Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 11, Number 5 (2015), pp. 3565-3581

http://www.ripublication.com

An Entropic Economic Order Quantity Model for Deterioration of Perishable Items with Cubic Demand Rate and its Fuzzy Environment

[1]G.Santhi and [2]K.Karthikeyan

[1]Research Associate, Statistics & O.R division, SAS,
VIT University, Vellore-14, T.N,India.
[2]Associate Professor, Statistics & O.R division, SAS,
VIT University, Vellore-14, T.N,India.

Abstract

A new type of replenishment policy is suggested in an entropic order quantity (EnOQ) model for deterioration of perishable items with time dependent demand rate. This model represents the cubic demand rate with entropy cost, particularly over a finite time horizon. Its main aim lies in the need for an entropic cost of the cycle time is a key feature of specific perishable items like fruits, vegetables, milk, food grains, fishes, bakery items, food stuffs etc. To handle this multiplicity of purposes in a pragmatic approach entropic order quantity model with cubic demand rate of perishable products to optimize its payoff is proposed. Using the effects of entropy cost and without entropy cost the profit-maximization models are formulated. The objective of this paper is to determine the cycle length and the replenishment order quantity so that the profit is maximized. Fuzziness is introduced by considering ordering cost and purchasing cost parameters to be fuzzy numbers. Signed distance method is used to defuzzification of the profit function. Finally, numerical examples and sensitivity analysis of the developed models are presented to illustrate the crisp and fuzzy cases in EnOQ and EOQ separately. In comparative analysis, we observed that the order quantity is more in EnOQ model but profit is approximately less in comparison to the Economic Order Quantity (EOQ) model.

Keywords: Economic order quantity; Cubic demand; Deterioration; Entropy cost; Profit maximization; Fuzzy number; Signed distance method;

1. Introduction

The traditional inventory model was that the items preserved their physical characteristics while they were kept stock or stored of goods. Although the economic order quantity (EOQ) formula has been widely used by practitioners as a decision-making tool for the control of inventory. The extensions to the economic order quantity model are introducing the entropy cost. The main criterion in classical inventory models is minimization of long-run average cost per unit time. The costs considered are generally fixed and variable holding cost, ordering cost and disposal cost. Costs associated with disorder in the system tied up in inventory are accounted for by including an entropy cost in the total costs. The entropy is frequently defined as the amount of disorder in a system. Its main objective lies in the need for an entropic cost of the cycle time is a key feature of specific perishable items like fruits, vegetables, food stuffs, bakery items, milk, food grains, fishes etc. As markets have become more and more competitive disorder has become a main characteristic of new productive systems that are effective in complex, dynamic and uncertain environments. This article introduced the concept of entropy cost to account for the hidden cost such as the additional managerial cost that is required to control the improvement process. Perishability product is an important feature of inventory control. In general deteriorationmay be considered as the result of various effects on stock, some of which are decay, change, spoilage,damage, decreasing usefulness and many more. While kept in store fruits, food grains, milk, vegetables, food stuffs, bakery items etc. suffer from reduction by decent spoilage. The decaying products are of two types which are the product deteriorate from the very beginning and the products start to deteriorate after a certain time.

