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On the degenerate Frobenius-Genocchi polynomials

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Abstract

Kim (2015) constructed the degenerate Frobenius-Euler polynomials and numbers and studied some identities of these polynomials. In this paper, by the same motivation, we define the degenerate Frobenius-Genocchi polynomials and investigate some new and interesting properties of these polynomials.

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1. Introduction

As is well known, the Bernoulli polynomials are defined by the generating function to be

$$\frac{t}{e^t-1}e^{xt}=\sum_{n=0}^{\infty}B_n(x)\frac{t^n}{n!} \qquad \text{(see [3 − 5, 11, 13, 15])} \tag{1}$$

and the Euler polynomials are defined by the generating function to be

$$\frac{2}{e^t+1}e^{xt}=\sum_{n=0}^{\infty}E_n(x)\frac{t^n}{n!} \qquad \text{(see [5, 14, 16, 18, 19, 25, 26]).} \tag{2}$$

When $x=0$, $B_n=B_n(0)$ and $E_n=E_n(0)$ are called the Bernoulli numbers and the Euler numbers respectively.

We consider a finite sum as follows:

$$1^k+2^k+\cdots+n^k \qquad \text{for all } k\in\mathbb{N}. \tag{3}$$

We also note that

$$\begin{aligned}
\sum_{l=0}^{n}e^{lt} &= \frac{1}{e^t-1}\left(e^{(n+1)t}-1\right) &(4)\\
&= \frac{1}{t}\left(\frac{t}{e^t-1}e^{(n+1)t}-\frac{t}{e^t-1}\right) &(5)\\
&= \frac{1}{t}\left(\sum_{m=0}^{\infty}B_m(n+1)\frac{t^m}{m!}-\sum_{m=0}^{\infty}B_m\frac{t^m}{m!}\right) &(6)\\
&= \frac{1}{t}\sum_{m=1}^{\infty}(B_m(n+1)-B_m)\frac{t^m}{m!} &(7)\\
&= \sum_{m=0}^{\infty}\frac{B_{m+1}(n+1)-B_{m+1}}{m+1}\frac{t^m}{m!} &(8)
\end{aligned}$$

and

$$\sum_{l=0}^{n}e^{lt}=\sum_{m=0}^{\infty}\left(\sum_{m=0}^{n}l^m\right)\frac{t^m}{m!}. \tag{9}$$

From (4) and (5), we obtain the following theorem.

Theorem 1.1. Let $n,m\in\mathbb{N}\cup\{0\}$. Then we have

$$\sum_{m=0}^{n}l^m=\frac{B_{m+1}(n+1)-B_{m+1}}{m+1}. \tag{10}$$

In order to calculate $\sum_{l=0}^{m}(-1)^l l^n$, we first note that

$$\begin{aligned}
2\sum_{l=0}^{n}(-1)^l e^{lt} &= \frac{2}{e^t+1}\left((-1)^n e^{(n+1)t}+1\right) &(11)\\
&= \frac{2}{e^t+1}(-1)^n e^{(n+1)t}+\frac{2}{e^t+1} &(12)\\
&= (-1)^n\sum_{m=0}^{\infty}E_m(n+1)\frac{t^m}{m!}+\sum_{m=0}^{\infty}E_m\frac{t^m}{m!} &(13)\\
&= \sum_{m=0}^{\infty}\left(\{(-1)^n E_m(n+1)+E_m\right\}\frac{t^m}{m!}. &(14)
\end{aligned}$$

From (5) and (7), we obtain the following theorem.

