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A Deterministic Production Inventory Model For Buyer-Manufacturer With Quantity Discount And Completely Backlogged Shortages For Fixed Life Time Product

[1]M. Ravithammal [2]R. Uthayakumar [3]S. Ganesh

1. Research Scholar, Department of Mathematics, Sathyabama University, Chennai-600 119, Tamil Nadu, India.
2. Professor, Department of Mathematics, Gandhigram Rural Institute Deemed University, Dindigul-624 302. Tamil Nadu, India.
3. Professor, Department of Mathematics, Sathyabama University, Chennai-600 119, Tamil Nadu, India.
Email: 1. ravith_vahab1@yahoo.co.in, 2. uthayagri@gmail.com, 3. sganesh19@yahoo.com
Corresponding Author E-mail: ravith_vahab1@yahoo.co.in

ABSTRACT

This paper deals with a deterministic production inventory model for buyer-manufacturer with quantity discount for fixed life time product. Shortages are allowed in each cycle and backlogged them completely. The models with and without coordination are studied. The objective of the study is to find optimum multiples of order (m^*, n^*) so that the optimal savings percentage is maximized. In addition, the centralized decision-making model is determining the effectiveness of the proposed coordination quantity discount model. The model reveals that the benefit for the manufacturer is more than the benefit of the buyer with coordination strategy. It is proved that the quantity discount is the best strategy to achieve system optimization and win-win outcome. Finally, a numerical example is discussed to test the model which is illustrated graphically also.

KEY WORDS Production, Inventory, Quantity discount, completely backlogged shortages, Coordination, Fixed life time products

1. INTRODUCTION

The inventory strategies and cost benefits do vary with different business situations. The decision on inventory is very complicated in case of products with limited shelf life such as food products, medicines etc., A business scenario of a buyer purchasing fixed life time products from manufacturer is the focus of the research. Strategic decision making is vital to the business organizations as it involves huge cost and time. The decisions also affect the image of the organization. The optimum inventory strategy is a type of production strategy that is helpful for the manufacturers and buyer to improve their benefit position and used to derive an inventory plan. The competitive position of both the manufacturer and buyer improves the model suggested in the paper is followed.

Liu and Shi [15] classified perishability and deteriorating inventory models into two major categories namely decay models and finite life time models. The first model deals with the inventory that deteriorates and reduces in quantity continuously in proportion with time. The second model assumes a limited life time for each item. It is further classified into two subcategories namely fixed finite lifetime model and random finite lifetime model. Fixed life time items model deals with the perishable items while random life time model deals with probability distribution such as exponential and Erlang distribution. Fries [5] developed optimal order policies for a perishable commodity with fixed life time. Nandakumar and Morton [18] analyzed near myopic heuristic for the fixed life perishability problem. Liu and Lian [14] considered (s,S) inventory model with fixed life time. Lian and Liu [13] developed single stage inventory models for fixed life time perishable problem. An optimal and issuing policy for a two-stage inventory system for perishable products was performed by Fujiwara et al. [6]. Kanchana and Anulark [11] developed an approximate periodic model for fixed-life perishable product in a two-echelon inventory distribution system.

Manufacturer and buyer can cooperate by adopting new purchasing schemes when manufacturer is offering discount to the buyer**.** So in a coordination strategy quantity discount mechanisms are necessary to maximize the profit of the manufacturer. Goyal and Gupta [7] studied integrated inventory model for buyer-vendor coordination. Single supplier multiple cooperative retailers inventory model with quantity discount and permissible delay in payments was performed Saoussen Krichen et al. [20]. Mahdi Tajbakhsh et al. [17] developed an inventory model with random discount offering. Hung-Chi Chang [10] reviewed a comprehensive note on an economic order quantity with imperfect quality and quantity discounts.

Taft [22] was the first proposer who gave economic production quantity inventory model for a single product-single stage manufacturing system. The multi products-single manufacturing system was researched and derived by Eilon [4] and Rogers [19]. Bomberger [3], Madigan [16], Stankard and Gupta [21], Hodgson [9] and Baker [1] are the pioneers who handle the problems deal with multi products on a single a machine. In recent years, many researchers have investigated on economic production quantity inventory models. Kaj-Mikael Bjork [12] analyzed a multi item fuzzy economic production quantity problem with a finite production rate. Biswajit Sarkar and Ilkyeong Moon [2] had researched an EPQ model with inflation in an imperfect production system. Gede Agus widyadana and Hui Ming Wee [8] developed optimal deteriorating items production inventory models with random machine breakdown and stochastic repair time.

The proposed model deals with a deterministic production inventory model for buyer and manufacturer with the quantity discount and completely backlogged shortages for fixed life time products. If we ignore production and shortage then we get the model by Yongrui et al. [23] which is considered a particular case in our model. In the coordination strategy, the buyers cost is stagnant while manufacturer's cost is able to reduce. If buyer is benefited not only with quantity discount but also some percentage of shares from manufacturer's total cost. In order to help the buyer decision maker can involve in centralized model to share the profit between the manufacturer and the buyer.

The detailed description of this paper is as follows. In section 2, notations, assumptions, decentralized models with and without coordination and centralized models are given. Analytically easily understandable solutions are obtained in these models. It is proved that the quantity discount is the best strategy to achieve system optimization and win-win outcome. In section 3 a numerical example is given in detail to illustrate the models. Finally conclusion and summary are presented.

2. MODEL FORMULATION

In this section, decentralized models with coordination and without coordination are analyzed. In the coordination model quantity discount is offered by the manufacturer and a centralized decision-making scenario is also formulated. The following assumptions and notations are used throughout this paper to develop the proposed model.

2.1 Notations

D -Annual demand of the buyer

L -Life time of product

P -Production rate for manufacturer (P > D)

k_1, k_2 -Manufacturer and buyer's setup costs per order, respectively

h_1, h_2 -Manufacturer and buyer's holding costs, respectively

p_1, p_2 -Delivered unit price paid by the manufacturer and the buyer respectively

s_1, s_2 -Manufacturer and the buyer's shortage costs, respectively

Q_1 -Amount remains in the inventory after satisfying the shortage demand

Q_2 -Amount which is immediately taken to satisfy unfilled demand (Shortage period)

Q_0 -Buyer's EOQ such that $Q_0 = Q_1 + Q_2$

α -Manufacturer and buyer' negotiation percentage

m -Manufacturer order multiple in the absence of any coordination

n -Manufacturer order multiple under coordination

K -Buyer's order multiple under coordination. KQ_0 buyer's new order quantity

$d(K)$ -Denotes the per unit dollar discount to the buyer if he orders KQ_0 every time

t_1 -The time during which items are drawn from the inventory as needed

t_2 -The time during which demands are being accumulated but not filled (i.e., Shortage period)
t -The total time period such that $t = t_1 + t_2$

2.2 Assumptions

(1) Demand is constant
(2) Shortages are allowed and completely backlogged for both the manufacturer and buyer
(3) Manufacturer produces the product continuously until the buyer shortage completes, $s_2 \geq s_1$
(4) Production rate is greater than demand, $P > D$
(5) All items ordered by the manufacturer arrive fresh and new. i.e., their age equals zero.
(6) The buyer's holding cost is higher than $\left(\frac{P}{P-D}\right)$ times of manufacturer holding cost and the buyer's shortage cost is higher than $\left(\frac{P}{P-D}\right)$ times of manufacturer shortage cost. i.e., $h_2 \geq \left(\frac{P}{P-D}\right) h_1$ and $s_2 \geq \left(\frac{P}{P-D}\right) s_1$

2.3 Model formulation for the system without coordination

In the model without coordination, the buyer's annual average inventory holding cost is $\frac{h_2 Q_1^2}{2Q_0}$ and the annual shortage cost is $\frac{s_2 Q_2^2}{2Q_0}$.Therefore the buyer's total annual cost is defined as

$$TC_r = \frac{Dk_2}{mQ_0} + \frac{h_2 Q_1^2}{2Q_0} + \frac{s_2 Q_2^2}{2Q_0}$$
$$= \frac{Dk_2}{mQ_0} + \frac{h_2 Q_1^2}{2Q_0} + \frac{s_2 (Q_0-Q_1)^2}{2Q_0} \text{ (since } Q_2 = Q_0 - Q_1)$$

Differentiate partially with respect to Q_1 and Q_0 we get $Q_1 = \frac{s_2 Q_0}{h_2+s_2}$ and $Q_0 = \sqrt{2Dk_2}\sqrt{\frac{h_2+s_2}{h_2 s_2}}$.