Goyal (1985) proposed an EOQ model under conditions of permissible delay in payments. According to Raafat (1991) EOQ model assuming time value of money, deterioration rate, shortages and probabilistic number or an exponential function. Richter (1996a) analyzed the Economic order quantity model with waste disposal and used product collection rates. According to Richter (1996b) find an EOQ model with repair and waste disposal and used product collection rates. Whewell (1997) have used business process management. Chang et al. (1998) developed a triangular fuzzy model for inventory with backorder quantity. Goyal and Giri (2001) proposed an recent trends of the modeling in deteriorating items inventory. Chang (2004) established an inventory model taking deterioration under inflation when supplier credits linked to order quantity. Jaber et al., (2004) used the behavior of production systems thoroughly resembles that of physical systems. Dye et al. (2005) developed an inventory control with fuzzy lead-time and dynamic demand. Urban (2005) examined is used stock development inventory and its outline can be found in the review and inventory level dependent demand by considering a periodic review model. Jaber (2006) developed an imperfect production process with quality corrective interruptions and reduction in setups. Jaber et al., (2006) the developed model is investigated in a two-level supply chain coordination. Abad (2008) are considered optimal pricing and EOQ under the partial backlogging and shortages with lost sale costs. According to Jaber (2007) an economic lot size problem with permissible delay in payments. Chung et al., (2007) analyzed the EOQ model to determine retailers optimal with imperfect quality items considering permissible delay in payments. Eroglu et al., (2007) used the EOQ model with defective items and shortages.Jaber et al., (2008) studied a defective items permitting to a learning curve, the inspection rate was much higher than the demand rate and the percentage defectives per delivery reduce to a small value. Recently, Pattnaik(2012) developed an entropic order quantity model for perishable items with two component demand and without shortages.

In this paper, we developed an instantaneous inventory model to investigate the effect of the approximation made by using the average payoff while determining the optimal values of the policy variables. This paper focuses exclusively on the cost of entropy with cubic demand rate and holding cost is taken as a linear function of time. we have considered that the ordering cost and purchasing cost parameters are uncertain. Finally the ordering cost and purchasing cost parameters are fuzzified as the triangular fuzzy numbers. Signed distance method is used to defuzzification of the profit function.

2.Preliminaries

Definition2.1.A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in A, \mu_{\tilde{A}}(x) \in [0,1]\}$. In the pair $(x, \mu_{\tilde{A}}(x))$, the first element x belong to the classical set *A,* the second element $\mu_{\tilde{A}}(x)$, belong to the interval [0,1] called membership function.

Definition 2.2.A fuzzy set $\tilde{A} = (a_1, a_2, a_3)$ on*R*,where $a_1 \le a_2 \le a_3$ is called a triangular fuzzy number if its membership function $\mu_{\tilde{A}}(x)$ isgiven by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{if } x<a_1 \\ \dfrac{x-a_1}{a_2-a_1} & \text{if } a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2} & \text{if } a_2 \le x \le a_3 \\ 0 & \text{if } x>a_3 \end{cases}$$

Definition 2.3. If $\tilde{A} = (a_1, a_2, a_3)$ is a triangular fuzzy number then the signed distance of $\tilde{A}$ is defined as $d(\tilde{A}, 0) = \frac{1}{4}(a_1 + 2a_2 + a_3)$.

3. Notation and assumptions

3.1. Notations

The notations are used throughout this article are given as follows:

$D(t)$ demand rate is cubic function of time

θ	the deterioration parameter
HC	holding cost per unit time, $H(t) = h + \beta t$, $h > 0$, $\beta > 0$
C_0	the unit purchasing price per item
s	the unit selling price, where $s \geq C_0$
A	ordering cost per unit order is known and constant
$\tilde{C}_0$	the fuzzy purchasing price per item
$\tilde{A}$	the fuzzy ordering cost per order
$I(t)$	the inventory level at time t, where $t \in [0, T]$
Q	the maximum inventory level for entropic order quantity
Q_1	the maximum inventory level for economic order quantity
T, T_1	the cycle lengths for the above two respective cases.
TP	the total profit of an inventory system.
$\tilde{TP}$	the fuzzy total profit of an inventory system

3.2. Assumptions

The following assumptions are made to develop this article:

1) Inventory system is included only one item.
2) The demand rate is time dependent cubic, i.e., $D(t) = a + bt + ct^2 + dt^3$, where a, b, c and d are the positive constants.
3) Deterioration rate θ is constant and ($0 \leq \theta \leq 1$).
4) Shortages are not permitted.
5) Lead time is zero or negligible.
6) The time horizon is finite.
7) Replenishment rate is infinite.
8) The holding cost $H(t)$ per unit time is time dependent and it is assumed $H(t) = h + \beta t$,
9) The entropy generation rate must satisfy $S = \frac{d\sigma(t)}{dt}$ where $\sigma(t)$ is the total entropy generated by time t and S is the rate at which entropy is generated. Entropy cost is computed by dividing the total commodity flow in a cycle of duration T. The total entropy generated over time T as $\sigma(T) = \int_0^T S dt$; $S = \frac{R(I(t))}{s}$ Entropy cost per cycle is $EC = EC(\mathrm{WD}) = \frac{Q_{WD}}{\sigma(T)}$ EC is measured in an appropriate price unit with deterioration respectively.