Theorem 1.2. Let $n, m \in \mathbb{N}\cup\{0\}$. Then we have

$$\begin{aligned}
2\sum_{l=0}^{n}(-1)^l e^{lt} &= (-1)^m E_m(n+1)+E_m &(15)\\
&= \begin{cases} E_m(n+1)+E_m & \text{if } n\equiv 1 \pmod 2\\ E_m-E_m(n+1) & \text{if } n\equiv 0 \pmod 2).\end{cases} &(16)
\end{aligned}$$

By (8), we get

$$\begin{aligned}
2 &= \left(\sum_{n=0}^{\infty}E_n\frac{t^n}{n!}\right)\left(e^t+1\right) &(17)\\
&= \left(\sum_{n=0}^{\infty}E_n\frac{t^n}{n!}\right)\left(\sum_{l=0}^{\infty}\frac{t^l}{l!}+1\right) &(18)\\
&= \sum_{m=0}^{\infty}\left(\sum_{n=0}^{m}\binom{m}{n}E_m+E_n\right)\frac{t^n}{n!}. &(19)
\end{aligned}$$

From (9), we obtain the following theorem.

Theorem 1.3. Let $n, m \in \mathbb{N}\cup\{0\}$. Then we have

$$\sum_{m=0}^{n}\binom{m}{n}E_m(n+1)+E_n=\begin{cases}2 & \text{if } n=0\\ 0 & \text{if } n>0.\end{cases} \tag{20}$$

By (10), we calculate the followings:

$$E_0=1,\ E_1=-\frac{1}{2},\ E_2=0,\ E_3=\frac{1}{4},\ E_4=0,\cdots \tag{21}$$

From (11), we see that for $n \in \mathbb{N} \cup \{0\}$

$$E_{2n} = 0 \text{ and } E_{2n+1} \in \mathbb{Q}. \tag{22}$$

Recall that the Genocchi numbers are defined by the generating function to be

$$\frac{2t}{e^t + 1} = \sum_{n=0}^{\infty} G_n \frac{t^n}{n!} \qquad \text{(see [1])} \tag{23}$$

By (13), we get

$$G_0 = 0 \tag{24}$$

and

$$\begin{aligned} 2t &= \left(\sum_{n=0}^{\infty} G_n \frac{t^n}{n!}\right)\left(e^t + 1\right) && (25) \\ &= \left(\sum_{n=0}^{\infty} G_n \frac{t^n}{n!}\right)\left(\sum_{l=0}^{\infty} \frac{t^l}{l!} + 1\right) && (26) \\ &= \sum_{m=0}^{\infty} \left(\sum_{n=0}^{m} \binom{m}{n} G_m + G_n\right) \frac{t^n}{n!}. && (27) \end{aligned}$$

From (14) and (15), we obtain the following theorem.

Theorem 1.4. Let $m \in \mathbb{N}$. Then we have

$$G_0 = 0$$

and

$$\sum_{m=0}^{n} \binom{m}{n} G_m(n+1) + G_n = \begin{cases} 2 & \text{if } n = 1 \\ 0 & \text{if } n > 1. \end{cases} \tag{28}$$

By (16), we calculate the followings:

$$G_1 = 1, \; G_2 = -1, \; G_3 = 0, \; G_4 = 1, \cdots \tag{29}$$

From (17), we see that for $n \in \mathbb{N}$

$$G_{2n+1} = 0 \text{ and } G_{2n} \in \mathbb{Z}. \tag{30}$$

From (12) and (18), we see that

$$\frac{2}{e^t + 1} e^{xt} = \sum_{n=0}^{\infty} E_n(x) \frac{t^n}{n!} \in \mathbb{Q}([t]) \tag{31}$$

and

$$\frac{2t}{e^t+1}e^{xt}=\sum_{n=0}^{\infty}G_n(x)\frac{t^n}{n!}\in\mathbb{Z}([t]), \tag{32}$$

where $R([t])$ is the set of all polynomials in an indeterminate t with coefficients in a ring F and F is either $\mathbb{Q}$ or $\mathbb{Z}$.

