 1-month horizon, at the 0.1%, 1%, 5% and 10% significance levels.

Table 6. Value-at-Risk (VaR) and conditional Value-at-Risk (CVaR) estimates

Index	Distribution	Significance α	VaR (RM)	CVaR (RM)
FBMHS	Normal	0.1%	2,254.00	22,981.00
	mixture	1%	1,749.00	2,068.00
		5%	311.00	1,465.00
		10%	819.00	1,765.00
FBMS	Normal	0.1%	2,158.00	22,958.00
	mixture	1%	1,629.00	4,814.00
		5%	1,019.00	1,324.00
		10%	663.00	1,043.00
FBMMES	Normal	0.1%	938.50	988.60
		1%	714.22	782.52
		5%	514.13	605.63
		10%	407.46	515.27

Notes:

FBMHS: FTSE Bursa Malaysia Hijrah Shariah Index

FBMS: FTSE Bursa Malaysia EMAS Shariah Index

FBMMES: FTSE Bursa Malaysia ACE Index

5. Risk Model Validation

In this section, we present approaches to Value-at-Risk (VaR) and conditional Value-at-Risk (CVaR) model validation through out-of-sample forecast evaluation techniques that are commonly termed as back-test. Note that, failure of a back-test indicates model misspecification and/or there are large estimation errors. We back-test our risk models using the [3] test for unconditional coverage and a method for back-testing conditional Value-at-Risk (CVaR) due to [4].

Test for unconditional coverage test of the null hypothesis that the actual number of violations is equal to the expected number of violations. While back-testing conditional Value-at-Risk (CVaR) test the null hypothesis that the conditional Value-at-Risk (CVaR) does not consistently underestimate the true potential for losses beyond the Value-at-Risk (VaR). At the end of each risk horizon, we calculate the actual profits/losses for the stocks. An exceedance occurs if the loss is greater than the estimate Value-at-Risk (VaR) for the risk horizon. These exceedances are the inputs to the standard tests of unconditional coverage and back-testing conditional Value-at-Risk (CVaR).

An unconditional coverage test, introduced by [3], is a test of the null hypothesis that the indicator function, which follow an iid Bernoulli process, has a constant success probability equal to the significance level of the Value-at-Risk (VaR), α . The test statistics for unconditional coverage is a likelihood ratio statistic given by

$$LR = \frac{\pi_{\text{exp}}^{n_1} (1 - \pi_{\text{exp}})^{n_0}}{\pi_{\text{obs}}^{n_1} (1 - \pi_{\text{obs}})^{n_0}}; \quad -2 \ln LR \sim \chi_1^2. \quad (5.1)$$

where π_{exp} : the expected proportion of exceedances; π_{obs} : the observed number of exceedances (that lie in the prescribed interval of the distribution); n_1 : the observed number of exceedances (the number of violations: the number of returns that lie inside the interval); n : the sample size of the back-test; n_0 : the number of returns with indicator 0 (good returns: the number of returns that lie outside the interval). Note that

$n_0 = n - n_1$, $\pi_{\text{exp}} = \alpha$ and $\pi_{\text{obs}} = \frac{n_1}{n}$. The asymptotic distribution of $-2 \ln LR$ is chi-squared with one degree of freedom.

Table 7 reports the unconditional coverage test for the series. We perform an unconditional coverage test on the 0.1%, 1%, 5% and 10% monthly Value-at-Risk (VaR). The back-test is based on 50 observations. We fail to reject at the 1%, 5% and 10% significance level, the null hypothesis that Normal mixture Value-at-Risk (NM-VaR) and Normal VaR (N-VaR) model for all three series. For FTSE Bursa Malaysia Hijrah Shariah Index, we fail to reject at the 0.1% significance level, the null hypothesis that Normal mixture Value-at-Risk (NM-VaR) model is accurate in the sense that the total number of exceedances is close to the expected number.

Table 7. Unconditional coverage test

Index	Statistics	Significance level			
		0.1%	1%	5%	10%
	n	50	50	50	50
FBMHS	n_0	49	48	47	45
(Normal	n_1	1	2	3	5
mixture	π_{exp}	0.1%	1%	5%	10%
distribution)	π_{obs}	2%	4%	6%	10%
	$\ln(LR)$	-2.0548	-1.2955	-0.0496	0.0000
	Test statistics	4.1096	2.5911	0.0992	0.0000
FBMS	n_0	48	48	45	44
(Normal	n_1	2	2	5	6
mixture	π_{exp}	01%	1%	5%	10%
distribution)	π_{obs}	4%	4%	10%	12%
	$\ln(LR)$	-5.4663	-1.2955	-1.0327	-0.1051
	Test statistics	10.9327	2.5911	2.0654	0.2102
FBMMES	n_0	50	50	47	45
(Normal	n_1	0	0	3	5
distribution)	π_{exp}	0.1%	1%	5%	10%
	π_{obs}	0%	0%	6%	10%
	$\ln(LR)$	-	-	-0.0496	0.0000

	Test statistics	-	-	0.0992	0.0000
	$\chi^2_{\alpha,1}$	10.8276	6.6349	3.8415	2.7055

Notes:

FBMHS: FTSE Bursa Malaysia Hijrah Shariah Index

FBMS: FTSE Bursa Malaysia EMAS Shariah Index

FBMMES: FTSE Bursa Malaysia ACE Index

Since the Normal linear Value-at-Risk (VaR) model is the best risk model for FTSE Bursa Malaysia ACE Index, we perform the bias test for validating this risk model. An approximate 99.9%, 99%, 95% and 90% confidence interval for the bias parameter is calculated. We use the maximum number of observations in the back-test, $T = 50$, hence $\sqrt{2T} = \sqrt{2*50} = 10$. Hence,