4. Mathematical model formulation

Let $I(t)$ be the inventory level at any time t, the differential equation representing the instantaneous state over $(0,T)$ is given by in Fig.1.

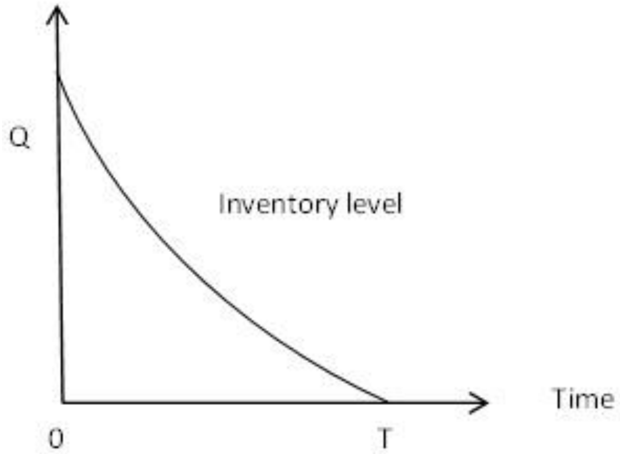

Fig.1. Graphical representation of inventory system

$$\frac{dI(t)}{dt}+\theta I(t)=-D(t),\ 0\le t\le T \tag{4.1}$$

$$\frac{dI(t)}{dt}+\theta I(t)=-(a+bt+ct^2+dt^3),\ o\le t\le T \tag{4.2}$$

The solution of above equation (4.2) is

$$I(t)=\left[a(T-t)+\frac{(b+a\theta)(T^2-t^2)}{2}+\frac{(c+b\theta)(T^3-t^3)}{3}+\frac{(d+c\theta)(T^4-t^4)}{4}+\frac{d\theta(T^5-t^5)}{5}\right],0\le t\le T \tag{4.3}$$

Withboundary conditions $I(0)=Q$ and $I(T)=0$,

We get the optimum order quantity is given by

$$I(0)=Q=aT+\frac{(b+a\theta)T^2}{2}+\frac{(c+b\theta)T^3}{3}+\frac{(d+c\theta)T^4}{4}+\frac{(d\theta)T^5}{5} \tag{4.4}$$

The total profit (TP) per unit time consists of the following components:

(i) Inventory holding cost per cycle [0, T] is given by $HC=\frac{1}{T}\int_0^T(h+\beta t)I(t)\,dt$

$$HC=\left[\begin{array}{l}\frac{haT}{2}+\frac{h(b+a\theta)T^2}{3}+\frac{h(c+b\theta)T^3}{4}+\frac{h(d+c\theta)T^4}{5}+\frac{hd\theta T^5}{6}+\frac{\beta aT^2}{6}\\+\frac{\beta(b+a\theta)T^3}{8}+\frac{\beta(c+b\theta)T^4}{10}+\frac{\beta(d+c\theta)T^5}{12}+\frac{\beta d\theta T^6}{14}\end{array}\right] \tag{4.5}$$

(ii) Ordering Cost (OC) per cycle [0, T] is given by $OC = \dfrac{A}{T}$ (4.6)

(iii) Purchase cost (PC) per cycle [0, T] is given by $PC = \dfrac{C_o Q}{T}$ (4.7)

(iv) Entropy cost (EC) per cycle [0, T] is given by

$$EC = EC_{\text{WD}} = \frac{Q_{WD}}{\sigma(T)} = \frac{1}{T}\left(\frac{Q}{\sigma(T)}\right)$$

$$\textit{where}\quad \sigma(T) = \int_0^T S dt = \int_0^T \frac{\text{a+bt+ct}^2 + dt^3}{s}$$

$$EC = \frac{sQ}{T\left(aT + \frac{bT^2}{2} + \frac{cT^3}{3} + \frac{dT^4}{4}\right)} \tag{4.8}$$