For $u\in\mathbb{C}$ with $u\neq 1$, the Frobenius-Genocchi polynomials are defined by the generating function to be

$$\frac{(1-u)t}{e^t-u}e^{xt}=\sum_{n=0}^{\infty}GH_n(x|u)\frac{t^n}{n!}. \tag{33}$$

When $x=0$, $GH_n(u)=GH_n(0|u)$ are called the Frobenius-Genocchi numbers. In particular, if $u=-1$, we have

$$\sum_{n=0}^{\infty}GH_n(x|-1)\frac{t^n}{n!} = \frac{2t}{e^t+1}e^{xt} \tag{34}$$

$$= \sum_{n=0}^{\infty}G_n(x)\frac{t^n}{n!}, \tag{35}$$

where $G_n(x)$ are the Genocchi polynomials. In fact, $GH_n(x|-1)=G_n(x)$ for all $n\in\mathbb{N}\cup\{0\}$. We observe that

$$\frac{1-u}{e^t-u}e^{xt}=\sum_{n=0}^{\infty}H_n(x|u)\frac{t^n}{n!}. \tag{36}$$

where $H_n(x|u)$ are the Frobenius-Euler polynomials (see [21]). When $x=0$, $H_n(u)=H_n(0|u)$ are called the Frobenius-Euler numbers.

In recent years, many researchers have studied various types of special polynomials, for examples, Barnes-type degenerate Euler polynomials, the degenerate Frobenius-Euler polynomials, Daehee polynomials, Changhee polynomials, and Boole polynomials etc. (see [1,6-10, 17, 20-24]). Thus, our motivation in this paper is to define the degenerate Frobenius-Genocchi polynomials and to investigate some new and interesting properties of these polynomials.

2. Properties of Frobenius-Genocchi polynomials

We note that the Stirling number of the first kind is defined as

$$(x)_n=\sum_{l=0}^{n}S_1(n,l)x^l, \qquad (n\geq 0), \tag{37}$$

where $(x)_n = x(x-1)\cdots(x-n+1)$ and $(x)_0 = 1$, and the Stirling number of the second kind is defined as

$$(e^t - 1)^n = n!\sum_{l=n}^{n} S_2(l,n)\frac{t^l}{l!}. \tag{38}$$

By (22) and (23), we get

$$\begin{aligned} t\sum_{n=0}^{\infty} H_n(x|u)\frac{t^n}{n!} &= \frac{(1-u)t}{e^t - u}e^{xt} \\ &= \sum_{n=0}^{\infty} GH_n(x|u)\frac{t^n}{n!}. \end{aligned} \tag{39}$$

By comparing coefficients on the both sides of (26), we get

$$GH_0(x|u) = 0. \tag{40}$$

By (26) and (27), we get

$$\sum_{n=0}^{\infty} H_n(x|u)\frac{t^n}{n!} = \frac{1}{t}\sum_{n=1}^{\infty} GH_n(x|u)\frac{t^n}{n!} \tag{41}$$

$$= \sum_{n=1}^{\infty} GH_n(x|u)\frac{t^{n-1}}{n!} \tag{42}$$

$$= \sum_{n=0}^{\infty} \frac{GH_{n+1}(x|u)}{n+1}\frac{t^n}{n!}. \tag{43}$$

From (28), we obtain the following theorem

Theorem 2.1. Let $n \in \mathbb{N} \cup \{0\}$ and $u \in \mathbb{C}$ with $u \neq 1$. Then we have

$$H_n(x|u) = \frac{GH_{n+1}(x|u)}{n+1}. \tag{44}$$

From (21), we note that

$$\frac{(1-u)t}{e^t - u} = \sum_{n=0}^{\infty} GH_n(u)\frac{t^n}{n!}. \tag{45}$$

By (30), we get

$$\begin{aligned}(1-u)t &= \left(\sum_{n=0}^{\infty} GH_n(u)\frac{t^n}{n!}\right)\left(e^t-u\right) &(46)\\ &= \left(\sum_{m=0}^{\infty} GH_m(u)\frac{t^m}{m!}\right)\left(\sum_{l=0}^{\infty}\frac{t^l}{l!}-u\right) &(47)\\ &= \sum_{n=0}^{\infty}\left(\sum_{m=0}^{n}\frac{GH_m(u)}{n-m}\right)\frac{t^n}{n!}-u\sum_{n=0}^{\infty}GH_n(u)\frac{t^n}{n!} &(48)\\ &= \sum_{n=0}^{\infty}\left(\sum_{m=0}^{n}\binom{n}{m}GH_m(u)-uGH_n(u)\right)\frac{t^n}{n!}. &(49)\end{aligned}$$

From (31), we obtain the following theorem.