Hence, the buyer's optimum annual cost $TC_r^* = \sqrt{2Dk_2h_2}\sqrt{\frac{s_2}{h_2+s_2}}$ and order quantity $Q_0 = \sqrt{2Dk_2}\sqrt{\frac{h_2+s_2}{h_2 s_2}}$. The manufacturer order size is mQ_0, since he faced with a constant demand at fixed intervals $t_0 = {Q_0}/{D} = \sqrt{\frac{2k_2}{Dh_2}}\sqrt{\frac{h_2+s_2}{s_2}}$.

In this case, the manufacturer's average inventory is $\frac{(m-1)Q_1+(m-2)Q_1+ \ldots + Q_1+ 0Q_1}{m}$ $= \frac{(m-1)Q_1}{2}$, the annual average inventory holding cost is $\frac{(m-1)h_1 Q_1^2}{2Q_0}\left(\frac{P}{P-D}\right)$ and the annual shortage cost is $\frac{(m-1)s_1 Q_2^2}{2Q_0}\left(\frac{P}{P-D}\right)$.

Therefore the total annual cost for the manufacturer is defined as

$$TC_v(m) = \frac{Dk_1}{mQ_0} + \frac{(m-1)h_1 Q_1^2}{2Q_0}\left(\frac{P}{P-D}\right) + \frac{(m-1)s_1 Q_2^2}{2Q_0}\left(\frac{P}{P-D}\right)$$

$$= \frac{Dk_1}{m\sqrt{2Dk_2}\sqrt{\frac{h_2+s_2}{h_2 s_2}}} + \frac{\left(\frac{P}{P-D}\right)(m-1)(s_2Q_0\ /\ (h_2+s_2))^2 h_1}{2Q_0} + \frac{\left(\frac{P}{P-D}\right)(m-1)s_1 (h_2Q_0\ /\ (h_2+s_2))^2}{2Q_0}$$

(since $Q_1 = \frac{s_2 Q_0}{h_2+s_2}$ and $Q_2 = Q_0 - Q_1$)

$$= \frac{k_1}{m}\sqrt{\frac{Dh_2 s_2}{2k_2 (h_2+s_2)}} + (m-1)\left(\frac{P}{P-D}\right)\frac{h_1 s_2^2 + s_1 h_2^2}{(h_2+s_2)^2}\sqrt{\frac{Dk_2 (h_2+s_2)}{2h_2 s_2}}$$

Now the manufacturer's problem without coordination can be formulated as follows

$$\min TC_v(m)$$

$$\text{s.t}\begin{cases} mt_0 \le L, \\ m \ge 1, \end{cases} \qquad (1)$$

here $mt_0 \le L$ indicates that items are not overdue before they are sold up by the buyer.

Theorem 1

Let m^* be the optimum of (1), $m^* \ge 1$. If $L^2 \ge \frac{2k_2 (h_2+s_2)}{Dh_2 s_2}$ then $m^* = \min\left\{\left\lceil \sqrt{\frac{k_1\left(1-\frac{D}{P}\right)(s_2 h_2^2 + h_2 s_2^2)}{k_2 (h_1 s_2^2 + s_1 h_2^2)} + \frac{1}{4}} - \frac{1}{2}\right\rceil, \left\lceil \frac{L}{\sqrt{\frac{2k_2}{Dh_2}}\sqrt{\frac{h_2+s_2}{s_2}}} \right\rceil\right\}$ where $\lceil x \rceil$ is the least integer greater than or equal to x.

Proof

$TC_v(m)$ is strictly convex in m because $\frac{d^2 TCv(m)}{dm^2} = \frac{2k_1}{m^3}\sqrt{\frac{2Dh_2 s_2}{k_2 (h_2+s_2)}} > 0$. Let m_1^* be the optimum of $\min TC_v(m)$, then $m_1^* = \max\{m_3, 1\}$

where $m_3 = \min\{m \mid TC_v(m) \le TC_v(m+1)\}$

$$= \min\left\{m \mid m(m+1) \ge \frac{k_1\left(1-\frac{D}{P}\right)h_2 s_2 (h_2+s_2)}{k_2(h_1 s_2^2 + s_1 h_2^2)}\right\}$$

$$= \left\lceil \sqrt{\frac{k_1\left(1-\frac{D}{P}\right)(s_2 h_2^2 + h_2 s_2^2)}{k_2 (h_1 s_2^2 + s_1 h_2^2)} + \frac{1}{4}} - \frac{1}{2}\right\rceil$$

Now $m_1^* = \max\{m_3, 1\}$

$$= \left\{\left\lceil \sqrt{\frac{k_1\left(1-\frac{D}{P}\right)(s_2 h_2^2 + h_2 s_2^2)}{k_2 (h_1 s_2^2 + s_1 h_2^2)} + \frac{1}{4}} - \frac{1}{2}\right\rceil, 1\right\}$$

$$= \left[\sqrt{\frac{k_1 \left(1-\frac{D}{P}\right)(s_2\, h_2^2 + h_2\, s_2^2)}{k_2\,(h_1\, s_2^2 + s_1\, h_2^2)} + \frac{1}{4}} - \frac{1}{2} \right] \geq 1 \text{ (since } m \geq 1\text{)}$$

Now put $t_0 = \sqrt{\frac{2k_2}{Dh_2}}\sqrt{\frac{h_2+s_2}{s_2}}$ into $mt_0 \leq L$ we get, $m\sqrt{\frac{2k_2}{Dh_2}}\sqrt{\frac{h_2+s_2}{s_2}} \leq L$.

Take $m_2^* = \frac{L}{\sqrt{\frac{2k_2}{Dh_2}}\sqrt{\frac{h_2+s_2}{s_2}}} \geq 1$, since $L^2 \geq \frac{2k_2\,(h_2+s_2)}{Dh_2 s_2}$.

Here $TC_v(m)$ is a convex function. If $m_1^* \leq m_2^*$ then $m^* = m_1^*$ else $m^* = m_2^*$.

Hence if $L^2 \geq \frac{2k_2\,(h_2+s_2)}{Dh_2 s_2}$, then

$$m^* = min\left\{ \left[\sqrt{\frac{k_1 \left(1-\frac{D}{P}\right)(s_2\, h_2^2 + h_2\, s_2^2)}{k_2\,(h_1\, s_2^2 + s_1\, h_2^2)} + \frac{1}{4}} - \frac{1}{2} \right], \left[\frac{L}{\sqrt{\frac{2k_2}{Dh_2}}\sqrt{\frac{h_2+s_2}{s_2}}} \right] \right\} \quad (2)$$

Remark 1: Without coordination strategy the manufacturer's optimized total cost is $TC_v(m^*)$, and place $\frac{D}{m^*\sqrt{2Dk_2}\sqrt{\frac{h_2+s_2}{h_2 s_2}}}$ orders each year with an interval $\frac{m^*\sqrt{2Dk_2}\sqrt{\frac{h_2+s_2}{h_2 s_2}}}{D}$.