(v) Sales revenue (SR) per cycle [0, T] is given by

$$SR = \frac{1}{T}\,\text{s}\left[\int_0^T \text{a+bt+ct}^2 + dt^3\right] \tag{4.9}$$

The total profit per unit time is given by

$$TP(T) = SR - HC - OC - PC - EC$$

After make integration and some manipulation of the relevant costs, we get the total profit unit of time is

$$TP = \left[s\left(a + bT + \frac{cT^2}{2} + \frac{dT^3}{3}\right) - \left(\begin{array}{l} \frac{haT}{2} + \frac{h(b+a\theta)T^2}{3} + \frac{h(c+b\theta)T^3}{4} \\ + \frac{h(d+c\theta)T^4}{5} + \frac{hd\theta T^5}{6} + \frac{\beta a T^2}{6} \\ + \frac{\beta(b+a\theta)T^3}{8} + \frac{\beta(c+b\theta)T^4}{10} + \\ \frac{\beta(d+c\theta)T^5}{12} + \frac{\beta d\theta T^6}{14} \end{array} \right) - \frac{sQ}{T\left(aT + \frac{bT^2}{2} + \frac{cT^3}{3} + \frac{dT^4}{4}\right)} - A - C_o Q \right] \tag{4.10}$$

In the above equation (4.10), no constraints will be imposed. The only constraint is the non-negative restriction for *T*.

Thus we have to determine T from the maximization problem

$$\textit{Maximize } TP(T)$$
$$\forall T \geq 0 \tag{4.11}$$

Case-1.The Profit Maximization for EOQ Model

In this situation the entropy cost is ignored, so the order level and total profit per unit time is obtained from (4.4) and (4.10) by substituting $T=T_1$ and $EC=0$.

$$Q_1 = aT_1 + \frac{(b+a\theta)T_1^2}{2} + \frac{(c+b\theta)T_1^3}{3} + \frac{(d+c\theta)T_1^4}{4} + \frac{d\theta T_1^5}{5} \tag{4.12}$$

$$TP(T_1) = SR - HC - OC - PC$$

$$TP(T_1) = \left[s\left(a + bT_1 + \frac{cT_1^2}{2} + \frac{dT_1^3}{3}\right) - \left(\begin{array}{l} \frac{haT_1}{2} + \frac{h(b+a\theta)T_1^2}{3} + \frac{h(c+b\theta)T_1^3}{4} \\ + \frac{h(d+c\theta)T_1^4}{5} + \frac{hd\theta T_1^5}{6} + \frac{\beta a T_1^2}{6} \\ + \frac{\beta(b+a\theta)T_1^3}{8} + \frac{\beta(c+b\theta)T_1^4}{10} + \\ \frac{\beta(d+c\theta)T_1^5}{12} + \frac{\beta d\theta T_1^6}{14} \end{array} \right) - A - C_o Q_1 \right] \quad (4.13)$$

In the above equation (4.13), no constraints will be imposed. The only constraint is the non-negative restriction for T_1.

Thus we have to determine T_1 from the maximization problem

$$\begin{array}{l} \textit{Maximize } TP(T_1) \\ \forall T_1 \geq 0 \end{array} \quad (4.14)$$

5. The fuzzy mathematical model

In this section we present the fuzzy formulation of the model.

To develop the inventory model in fuzzy environment, Let us consider that the ordering cost and purchasing cost parameters are uncertain. We represent them bythe triangular fuzzy numbers as

$$\tilde{A} = (A-\Delta_1, A, A+\Delta_2) \qquad \textit{where } 0<\Delta_1<A \textit{ and } 0<\Delta_1\Delta_2$$

$$\tilde{C}_0 = (C_0-\Delta_3, C_0, C_0+\Delta_4) \qquad \textit{whre } 0<\Delta_3<C_0 \textit{ and } 0<\Delta_3\Delta_4$$

The variables $\Delta_1, \Delta_2, \Delta_3$ and Δ_4 are determined by the decision maker based on the uncertainty of the problem.