Theorem 2.2. Let $n \in \mathbb{N}$ and $u \in \mathbb{C}$ with $u \neq 1$. Then we have

$$GH_0(u) = 0, \tag{50}$$

and

$$\sum_{m=0}^{n}\binom{n}{m}GH_m(u)-uGH_n(u) = (1-u)\delta_{1,n}, \tag{51}$$

where $\delta_{1,n}$ is the Kronecker's symbol.

By (21), we note that

$$\begin{aligned}\sum_{n=0}^{\infty} GH_n(x|u)\frac{t^n}{n!} &= \frac{(1-u)t}{e^t-u}e^{xt} &(52)\\ &= \left(\sum_{l=0}^{\infty} GH_l(u)\frac{t^l}{l!}\right)\left(\sum_{m=0}^{\infty}x^m\frac{t^m}{m!}\right) &(53)\\ &= \sum_{n=0}^{\infty}\left(\sum_{l=0}^{n}GH_l(u)\frac{x^{n-l}n!}{l!(n-l)!}\right)\frac{t^n}{n!} &(54)\\ &= \sum_{n=0}^{\infty}\left(\sum_{l=0}^{n}GH_l(u)\binom{n}{l}x^{n-l}\right)\frac{t^n}{n!}. &(55)\end{aligned}$$

From (34), we obtain the following theorem.

Theorem 2.3. Let $n \in \mathbb{N}\cup\{0\}$ and $u \in \mathbb{C}$ with $u \neq 1$. Then we have

$$GH_n(x|u) = \sum_{l=0}^{n} GH_l(u)\binom{n}{l}x^{n-l} = (GH(u)+x)^n, \tag{56}$$

with the usual convolution about replacing $GH^l(u)$ by $GH_l(u)$.

Note that if we take $u = -1$, by (35), we have

$$\begin{aligned} G_n(x) &= GH_n(x|-1) && (57)\\ &= \sum_{l=0}^{n} GH_l(-1)\binom{n}{l}x^{n-l} && (58)\\ &= \sum_{l=0}^{n} G_l\binom{n}{l}x^{n-l} && (59)\\ &= (G+x)^n && (60) \end{aligned}$$

with the usual convolution about replacing G^l by G_l.

3. Degenerate Frobenius-Genocchi polynomials

For $u \in \mathbb{C}$ with $u \neq 1$, we consider the degenerate Frobenius-Euler polynomials which are given by the generating function to be

$$\frac{1-u}{(1+\lambda t)^{\frac{1}{\lambda}} - u}(1+\lambda t)^{\frac{x}{\lambda}} = \sum_{n=0}^{\infty} h_{n,\lambda}(x|u)\frac{t^n}{n!} \qquad \text{(see [21]).} \tag{61}$$

In the reference ([21]), they obtained some interesting results of these polynomials. By the same motivation, we define the degenerate Frobenius-Genocchi polynomials which are given by the generating function to be

$$\frac{(1-u)t}{(1+\lambda t)^{\frac{1}{\lambda}} - u}(1+\lambda t)^{\frac{x}{\lambda}} = \sum_{n=0}^{\infty} Gh_{n,\lambda}(x|u)\frac{t^n}{n!}. \tag{62}$$

When $x = 0$, $Gh_{n,\lambda}(u) = Gh_{n,\lambda}(0|u)$ are called the degenerate Frobenious-Genocchi numbers. By (37), we get

$$\begin{aligned} \sum_{n=0}^{\infty} Gh_{n,\lambda}(x|u)\frac{t^n}{n!} &= t\frac{1-u}{(1+\lambda t)^{\frac{1}{\lambda}} - u}(1+\lambda t)^{\frac{x}{\lambda}} && (63)\\ &= t\sum_{n=0}^{\infty} h_{n,\lambda}(x|u)\frac{t^n}{n!} && (64)\\ &= \sum_{n=0}^{\infty} h_{n,\lambda}(x|u)\frac{t^{n+1}}{n!}. && (65) \end{aligned}$$