Manufacturer order size is $m^*\sqrt{2Dk_2}\sqrt{\frac{h_2+s_2}{h_2 s_2}}$.

2.4 Model formulation for system with coordination

The model formulation is similar to Yongrui et al. [23] which is considered as a particular case in our model. If we ignore production and shortage in proposed model then we get the model by Yongrui et al. [23]. Based on quantity discount coordination strategy, the manufacturer asks the buyer to change his lot size by $KQ_0, (K > 0)$. If the buyer's new order is KQ_0 then the manufacturer offers quantity discount at a discount factor $d(K)$, then manufacturer order quantity is nKQ_0, where n > 0.

The total cost $TC_v(n)$ of the manufacturer contains the ordering cost $\frac{Dk_1}{nKQ_0}$, the inventory holding cost $\frac{(n-1)\left(\frac{P}{P-D}\right)h_1\, KQ_1^2}{2Q_0}$, the shortage $\frac{(n-1)\left(\frac{P}{P-D}\right)s_1\, KQ_2^2}{2Q_0}$ and the buyer's quantity discount $Dd(K)p_2$. Thus

$$TC_v(n) = \frac{Dk_1}{nKQ_0} + \frac{(n-1)\left(\frac{P}{P-D}\right)h_1\, KQ_1^2}{2Q_0} + \frac{(n-1)\left(\frac{P}{P-D}\right)s_1\, KQ_2^2}{2Q_0} + p_2 Dd(K) \quad (3)$$

The Manufacturer problem with coordination can be developed as follows

$$min\, TC_v(n)$$

$$\text{subject to} \begin{cases} nKt_0 \leq L, \\ \frac{Dk_2}{KQ_0} + \frac{Kh_2\, Q_1^2}{2Q_0} + \frac{Ks_2 Q_2^2}{2Q_0} - \sqrt{2Dk_2\, h_2}\sqrt{\frac{s_2}{h_2+s_2}} \leq p_2 Dd(K), \\ n \geq 1, \end{cases} \quad (4)$$

The first constraint of equation (4) indicates that, items are not overdue before they are sold and the second constraint indicates that, the total cost will be reduced to the coordination scheme.

Theorem 2

Let m^* be the optimum of (1), and n^* be the optimum of (4), then we have

$$TC_v(n^*) \leq TC_v(m^*) \tag{5}$$

Proof

The quantity discount factor $p_2Dd(K)$ is just the compensation to the buyer by the manufacturer which is a part of the manufacturer costs. $p_2Dd(K)$ takes smallest value only the second constraint of (4) is an equation. If the total cost of manufacturer under coordination is minimized, the above inequality must be an equation.

i.e., $\frac{Dk_2}{KQ_0} + \frac{Kh_2 Q_1^2}{2Q_0} + \frac{Ks_2Q_2^2}{2Q_0} - \sqrt{2Dk_2h_2}\sqrt{\frac{s_2}{h_2+s_2}} = p_2Dd(K)$

$$d(K) = \frac{\frac{Dk_2}{KQ_0} + \frac{Kh_2 Q_1^2}{2Q_0} + \frac{Ks_2Q_2^2}{2Q_0} - \sqrt{2Dk_2h_2}\sqrt{\frac{s_2}{h_2+s_2}}}{p_2 D} \tag{6}$$

If $K = 1$, then $d(1) = \frac{\sqrt{2Dk_2h_2}\sqrt{\frac{s_2}{h_2+s_2}} - \sqrt{2Dk_2h_2}\sqrt{\frac{s_2}{h_2+s_2}}}{p_2 D} = 0$

Thus, if $K = 1$, then (4) and (1) are same. i.e., (1) is special case of (4). Hence $TC_v(n^*) \leq TC_v(m^*)$

Remark 2: The above theorem concludes that the optimum total cost under coordination is less in comparison with absence of coordination, so the manufacturer will gain, if the buyer orders KQ_0 every time.

Now replace the value of $d(K)$ in $TC_v(n)$

$$TC_v(n) = \frac{Dk_1}{nKQ_0} + \frac{(n-1)\left(\frac{P}{P-D}\right)h_1 KQ_1^2}{2Q_0} + \frac{(n-1)\left(\frac{P}{P-D}\right)s_1 KQ_2^2}{2Q_0} + \frac{Dk_2}{KQ_0} + \frac{Kh_2 Q_1^2}{2Q_0} + \frac{Ks_2Q_2^2}{2Q_0} - \sqrt{2Dk_2h_2}\sqrt{\frac{s_2}{h_2 + s_2}}$$

$$= \frac{D}{KQ_0}\left(\frac{k_1}{n} + k_2\right) + \frac{K}{2}\left(\frac{(n-1)\left(\frac{P}{P-D}\right)s_2^2Q_0h_1}{(h_2+s_2)^2} + \frac{s_2^2Q_0h_2}{(h_2+s_2)^2}\right) + \frac{K}{2}\left(\frac{(n-1)\left(\frac{P}{P-D}\right)h_2^2Q_0s_1}{(h_2+s_2)^2} + \frac{h_2^2Q_0s_2}{(h_2+s_2)^2}\right) - \sqrt{2Dk_2h_2}\sqrt{\frac{s_2}{h_2+s_2}} \tag{7}$$

Let K^* be the minimum of $TC_v(n)$. For optimality $\frac{dTC_v(n)}{dK} = 0$

$$\text{Therefore } K^*(n) = \frac{h_2+s_2}{Q_0}\sqrt{\frac{2D(\frac{k_1}{n} + k_2)}{\left(\frac{P}{P-D}\right)(n-1)(s_2^2+s_1 h_2^2) + h_2s_2(h_2+s_2)}} \tag{8}$$

Now $nKt_0 \leq L$ we have, $\left(\frac{k_1}{n} + k_2\right) n^2 \leq L^2D\left(\frac{(n-1)\left(\frac{P}{P-D}\right)(h_1 s_2^2+s_1 h_2^2) + h_2s_2(h_2+s_2)}{2(h_2+s_2)^2}\right)$

$$\text{Take } g(n) = -k_2 n^2 + \left(\left(\frac{DL^2}{2}\right)\left(\frac{P}{P-D}\right)\frac{(h_1 s_2^2+s_1 h_2^2)}{(h_2+s_2)^2} - k_1\right)n + \frac{DL^2}{2}\left(\frac{s_2^2(h_2-\left(\frac{P}{P-D}\right)h_1)+h_2^2(s_2-\left(\frac{P}{P-D}\right)s_1)}{(h_2+s_2)^2}\right) \geq 0 \tag{9}$$

Substitute $K^*(n)$ in equation (7) we get

$$= \sqrt{\frac{2D(\frac{k_1}{n} + k_2)}{(h_2+s_2)^2}\left[\left(\frac{P}{P-D}\right)(n-1)(h_1 s_2^2 + s_1 h_2^2) + h_2s_2(h_2 + s_2)\right]} - \sqrt{\frac{2Dh_2k_2s_2}{(h_2+s_2)}} \tag{10}$$

Hence equation (4) is equivalent to

$$min\,TC_v(n)\sqrt{\frac{2D(\frac{k_1}{n}+k_2)}{(h_2+s_2)^2}\left[\left(\frac{P}{P-D}\right)(n-1)(h_1 s_2^2+s_1 h_2^2)\;+\;h_2 s_2(h_2+s_2)\right]}-\sqrt{\frac{2Dh_2k_2s_2}{(h_2+s_2)}}$$

subject to $\begin{cases} g(n) \geq 0, \\ n \geq 1 \end{cases}$ (11)

Now (11) becomes, $min\,\widetilde{TC}_v(n) = \frac{D(\frac{k_1}{n}+k_2)}{(h_2+s_2)}\left[\left(\frac{P}{P-D}\right)(n-1)(h_1 s_2^2+s_1 h_2^2)+h_2 s_2(h_2+s_2)\right]$

subject to $\begin{cases} g(n) \geq 0, \\ n \geq 1 \end{cases}$ (12)

Since $\sqrt{x}$ is a strictly increasing function for $x \geq 0$.