Therefore the total profit per unit time is a fuzzy sense is given by

$$\tilde{TP} = \left[s\left(a + bT + \frac{cT^2}{2} + \frac{dT^3}{3}\right) - \left(\begin{array}{l} \frac{\alpha a T}{2} + \frac{\alpha(b+a\theta)T^2}{3} + \frac{\alpha(c+b\theta)T^3}{4} \\ + \frac{\alpha(d+c\theta)T^4}{5} + \frac{\alpha d\theta T^5}{6} + \frac{\beta a T^2}{6} \\ + \frac{\beta(b+a\theta)T^3}{8} + \frac{\beta(c+b\theta)T^4}{10} + \\ \frac{\beta(d+c\theta)T^5}{12} + \frac{\beta d\theta T^6}{14} \end{array} \right) - \frac{sQ}{T\left(aT + \frac{bT^2}{2} + \frac{cT^3}{3} + \frac{dT^4}{4}\right)} - \tilde{A} - \tilde{C}_o Q \right] \quad (5.1)$$

Then the signed distance of $\tilde{A}$ *and* $\tilde{C}_0$ are given by

$$d(\tilde{A}, \tilde{0}) = A + \frac{1}{4}(\Delta_2 - \Delta_1)$$

$$d(\tilde{C}_0, \tilde{0}) = C_0 + \frac{1}{4}(\Delta_4 - \Delta_3)$$

Now, we defuzzify $TP(T)$ using signed distance method. The signed distance of $\tilde{TP}(T)$ *to* $\tilde{0}$ is given by

$$\tilde{TP}(T)=\left[\begin{array}{l} s\left(a+bT+\frac{cT^2}{2}+\frac{dT^3}{3}\right)-\left(\begin{array}{l}\frac{haT}{2}+\frac{h(b+a\theta)T^2}{3}+\frac{h(c+b\theta)T^3}{4}+\frac{h(d+c\theta)T^4}{5}\\ +\frac{hd\theta T^5}{6}+\frac{\beta aT^2}{6}+\frac{\beta(b+a\theta)T^3}{8}+\frac{\beta(c+b\theta)T^4}{10}+\\ \frac{\beta(d+c\theta)T^5}{12}+\frac{\beta d\theta T^6}{14}\end{array}\right)\\ -\frac{sQ}{T\left(aT+\frac{bT^2}{2}+\frac{cT^3}{3}+\frac{dT^4}{4}\right)}-\left(A+\frac{1}{4}(\Delta_2-\Delta_1)\right)-\left(C_0+\frac{1}{4}(\Delta_4-\Delta_3)\right)Q\end{array}\right] \quad (5.2)$$

In the above equation (5.2), no constraints will be imposed. The only constraint is the non-negative restriction for T.

Thus we have to determine T from the fuzzy maximization problem

$$\text{Maximize } \tilde{TP}(T)$$
$$\forall T\geq 0 \quad (5.3)$$

Case - 3. The Profit Maximization for fuzzy EOQ Model

In this case the entropy cost is ignored, so the order level and total profit per unit time is obtained from (4.4) and (4.10) by substituting $T=T_1$ and $EC=0$.

$$Q_1=aT_1+\frac{(b+a\theta)T_1^2}{2}+\frac{(c+b\theta)T_1^3}{3}+\frac{(d+c\theta)T_1^4}{4}+\frac{d\theta T_1^5}{5} \quad (5.4)$$