Thus, by comparing coefficients on the both sides (39), we get

$$Gh_{0,\lambda}(x|u) = 0 \tag{66}$$

and hence

$$\sum_{n=0}^{\infty} Gh_{n,\lambda}(x|u)\frac{t^n}{n!} = \sum_{n=1}^{\infty} Gh_{n,\lambda}(x|u)\frac{t^n}{n!} \tag{67}$$

$$= \sum_{n=0}^{\infty} \frac{Gh_{n+1,\lambda}(x|u)}{n+1}\frac{t^{n+1}}{n!}. \tag{68}$$

By (39) and (41), we get

$$\sum_{n=0}^{\infty} h_{n,\lambda}(x|u)\frac{t^n}{n!} = \frac{1}{t}\sum_{n=0}^{\infty} \frac{Gh_{n+1,\lambda}(x|u)}{n+1}\frac{t^{n+1}}{n!} \tag{69}$$

$$= \sum_{n=0}^{\infty} \frac{Gh_{n+1,\lambda}(x|u)}{n+1}\frac{t^{n}}{n!}. \tag{70}$$

By comparing of coefficients on the both sides of (42), we obtain the following theorem.

Theorem 3.1. Let $n \in \mathbb{N} \cup \{0\}$ and $u \in \mathbb{C}$ with $u \neq 1$. Then we have

$$h_{n,\lambda}(x|u) = \frac{Gh_{n+1,\lambda}(x|u)}{n+1}. \tag{71}$$

We note that

$$\left(\frac{x}{\lambda}\right)_m \lambda^m = \left(\frac{x}{\lambda}\right)\left(\frac{x}{\lambda}-1\right)\cdots\left(\frac{x}{\lambda}-(m-1)\right)\lambda^m \tag{72}$$

$$= (x|\lambda)_m, \tag{73}$$

where $(x|\lambda)_m = x(x-\lambda)\cdots(x-(m-1)\lambda)$ for all $m \in \mathbb{N}$. By (37), we get

$$\sum_{n=0}^{\infty} Gh_{n,\lambda}(x|u)\frac{t^n}{n!} \tag{74}$$

$$= t\frac{1-u}{(1+\lambda t)^{\frac{1}{\lambda}}-u}(1+\lambda t)^{\frac{x}{\lambda}} \tag{75}$$

$$= \left(\sum_{l=0}^{\infty} Gh_{l,\lambda}(u)\frac{t^l}{l!}\right)\left(\sum_{m=0}^{\infty}\binom{\frac{x}{\lambda}}{m}\lambda^{-m}t^m\right) \tag{76}$$

$$= \left(\sum_{l=0}^{\infty} Gh_{l,\lambda}(u)\frac{t^l}{l!}\right)\left(\sum_{m=0}^{\infty}\left(\frac{x}{\lambda}\right)_m\frac{1}{m!}\lambda^{-m}t^m\right) \tag{77}$$

$$= \sum_{n=0}^{\infty}\left(\sum_{l=0}^{n}\binom{n}{l}Gh_{l,\lambda}(u)(x|\lambda)_{n-l}\right)\frac{t^n}{n!}. \tag{78}$$

By comparing the coefficients on the both sides of (45), we obtain the following theorem.