Here $g(n)$ is strictly concave and $\widetilde{TC}_v(n)$ is convex because $\widetilde{TC}_v''(n) = \frac{2Dk_1\left[s_2^2(h_2-(\frac{P}{P-D})h_1)+h_2^2(s_2-(\frac{P}{P-D})s_1)\right]}{n^3} > 0$ and $g''(n) = -2k_2 < 0$ when $h_2 \geq \left(\frac{P}{P-D}\right)h_1$, $s_2 \geq \left(\frac{P}{P-D}\right)s_1$.

Proposition 1

Let n_1^* be the minimum of $\widetilde{TC}_v(n)$ for $n \geq 1$, then n_1^* is equal to

$$\begin{cases} \left[\sqrt{\frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2(h_2-(\frac{P}{P-D})h_1)+h_2^2(s_2-(\frac{P}{P-D})s_1)\right]}{k_2\,(h_1 s_2^2+s_1 h_2^2)}+\frac{1}{4}}-\frac{1}{2}\right], & \frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2(h_2-(\frac{P}{P-D})h_1)+h_2^2(s_2-(\frac{P}{P-D})s_1)\right]}{k_2\,(h_1 s_2^2+s_1 h_2^2)} \geq 2 \\ 1, & otherwise \end{cases} \quad (13)$$

Proof

$\widetilde{TC}_v(n_1^*) \leq min\left\{\widetilde{TC}_v(n_1^*-1), \widetilde{TC}_v(n_1^*+1)\right\}$ holds for $n \geq 1$, since n_1^* is the minimum of $\widetilde{TC}_v(n)$.

Now $\widetilde{TC}_v(n_1^*) - \widetilde{TC}_v(n_1^*-1) \leq 0$, we have $\frac{-Dk_1\left[(h_2 s_2^2+s_2 h_2^2)-\left(\frac{P}{P-D}\right)(h_1 s_2^2+s_1 h_2^2)\right]}{n_1^*(n_1^*-1)} + \left(\frac{P}{P-D}\right)Dk_2\,(h_1 s_2^2+s_1 h_2^2) \leq 0$

i.e., $\left(n_1^*-\frac{1}{2}\right)^2 \leq \frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2(h_2-(\frac{P}{P-D})h_1)+h_2^2(s_2-(\frac{P}{P-D})s_1)\right]}{k_2\,(h_1 s_2^2+s_1 h_2^2)}+\frac{1}{4}$ (14)

Similarly, $\widetilde{TC}_v(n_1^*) - \widetilde{TC}_v(n_1^*+1) \leq 0$, we have

$\left(n_1^*+\frac{1}{2}\right)^2 \geq \frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2(h_2-(\frac{P}{P-D})h_1)+h_2^2(s_2-(\frac{P}{P-D})s_1)\right]}{k_2\,(h_1 s_2^2+s_1 h_2^2)}+\frac{1}{4}$ (15)

If $\frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2(h_2-(\frac{P}{P-D})h_1)+h_2^2(s_2-(\frac{P}{P-D})s_1)\right]}{k_2\,(h_1 s_2^2+s_1 h_2^2)}+\frac{1}{4} < 0$ then $\widetilde{TC}_v(n_1^*) \leq \widetilde{TC}_v(n_1^*+1)$ for given n, hence $n_1^* = 1$.

if $\frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2(h_2-(\frac{P}{P-D})h_1)+h_2^2(s_2-(\frac{P}{P-D})s_1)\right]}{k_2\,(h_1 s_2^2+s_1 h_2^2)}+\frac{1}{4} \geq 0$, by (14) & 15)

$$\sqrt{\frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)\right]}{k_2\left(h_1 s_2^2+s_1 h_2^2\right)}+\frac{1}{4}}-\frac{1}{2}\le n_1^*$$

$$\le\sqrt{\frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)\right]}{k_2\left(h_1 s_2^2+s_1 h_2^2\right)}+\frac{1}{4}}+\frac{1}{2}$$

Hence, $n_1^* = \left\lceil\sqrt{\frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)\right]}{k_2\left(h_1 s_2^2+s_1 h_2^2\right)}+\frac{1}{4}}-\frac{1}{2}\right\rceil$.

If $0 < \frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)\right]}{k_2\left(h_1 s_2^2+s_1 h_2^2\right)} < 2$, $n_1^* = 1$, Hence (13) holds.

Proposition 2

Let $n_{2(1)}^*$ and $n_{2(2)}^*$ be solution of (9) then the following are true.

1) If $\left(\frac{DL^2}{2}\left(\frac{P}{P-D}\right)\frac{(h_1 s_2^2+s_1 h_2^2)}{(h_2+s_2)^2}-k_1\right)^2+2DL^2k_2\left[\frac{s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)}{(h_2+s_2)^2}\right]<0$, or $\left(\frac{DL^2}{2}\left(\frac{P}{P-D}\right)\frac{(h_1 s_2^2+s_1 h_2^2)}{(h_2+s_2)^2}-k_1\right)^2+2DL^2k_2\left[\frac{s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)}{(h_2+s_2)^2}\right]\ge 0$ and $n_{2(1)}^* < 1$, then $g(n) < 0$ for $n \ge 1$.

2) If $\left(\frac{DL^2}{2}\left(\frac{P}{P-D}\right)\frac{(h_1 s_2^2+s_1 h_2^2)}{(h_2+s_2)^2}-k_1\right)^2+2DL^2k_2\left[\frac{s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)}{(h_2+s_2)^2}\right]\ge 0$ and $n_{2(1)}^* \ge 1$, then i) If $n_{2(2)}^* \ge 1$, $g(n) \ge 0$ for $\lceil n_{2(2)}^*\rceil \le n \le [n_{2(1)}^*]$, ii) If $n_{2(2)}^* < 1$ and $n_{2(1)}^* \ge 1$, $g(n) \ge 0$ for $1 \le n \le [n_{2(1)}^*]$.