$$\tilde{TP}(T_1)=SR-HC-OC-PC$$

$$\tilde{TP}(T_1)=\left[s\left(a+bT_1+\frac{cT_1^2}{2}+\frac{dT_1^3}{3}\right)-\left(\begin{array}{l}\frac{haT_1}{2}+\frac{h(b+a\theta)T_1^2}{3}+\frac{h(c+b\theta)T_1^3}{4}\\ +\frac{h(d+c\theta)T_1^4}{5}+\frac{hd\theta T_1^5}{6}+\frac{\beta aT_1^2}{6}\\ +\frac{\beta(b+a\theta)T_1^3}{8}+\frac{\beta(c+b\theta)T_1^4}{10}+\\ \frac{\beta(d+c\theta)T_1^5}{12}+\frac{\beta d\theta T_1^6}{14}\end{array}\right)-\tilde{A}-\tilde{C}_oQ_1\right]$$

$$\tilde{TP}=\left[s\left(a+bT_1+\frac{cT_1^2}{2}+\frac{dT_1^3}{3}\right)-\left(\begin{array}{l}\frac{haT_1}{2}+\frac{h(b+a\theta)T_1^2}{3}+\frac{h(c+b\theta)T_1^3}{4}\\ +\frac{h(d+c\theta)T_1^4}{5}+\frac{hd\theta T_1^5}{6}+\frac{\beta aT_1^2}{6}\\ +\frac{\beta(b+a\theta)T_1^3}{8}+\frac{\beta(c+b\theta)T_1^4}{10}+\\ \frac{\beta(d+c\theta)T_1^5}{12}+\frac{\beta d\theta T_1^6}{14}\end{array}\right)-\left(A+\frac{1}{4}(\Delta_2-\Delta_1)\right)-\left(C_0+\frac{1}{4}(\Delta_4-\Delta_3)\right)Q_1\right] \quad (5.5)$$

In the above equation (5.5), no constraints will be imposed. The only constraint is the non-negative restriction for T_1.

Thus we have to determine T_1 from the fuzzy maximization problem

$$\text{Maximize } \tilde{TP}(T_1)$$
$$\forall T_1\geq 0 \quad (5.6)$$

6. Numerical Examples

Example 1: Crisp EnOQ model

Let A=500, a=25, b=20, c=10, d=3, $\theta = 0.02$, $h = 0.5$, $\beta = 0.6$, $s = 10.0$, $C_0 = 4.0$, $\Delta_1 = 0.0005$, $\Delta_2 = 0.01$, $\Delta_3 = 0.0004$, *and* $\Delta_4 = 0.02$, in appropriate units. We get the optimal values are T=3.2571; Q=404.3869; Holding cost=350.9669; Ordering cost=153.5108; Purchasing cost=496.6221; Entropy cost=3.2073; Sales revenue=1188.5; Total profit=187.2269.

Example 2: Crisp EOQ model

Let A=500, a=25, b=20, c=10, d=3, $\theta = 0.02$, $h = 0.5$, $\beta = 0.6$, $s = 10.0$, $C_0 = 4.0$, $\Delta_1 = 0.0005$, $\Delta_2 = 0.01$, $\Delta_3 = 0.0004$, *and* $\Delta_4 = 0.02$, in appropriate units. We get the optimum values are T_1=3.2529; Q_1=403.0471; Holding cost=349.4405; Ordering cost=153.7090; Purchasing cost=495.6157; Sales revenue=1186.2; Total profit =187.3875.

Example 3: Fuzzy EnOQ model

Let A=500, a=25, b=20, c=10, d=3, $\theta = 0.02$, $h = 0.5$, $\beta = 0.6$, $s = 10.0$, $C_0 = 4.0$, $\Delta_1 = 0.0005$, $\Delta_2 = 0.01$, $\Delta_3 = 0.0004$, *and* $\Delta_4 = 0.02$, in appropriate units. We get the optimum values are T=3.2558; Q=403.9718; Holding cost=350.4939; Ordering cost=153.5721; Purchasing cost=496.3104; Entropy cost=3.2085; Sales revenue=1187.8; Total profit=186.6192.

Example 4:Fuzzy EOQ model

Let A=500, a=25, b=20, c=10, d=3, $\theta = 0.02$, $h = 0.5$, $\beta = 0.6$, $s = 10.0$, $C_0 = 4.0$, $\Delta_1 = 0.0005$, $\Delta_2 = 0.01$, $\Delta_3 = 0.0004$, *and* $\Delta_4 = 0.02$, in appropriate units. We get the optimum values are T_1=3.2516; Q_1=402.6331; Holding cost=348.9691; Ordering cost=153.7705; Purchasing cost=495.3046; Sales revenue=1185.4; Total profit=186.7798.