Theorem 3.2. Let $n \in \mathbb{N} \cup \{0\}$ and $u \in \mathbb{C}$ with $u \neq 1$. Then we have

$$Gh_{n,\lambda}(x|u) = \sum_{l=0}^{n} \binom{n}{l} Gh_{l,\lambda}(u)(x|\lambda)_{n-l}. \tag{79}$$

Note that

$$t\frac{1-u}{(1+\lambda t)^{\frac{1}{\lambda}} - u} = \sum_{n=0}^{\infty} Gh_{n,\lambda}(u)\frac{t^n}{n!}. \tag{80}$$

By (47), we get

$$\begin{aligned}
t(1-u) &= \left(\sum_{n=0}^{\infty} Gh_{n,\lambda}(u)\frac{t^n}{n!}\right)\left((1+\lambda t)^{\frac{1}{\lambda}} - u\right) && (81)\\
&= \left(\sum_{m=0}^{\infty} Gh_{m,\lambda}(u)\frac{t^m}{m!}\right)\left(\sum_{l=0}^{\infty}(1|\lambda)_l\frac{t^l}{l!} - u\right) && (82)\\
&= \sum_{n=0}^{\infty}\left(\sum_{m=0}^{n} Gh_{m,\lambda}(u)(1|\lambda)_{n-m}\binom{n}{m}\right)\frac{t^n}{n!} - u\sum_{n=0}^{\infty} Gh_{n,\lambda}(u)\frac{t^n}{n!} && (83)\\
&= \sum_{n=0}^{\infty}\left(\sum_{l=0}^{n} Gh_{l,\lambda}(u)(1|\lambda)_{n-l}\binom{n}{l} - uGh_{n,\lambda}(u)\right)\frac{t^n}{n!}. && (84)
\end{aligned}$$

By comparing coefficients on the both sides of (48), we obtain the following theorem.

Theorem 3.3. Let $n \in \mathbb{N}$ and $u \in \mathbb{C}$ with $u \neq 1$. Then we have

$$Gh_{0,\lambda}(u) = 0 \tag{85}$$

and

$$\sum_{l=0}^{n} Gh_{l,\lambda}(u)(1|\lambda)_{n-l}\binom{n}{l} - uGh_{n,\lambda}(u) = (1-u)\delta_{1,n}, \tag{86}$$

where $\delta_{1,n}$ is the Kronecker's symbol.

By replacing t by $\frac{1}{lambda}(e^{\lambda t} - 1)$ in (37), we get

$$\frac{1}{\lambda}(e^{\lambda t} - 1)\frac{1-u}{e^t - u}e^{xt} = \sum_{n=0}^{\infty} Gh_{n,\lambda}(x|u)\frac{\left(\frac{1}{\lambda}(e^{\lambda t} - 1)\right)^n}{n!} \tag{87}$$

From (49), we note that

$$\begin{aligned} & \frac{1-u}{e^t-u}e^{xt} && (88)\\ =\ & \sum_{n=1}^{\infty} Gh_{n,\lambda}(x|u)\frac{\left(\frac{1}{\lambda}(e^{\lambda t}-1)\right)^{n-1}}{n!} && (89)\\ =\ & \sum_{n=0}^{\infty} \frac{Gh_{n+1,\lambda}(x|u)}{n+1}\frac{\left(\frac{1}{\lambda}(e^{\lambda t}-1)\right)^{n}}{n!} && (90)\\ =\ & \sum_{n=0}^{\infty} \frac{Gh_{n+1,\lambda}(x|u)}{n+1}\frac{1}{n!}n!\lambda^{-n}\sum_{m=n}^{\infty} S_2(m,n)\lambda^m\frac{t^m}{m!} && (91)\\ =\ & \sum_{m=0}^{\infty}\left(\sum_{n=0}^{m} \frac{Gh_{n+1,\lambda}(x|u)}{n+1}\lambda^{m-n}S_2(m,n)\right)\frac{t^m}{m!}. && (92) \end{aligned}$$

From (23) and (52), we obtain the following theorem.

Theorem 3.4. Let $n \in \mathbb{N}$ and $u \in \mathbb{C}$ with $u \neq 1$. Then we have

$$H_n(x|u) = \sum_{n=0}^{m} \frac{Gh_{n+1,\lambda}(x|u)}{n+1}\lambda^{m-n}S_2(m,n). \tag{93}$$

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