Proof

If $g(n) = 0$, we have

$$n_{2(1)}^*=\frac{\left(\frac{DL^2}{2}\left(\frac{P}{P-D}\right)\frac{(h_1 s_2^2+s_1 h_2^2)}{(h_2+s_2)^2}-k_1\right)+\sqrt{\left(\frac{DL^2}{2}\left(\frac{P}{P-D}\right)\frac{(h_1 s_2^2+s_1 h_2^2)}{(h_2+s_2)^2}-k_1\right)^2+2DL^2k_2\left[\frac{s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)}{(h_2+s_2)^2}\right]}}{2k_2}$$

$$n_{2(2)}^*=\frac{\left(\frac{DL^2}{2}\left(\frac{P}{P-D}\right)\frac{(h_1 s_2^2+s_1 h_2^2)}{(h_2+s_2)^2}-k_1\right)-\sqrt{\left(\frac{DL^2}{2}\left(\frac{P}{P-D}\right)\frac{(h_1 s_2^2+s_1 h_2^2)}{(h_2+s_2)^2}-k_1\right)^2+2DL^2k_2\left[\frac{s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)}{(h_2+s_2)^2}\right]}}{2k_2}$$

Here $g(n)$ is a quadratic function, hence

1) If $\left(\frac{DL^2}{2}\left(\frac{P}{P-D}\right)\frac{(h_1 s_2^2+s_1 h_2^2)}{(h_2+s_2)^2}-k_1\right)^2+2DL^2k_2\left[\frac{s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)}{(h_2+s_2)^2}\right]<0$, then $g(n) < 0$ for $n \ge 1$.

2) If $\left(\frac{DL^2}{2}\left(\frac{P}{P-D}\right)\frac{(h_1 s_2^2+s_1 h_2^2)}{(h_2+s_2)^2}-k_1\right)^2+2DL^2k_2\left[\frac{s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)}{(h_2+s_2)^2}\right]\ge 0$ then $n_{2(1)}^*$ and $n_{2(2)}^*$ are real solutions of $g(n) = 0$ for $n \ge 1$,

i) If $n_{2(1)}^* < 1$, then $g(n) < 0$ for $n \ge 1$;

ii) If $n_{2(2)}^* \ge 1$ then $g(n) \ge 0$ for $\lceil n_{2(2)}^*\rceil \le n \le [n_{2(1)}^*]$

iii) If $n^*_{2(2)} < 1$ and $n^*_{2(1)} \geq 1$, then view of n is positive integer, $g(n) \geq 0$ for $1 \leq n \leq [n^*_{2(1)}]$

Theorem 3

If $h_2 \geq \left(\frac{P}{P-D}\right)h_1$, $s_2 \geq \left(\frac{P}{P-D}\right)s_1$and $n^*_{2(2)} \geq 1$ then 1) If $1 \leq n^*_1 \leq [n^*_{2(1)}]$, $n^* = n^*_1$ 2) If $n^*_1 > [n^*_{2(1)}]$, $n^* = [n^*_{2(1)}]$.

Proof

Here $\widetilde{TC}_v(n) \geq 0$, hence it is a convex function. Since n^*_1 is the minimum of $\widetilde{TC}_v(n)$ for $n \geq 1$, if $1 \leq n^*_1 \leq [n^*_{2(1)}]$ then $n^* = n^*_1$. If $n^*_1 > [n^*_{2(1)}]$, then $n^* \leq [n^*_{2(1)}]$. In this interval, $\widetilde{TC}_v(n)$ is decreased. Hence 2) holds.

Remark 3: Here $\widetilde{TC}_v(n)$ is strictly concave if the buyer unit holding cost and shortage cost is lesser than $\left(\frac{P}{P-D}\right)$ times of the manufacturer's. We will not give further discussion about this because this case is not common in practice.

Theorem 4

$K^*(n^*) > 1$, if $h_2 \geq \left(\frac{P}{P-D}\right)h_1$and $s_2 \geq \left(\frac{P}{P-D}\right)s_1$

Proof

$$K^*(n) = \frac{h_2 + s_2}{Q_0}\sqrt{\frac{2D\left(\frac{k_1}{n} + k_2\right)}{\left(\frac{P}{P-D}\right)(n-1)(h_1 s_2^2 + s_1 h_2^2) + h_2 s_2 (h_2 + s_2)}}$$

$$= \sqrt{\frac{(s_2 h_2^2 + h_2 s_2^2)\left(\frac{k_1}{n} + k_2\right)}{k_2\left[\left(\frac{P}{P-D}\right)(n-1)(h_1 s_2^2 + s_1 h_2^2) + (s_2 h_2^2 + h_2 s_2^2)\right]}}$$

I) If $\frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)\right]}{k_2\,(h_1 s_2^2+s_1 h_2^2)} \geq 2$, then

$$n^* = n^*_1 = \left[\sqrt{\frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)\right]}{k_2\,(h_1 s_2^2+s_1 h_2^2)} + \frac{1}{4}} - \frac{1}{2}\right].$$

Since $x \geq 0$, $\left[\sqrt{x + \frac{1}{4}} - \frac{1}{2}\right] \leq \sqrt{x} + 1$ holds, then $K^*(n)$ is a decreasing function of n.

To prove $K^*\left[\sqrt{\frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)\right]}{k_2\,(h_1 s_2^2+s_1 h_2^2)}} + 1\right] > 1$

i.e., $$\sqrt{\frac{(s_2 h_2^2+h_2 s_2^2)\left(\frac{k_1}{\sqrt{\frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)\right]}{k_2\left(h_1 s_2^2+s_1 h_2^2\right)}+1}}+k_2\right)}{k_2\left[\left(\frac{P}{P-D}\right)\left(\sqrt{\frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)\right]}{k_2\left(h_1 s_2^2+s_1 h_2^2\right)}}\right)\left(h_1 s_2^2+s_1 h_2^2\right)+\left(s_2 h_2^2+h_2 s_2^2\right)\right]}}$$

$$=\frac{(s_2 h_2^2+h_2 s_2^2)\left(k_1+k_2\left(\sqrt{\frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)\right]}{k_2\left(h_1 s_2^2+s_1 h_2^2\right)}+1}\right)\right)}{k_2\sqrt{\frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)\right]}{k_2\left(h_1 s_2^2+s_1 h_2^2\right)}+1}\left[\left(\left(\frac{P}{P-D}\right)\sqrt{\frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2\left(h_2-\left(\frac{P}{P-D}\right)k_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)\right]}{k_2\left(h_1 s_2^2+s_1 h_2^2\right)}}\right)\left(h_1 s_2^2+s_1 h_2^2\right)+\left(s_2 h_2^2+h_2 s_2^2\right)\right]}>1 \quad (16)$$

$$\Rightarrow (s_2 h_2^2+h_2 s_2^2)k_1+\left(k_2\left(s_2 h_2^2+h_2 s_2^2\right)\sqrt{\frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)\right]}{k_2\left(h_1 s_2^2+s_1 h_2^2\right)}}\right)$$

$$> k_2\left(s_1 h_2^2+h_1 s_2^2\right)\left(\frac{P}{P-D}\right)\frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)\right]}{k_2\left(h_1 s_2^2+s_1 h_2^2\right)}+k_2\left(s_2 h_2^2+h_2 s_2^2\right)\sqrt{\frac{\left(1-\frac{D}{P}\right)k_1\left[s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)\right]}{k_2\left(h_1 s_2^2+s_1 h_2^2\right)}}$$

$$\Rightarrow (s_2 h_2^2+h_2 s_2^2) > \left[s_2^2\left(h_2-\left(\frac{P}{P-D}\right)h_1\right)+h_2^2\left(s_2-\left(\frac{P}{P-D}\right)s_1\right)\right]$$

$$\Rightarrow s_2^2 h_1 + h_2^2 s_1 > 0 \quad (17)$$

(17) holds, when $h_1, h_2, s_1, s_2 > 0$

II) If $n^* = n_1^* = 1$, then we have $K^*(1) = \sqrt{\frac{k_1+k_2}{k_2}}$. Since k_1, k_2 both are positive, $K^*(1) > 1$.

III) If $n^* = \lceil n_{2(1)}^* \rceil$, when $n_1^* > \lceil n_{2(1)}^* \rceil$. $K^*(n)$ is a decreasing function so $K^*(\lceil n_{2(1)}^* \rceil) \geq K^*(n_1^*) > 1$.