1. the approximate 99.9% confidence interval is
 $(1 - 3.89/10, 1 + 3.89/10) = (0.6109, 1.3891)$
2. the approximate 99% confidence interval is
 $(1 - 2.58/10, 1 + 2.58/10) = (0.7424, 1.2576)$
3. the approximate 99% confidence interval is
 $(1 - 1.96/10, 1 + 1.96/10) = (0.8040, 1.1960)$
4. the approximate 99% confidence interval is
 $(1 - 1.64/10, 1 + 1.64/10) = (0.8355, 1.1645)$

For this test, the bias statistic \hat{b} is the standard deviation for standardized FTSE Bursa Malaysia ACE returns. In this case, $\hat{b} = 1.0716$. According to [2], if we obtain a value \hat{b} that lies below this interval the model may be under-predicting Value-at-Risk (VaR), and if \hat{b} lies above this interval it could indicate that the model over-predicts Value-at-Risk (VaR). From the above results, \hat{b} value lies in the intervals for all four approximate confidence interval, meaning that the Normal linear Value-at-Risk (VaR) model is the best risk model for FTSE Bursa Malaysia ACE Index.

[4] develop a methodology for back-testing conditional Value-at-Risk (CVaR) that is based on a time series of standardized exceedence residuals, defined as

$$\varepsilon_{t+1} = \begin{cases} \frac{-Y_{t+1} - CVaR_{1,\alpha,t}}{\hat{\sigma}_t}, & \text{if } Y_{t+1} < -VaR_{1,\alpha,t}, \\ 0, & \text{otherwise} \end{cases} \quad (5.2)$$

where $\hat{\sigma}_t$ is the forecast of the standard deviation of the return from time t to time $t+1$. So, the $\hat{\sigma}_t$ is the forecast that is made at time t .

The test is based on the observation that, if the process dynamics are correct and conditional Value-at-Risk (CVaR) is an unbiased estimate of the expectation in the tail below the Value-at-Risk (VaR), the standardized exceedance residuals should behave as a sample from an iid zero mean process. The null hypothesis is that ε_t has

zero mean, against the alternative that the mean is positive. A positive mean suggests that the conditional Value-at-Risk (CVaR) is too low, and underestimation of the conditional Value-at-Risk (CVaR) is what we want to guard against. So the test statistic is

$$t = \frac{\bar{\varepsilon}}{\text{est se}(\bar{\varepsilon})}. \quad (5.3)$$

here denoted t because it looks like a standard t ratio, where $\bar{\varepsilon}$ denotes the sample mean of the standardized exceedance residuals. The distribution of the test statistic is found using the standard bootstrap simulation introduced by [5].

Table 8 reports the exceedances and t ratio on standardized exceedance residuals. It summarizes the results, including the values of the conditional Value-at-Risk (CVaR) t statistics. We only use those dates for which the Value-at-Risk (VaR) is exceeded in the time series while the other observations are simply excluded.

Table 8. Standardized exceedance residuals

Index	Statistics	Significance level			
		0.1%	1%	5%	10%
	Exp num of exceedances	0.05	0.5	2.5	5.0
FBMHS	Num of exceedances	1	2	3	5
(Normal	Mean SER	-5.6784	-0.9167	-0.2884	-0.1803
mixture	Std dev SER	-	1.3572	1.3245	0.9547
distribution)	t	-	-0.6755	-0.2177	-0.1889
FBMS	Num of exceedances	2	2	5	6
(Normal	Mean SER	-26.8652	-2.0436	-0.1214	0.0497
mixture	Std dev SER	4.8237	1.0430	1.2173	1.0553
distribution)	t	-5.5694	-1.9593	-0.0997	0.0471
FBMMES	Num of exceedances	0	0	3	5
(Normal	Mean SER	-	-	-0.1450	-0.0859
distribution)	Std dev SER	-	-	0.2356	0.3650
	t	-	-	-0.6154	-0.2353

Notes:

FBMHS: FTSE Bursa Malaysia Hijrah Shariah Index

FBMS: FTSE Bursa Malaysia EMAS Shariah Index

FBMMES: FTSE Bursa Malaysia ACE Index

6. Conclusion

In this paper, we estimate the best-fitting distribution for returns and applied it in financial risk management analysis. We estimate the unconditional returns distribution, estimate and evaluate Value-at-Risk (VaR) and conditional Value-at-Risk (CVaR) for monthly series of FTSE Bursa Malaysia Hijrah Shariah Index, FTSE Bursa Malaysia EMAS Shariah Index and FTSE Bursa Malaysia ACE Index. We conclude that two-component Normal mixture distribution is the best-fitting

distribution for FTSE Bursa Malaysia Hijrah Shariah Index and FTSE Bursa Malaysia EMAS Shariah Index. Meanwhile, Normal distribution is the best-fitting distribution for FTSE Bursa Malaysia ACE Index. Next, we apply the best-fitting distributions model to estimate and evaluate the Value-at-Risk (VaR) and conditional Value-at-Risk (CVaR). We perform model validation for both risk measures using unconditional coverage test and back-testing conditional value at risk for model evaluation. We may conclude from the above analysis that using the Normal distribution can fit the monthly series of FTSE Bursa Malaysia ACE Index well. Meanwhile, the two-component Normal mixture distribution model can fit the data (FTSE Bursa Malaysia Hijrah Shariah Index and FTSE Bursa Malaysia EMAS Shariah Index) well and can perform better in estimating both Value-at-Risk (VaR) and conditional Value-at-Risk (CVaR) where it can capture the stylized facts of non-normality and leptokurtosis in returns distribution.

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