From (I) to (III), if $h_2 \geq \left(\frac{P}{P-D}\right)h_1$, $s_2 \geq \left(\frac{P}{P-D}\right)s_1$ then $K^*(n) > 1$.

Remark 4: Theorem (4) ensures that the buyer's order size is larger at coordination against the non-coordination, if $h_2 \geq \left(\frac{P}{P-D}\right)h_1$ and $s_2 \geq \left(\frac{P}{P-D}\right)s_1$.

2.5 Model formulation for system optimization

In this section, we analyzed the centralized system optimization model with a single decision-maker**.**

In the coordination strategy, the buyers cost is stagnant while manufacturer's cost is able to reduce. If buyer is benefited not only with quantity discount but also some percentage of shares from manufacturer's total cost. If there is a common decision maker for both the buyer and manufacturer, the profit sharing between manufacturer and buyer is made possible. The decision maker balances the benefit of both manufacturer and buyer. Otherwise manufacturer benefited more than the buyer. The objective is to minimize the total cost of the system. The model formulation for system optimization is similar to Yongrui et al. [23] which is considered as a particular case in our model. Consider Q be the buyer's order quantity, then the manufacturer order nQ every time, where Q and n are decision variables. The system optimization problem becomes

$$\min \mathrm{TC_s(n,Q)} = \frac{Dk_1}{nQ} + \frac{\left(\frac{P}{P-D}\right)(n-1)h_1 Q_1^2}{2Q} + \frac{\left(\frac{P}{P-D}\right)(n-1)s_1 Q_2^2}{2Q} + \frac{Dk_2}{Q} + \frac{h_2 Q_1^2}{2Q} + \frac{s_2 Q_2^2}{2Q}$$

$$\text{subject to} \begin{cases} n\frac{Q}{D} \leq L, \\ n \geq 1, \end{cases} \qquad (18)$$

Theorem 5
The proposed quantity discount strategy can achieve system coordination.

Proof
Since $\mathrm{TC_s(n,Q)}$ is convex in Q, $\mathrm{Q^*}$ be the optimum of $\mathrm{TC_s(n,Q)}$, by simple calculation

$$\left[\frac{\partial TC_s(n,Q)}{\partial Q}\right]_{(Q=Q^*)} = \left(\frac{Dk_1}{n} + Dk_2\right)\left(\frac{-1}{Q^{*2}}\right) + \frac{\left(\frac{P}{P-D}\right)(n-1)}{2(h_2+s_2)^2}\left(h_1 s_2^2 + s_1 h_2^2\right) + \frac{1}{2(h_2+s_2)^2}\left(h_2 s_2^2 + s_2 h_2^2\right) = 0 \qquad (19)$$

$$\mathrm{Q^*(n)} = (h_2 + s_2)\sqrt{\frac{2D\left(\frac{k_1}{n} + k_2\right)}{\left(\frac{P}{P-D}\right)(n-1)(h_1 s_2^2 + s_1 h_2^2) + h_2 s_2 (h_2 + s_2)}} \qquad (20)$$

Substitute $\mathrm{Q^*(n)}$ into (18), we get

$$\min \mathrm{TC_s(n)} = \sqrt{\frac{2D\left(\frac{k_1}{n} + k_2\right)}{(h_2+s_2)^2}\left[\left(\frac{P}{P-D}\right)(n-1)(h_1 s_2^2 + s_1 h_2^2) + h_2 s_2(h_2 + s_2)\right]}$$

subject to

$$\begin{cases} -k_2 n^2 + \left(\left(\frac{DL^2}{2}\right)\left(\frac{P}{P-D}\right)\frac{(h_1 s_2^2 + s_1 h_2^2)}{(h_2+s_2)^2} - k_1\right)n + \frac{DL^2}{2}\left(\frac{s_2^2\left(h_2 - \left(\frac{P}{P-D}\right)h_1\right) + h_2^2\left(s_2 - \left(\frac{P}{P-D}\right)s_1\right)}{(h_2+s_2)^2}\right) \geq 0, \\ n \geq 1, \end{cases} \qquad (21)$$

Since $\sqrt{x}$ is a strictly increasing function for $x \geq 0$.

Now (21) becomes, $\min \widetilde{TC}_s(n) = \frac{D\left(\frac{k_1}{n} + k_2\right)}{(h_2+s_2)}\left[\left(\frac{P}{P-D}\right)(n-1)(h_1 s_2^2 + s_1 h_2^2) + h_2 s_2(h_2 + s_2)\right]$

subject to

$$\begin{cases} -k_2 n^2 + \left(\left(\frac{DL^2}{2}\right)\left(\frac{P}{P-D}\right)\frac{(h_1 s_2^2 + s_1 h_2^2)}{(h_2+s_2)^2} - k_1\right)n + \frac{DL^2}{2}\left(\frac{s_2^2\left(h_2 - \left(\frac{P}{P-D}\right)h_1\right) + h_2^2\left(s_2 - \left(\frac{P}{P-D}\right)s_1\right)}{(h_2+s_2)^2}\right) \geq 0, \\ n \geq 1, \end{cases} \qquad (22)$$

Note that (22) is exactly the same as (12), so they have same optimum n^*.

By (11) and (21), $\mathrm{TC_s}(n^*) = \mathrm{TC_v}(n^*) + \sqrt{2Dk_2h_2}\sqrt{\frac{s_2}{h_2+s_2}}$ (23)

where $\sqrt{2Dk_2h_2}\sqrt{\frac{s_2}{h_2+s_2}}$ is the buyer's actual cost under coordination.

The Manufacturer optimal order quantity is equal in these two cases, i.e., $n^*K^*(n^*)Q_0 = n^*Q^*(n^*)$ (24)

By $K^*(n)$ and $Q^*(n)$, the buyer's optimal order quantity under coordination is equal to that under system optimization. i.e., $K^*(n^*)Q_0 = Q^*(n^*)$ (25)

From equation (23) to (25), we see that the quantity discount contract can achieve system coordination.

3. NUMERICAL EXAMPLE

In this section, numerical examples are presented to illustrate the performance of the quantity discount strategy proposed in previous sections. The sensitivity analysis of cost savings on parameters has been given. The buyer's saving in percentage $SP_b = 100\alpha(TC_v(m^*) - TC_v(n^*))/TC_r(m^*)$. The manufacturer's saving in percentage $SP_{M1} = 100(1-\alpha)(TC_v(m^*) - TC_v(n^*))/TC_v(m^*)$. The manufacturer's saving in percentage if he does not share the saving with the buyer $SP_{M2} = 100(TC_v(m^*) - TC_v(n^*))/TC_v(m^*)$. The system saving in percentage is $SP_s = 100\big(TC_v(m^*) - TC_v(n^*)\big)/\big(TC_v(m^*) + TC_r(m^*)\big)$.

Example Given P = 20,000 per year, D = 10,000 units per year, p_2 = 30$ per unit, α = 0.5, L = 0.25 year, k_1 = 300$ per order, k_2 = 100$ per order. The different values of h_1, s_1, h_2, s_2 and computational results are as specified in Table 1.

Table 1: Brief summary of the results for above example

h_1	h_2	s_1	s_2	K*(n)	d(K)	SP_b	SP_{M1}	SP_{M2}	SP_s
3	10	10	50	2.0000	0.0034	1.6667	1.6129	3.2258	1.6393
4	10	10	50	2.0000	0.0034	5.8333	5.2239	10.4478	5.5118
5	10	10	50	2.0000	0.0034	25.0000	16.6667	33.0000	20.0000
3	10	15	50	2.0000	0.0034	2.5000	2.3810	4.7619	2.4390
4	10	20	50	2.0000	0.0034	7.5000	6.5217	13.0435	6.9767
5	10	25	50	2.0000	0.0034	25.0000	16.6667	33.3333	20.0000
5	11	25	50	2.0000	0.0035	25.0000	16.6667	33.3333	20.0000
5	12	25	50	2.0000	0.0037	25.0000	16.6667	33.3333	20.0000
5	13	25	50	2.0000	0.0038	25.0000	16.6667	33.3333	20.0000
5	14	25	50	2.0000	0.0039	6.9196	6.0784	12.1569	6.4718
5	15	25	50	2.0000	0.0040	6.0897	5.4286	10.8571	5.7402
5	15	25	55	2.0000	0.0040	5.4654	4.9268	9.8537	5.1821
5	15	25	60	2.0000	0.0041	5.0000	4.5455	9.0909	4.7619
5	15	25	65	2.0000	0.0041	4.6474	4.2522	8.5044	4.4410
5	15	25	70	2.0000	0.0041	4.3768	4.0245	8.0489	4.1932
5	15	25	75	2.0000	0.0042	4.1667	3.8462	7.6923	4.0000
5	11	25	55	2.0000	0.0036	25.0000	16.6667	33.3333	20.0000
5	12	25	60	2.0000	0.0037	25.0000	16.6667	33.3333	20.0000
5	13	25	65	2.0000	0.0039	6.7308	5.9322	11.8644	6.3063
5	14	25	70	2.0000	0.0040	5.3571	4.8387	9.6774	5.0847
5	15	25	75	2.0000	0.0042	4.1667	3.8462	7.6923	4.0000

The computational result highlights, the proposed model was solved for different values of holding cost and shortage cost for manufacturer and buyer with fixed demand and production rate. For each choice of holding cost and shortage cost of the manufacture, the saving percentage was found to increase. In contrast, each choice of holding cost and shortage cost of the buyer, the saving percentage was found to decrease. Hence, in quantity discount coordination mechanism, if the buyer's holding cost is high, the manufacturer and buyer cannot gain more whereas if the manufacture's holding cost is high, the benefit is significant.

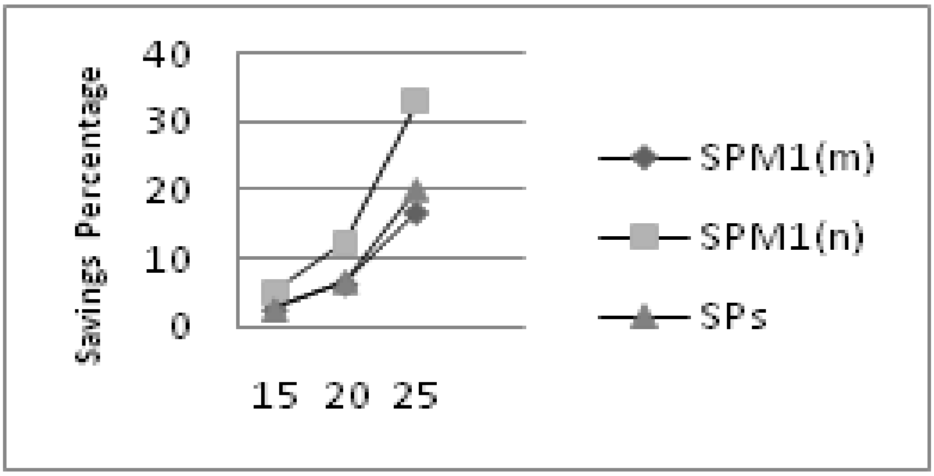


Fig 1: Effect of changes when s_1 and h_1 increases

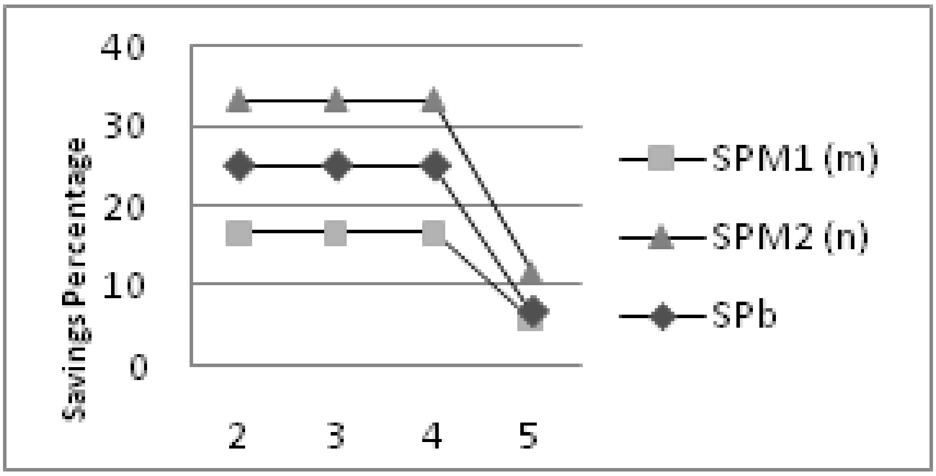


Fig 2: Effect of changes when h_2 increases and s_2 fixed

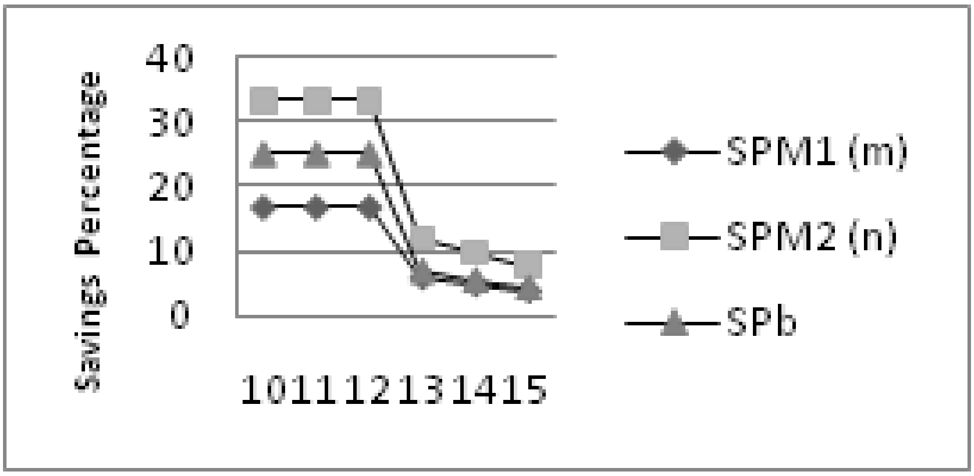


Fig 3: Effect of changes when h_2 and s_2 increases

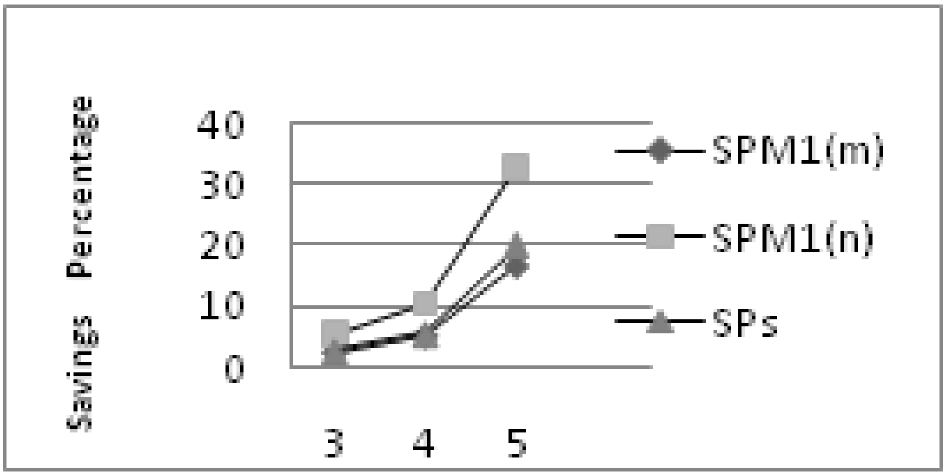


Fig 4: Effect of changes when h_1 increases and s_1 fixed

CONCLUSION

In this paper we have developed inventory model in which quantity discount coordination strategy with shortages for manufacturer and buyer supply chain of fixed life time product. The goal is to find optimal multiples of orders so as to maximize the saving percentage. The model dealt in this paper along with the numerical example brings into light certain specific findings, i.e., in quantity discount coordination mechanism, if the buyer's holding cost is high, the manufacturer and buyer can benefitted less whereas if the manufacture's holding cost is high, the gain is significant. A decision maker can be used to decide on the optimum percentage to be shared by the manufacturer with the buyer from his increased saving percentage. The proposed model concludes that both manufacturer and buyer are benefitted only when

coordination strategy is adopted. It proves that the quantity discount strategy attains system optimization. The system saving percentage aids to arrive at the decision.
The managerial implications are i) The manufacturer and buyer can decide optimum inventory level. It helps in making out production and inventory schedules ii) Increase in profits for both manufacturer and buyer is possible. The current model deals with single product. Future research is possible with multiple product and multi-supplier scenario.

REFERENCES

[1] Baker, K.R., 1970, "On Madigan's approach to the deterministic multi product production and inventory problem," Manage. Sci., 16(9), pp. 636-638.

[2] Biswajit Sarkar, Ilkyenog Moon, 2011, "An EPQ model with inflation in an imperfect production system," Applied Mathematics and Computation, 217, pp. 6159-6167.

[3] Bomberger, E.E., 1966, "A dynamic programming approach to a lot size scheduling problem," Management Science, 12(11), pp. 778-784.

[4] Eilon, S., 1957, "Scheduling for batch production," Prod. Eng. J., 36, pp. 549-570 and582.

[5] Fries, B., 1975, "Optimal order policies for a perishable commodity with fixed life time," Operations Research, 23, 46-61.

[6] Fujiwara, O., Soewandi, H., and Sedarage, D., 1997, "An optimal and issuing policy for a two-stage inventory system for perishable products," European Journal of Operational Research, 99, pp. 412-424.

[7] Goyal, S. K., and Gupta, Y.P., 1989, "Integrated inventory models: the buyer-vendor coordination," European Journal of Operational Research, 41, pp. 261-269.

[8] Gede Agus Widyadana, Hui Ming Wee, 2011, "Optimal deteriorating items production inventory models with random machine breakdown and stochastic repair time," Applied Mathematical Modeling, 35, pp. 395-3508.

[9] Hodgson, T.J., 1970, "Addendum to Stankard and Gupta's note on lot size scheduling," Management Science, 16(7), pp. 514-517.

[10] Hung-Chi Chang, 2011, "A comprehensive note on: An economic order quantity with imperfect quality and quantity discounts," Applied Mathematical Modeling, 35, pp. 5208-5216.

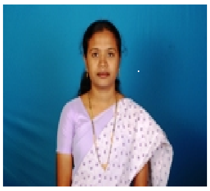
[11] Kanchana, K., and Anulark, T., 2006, "An approximate periodic model for fixed-life perishable product in a two-echelon inventory distribution system," International Journal of Production Economic, 100, pp. 101-115.

[12] Kaj-Mikael Bjork, 2012, "A multi item fuzzy economic production quantity problem with a finite production rate," International Journal of Production Economics, 135, pp. 702-707.

[13] Lian, Z., and Liu, L., 2001, "Continuous review perishable inventory systems: models and heuristics," IIE Transactions, 33, pp. 809-822

[14] Liu, L., and Lian, Z., 1999, "(s, S) model for inventory with fixed life time," Operations Research, 47(1), pp. 130-158.
[15] Liu, L., and Shi, D., 1999, "(s, S) model for inventory with exponential lifetimes and renewal demands," Naval Research Logistics, 46, pp. 39-56.
[16] Madigan, J.T., 1968, "Scheduling a Multi-product single machine system for an infinite planning period," Management Science, 14(11), pp. 713-719.
[17] Mahdi Tajbaksh, M., Chi-Guhn Lee, and Saeed Zolfaghari, 2011, "An inventory model with random discount offerings," Omega, 39, pp. 710-718.
[18] Nandakumar, P., and Morton, T.E., 1993, "Near myopic heuristic for the fixed life perishability problem," Management Science, 39, pp. 1490-1498.
[19] Rogers, J., 1958, "A computational approach to the economic lot scheduling problem," Management Science, 4(3), pp. 264-291.
[20] Saoussen Krichen, Awatef Laabidi, Fouad Ben Abdelaziz, 2011, "Single supplier multiple cooperative retailers inventory model with quantity discount and permissible delay in payments," Computers & Industrial Engineering, 60, pp. 164-172.
[21] Stankard, M. F., and Gupta, S. K., 1969, "A note on Bomberger's approach to lot size scheduling: Heuristic proposed, " Management Science, 15(7), pp. 449-452.
[22] Taft, E.W., 1918, " The most economical production lot," Iron Age, 101, pp. 1410-1412.
[23] Yongrui Duan, Jianwen Luo, Jiazhen Huo, 2010, " Buyer-vendor inventory coordination with quantity discount incentive for fixed lifetime product," International Journal of Production Economics, 128, pp. 351-357.

ABOUT THE AUTHORS



Mrs. M. Ravithammal is an Assistant Professor in the department of Mathematics, Vels University, Chennai. She is doing PhD in the area of Inventory Management and Control since 2013 in Sathyabama University, Chennai. She has published research articles in refereed journals and Conferences (National and International) in this area.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R. Uthayakumar was born in Dindigul, India in 1967. He received the Ph.D. Degree in Mathematics from Gandhigram Rural Institute-Deemed University, Gandhigram, India; in 2000. He is currently a Professor in Mathematics in Gandhigram Rural Institute-Deemed University, Gandhigram, India. His major research interests include Mathematical Modeling, Fractal Analysis, Fuzzy Spaces, Operations Research, Inventory Management and Control, Supply Chain Systems and Biomedical Signal Processing. He is an author and coauthor of more than 100 international refereed scientific journal articles and 50 papers published in proceedings of conferences.

S. Ganesh, Professor and Head of Department of Mathematics, Sathyabama University, Chennai has 23 Years of Experience in Teaching which includes 10 years of Research Experience. He completed his Doctorate from Anna University, Chennai. He has 24 publications in International Journals, 14 National Journals, 15 International Conferences and 12 National Conferences. He has been awarded Best Teacher Award twice in Sathyabama University. He has organized several International Conferences, National Conferences and International Workshops in Sathyabama University with funding from CSIR. He has organized several Mathematical Programmes for students to bring out their hidden talents. He is guiding 9 students in his area of Research of which three of them are completed his Viva. He has published several books on Engineering Mathematics for the welfare of the students. His aim is to become a true Professional in teaching and a good researcher in Math.