7. Sensitivity Analysis

We now study the sensitivity of the models developed in the effects of changes in the system parameters a, b, c, d, θ, h, β, s, and C_0 on the total profit per unit time *TP* by the four models. The sensitivity analysis is shown in Tables.

Table-1: Optimal values of Crisp Entropic Order Quantity (CEnOQ) models

parameters		Optimum values							
		T	Q	Holding cost	Ordering cost	Purchasing cost	Sales revenue	Entropy cost	Total profit
θ	0.02	3.2571	404.3869	350.9669	153.5108	496.6221	1188.5	3.2073	187.2269
	0.04	3.0902	368.0311	308.2217	161.8018	476.3849	1098.7	3.5079	151.9273
	0.06	2.9509	339.6113	275.8026	169.4398	460.3494	1028.1	3.7936	122.0063
	0.08	2.8321	316.7271	250.3721	176.5474	447.3388	970.9373	4.0670	96.0537
	0.10	2.7291	297.8785	229.9009	183.2106	436.5959	923.6235	4.3302	73.1384
a	23	3.2662	400.5562	350.3505	153.0831	490.5470	1173.6	3.1995	179.4129
	24	3.2617	402.4903	350.6781	153.2943	493.5957	1181	3.2033	183.3175
	25	3.2571	404.3869	350.9669	153.5108	496.6221	1188.5	3.2073	187.2269
	26	3.2526	406.3099	351.2896	153.7232	499.6740	1196	3.2112	191.1407
	27	3.2480	408.1954	351.5736	153.9409	502.7036	1203.4	3.2152	195.0593
b	18	3.2653	395.8865	344.7395	153.1253	484.9619	1160.4	3.1998	177.4112
	19	3.2612	400.1493	347.8659	153.3178	490.8001	1174.5	3.2035	182.3172
	20	3.2571	404.3869	350.9669	153.5108	496.6221	1188.5	3.2073	187.2269
	21	3.2532	408.6639	354.1159	153.6948	502.4762	1202.6	3.2109	192.1399
	22	3.2492	412.8843	357.2039	153.8840	508.2904	1216.6	3.2146	197.0565
c	8	3.2640	382.2782	329.9779	153.1863	468.4782	1121.3	3.2001	169.4997
	9	3.2605	393.3551	340.4984	153.3507	482.5702	1154.9	3.2037	178.3620
	10	3.2571	404.3869	350.9669	153.5108	496.6221	1188.5	3.2073	187.2269
	11	3.2540	415.4433	361.4627	153.6570	510.6862	1222.1	3.2106	196.0937
	12	3.2511	426.4998	371.9581	153.7941	524.7452	1255.6	3.2137	204.9625
d	1	3.2486	343.0973	286.2065	153.9125	422.4556	1012.3	3.2114	149.6232
	2	3.2534	373.7399	318.5700	153.6854	459.5069	1100.3	3.2090	168.4222
	3	3.2571	404.3869	350.9669	153.5108	496.6221	1188.5	3.2073	187.2269
	4	3.2601	435.0591	383.4123	153.3695	533.7984	1276.8	3.2061	206.0355
	5	3.2626	465.7575	415.9026	153.2520	571.0262	1365.2	3.2051	224.8469
h	0.3	3.4806	480.9051	373.4879	143.6534	552.6692	1318.1	3.0116	248.1540
	0.4	3.3654	440.1635	361.7621	148.5707	523.1634	1249.9	3.1092	216.2870
	0.5	3.2571	404.3869	350.9669	153.5108	496.6221	1188.5	3.2073	187.2269
	0.6	3.1554	372.9115	341.0820	158.4585	472.7280	1133.1	3.3056	160.6250
	0.7	3.0601	345.1953	332.1147	163.3934	451.2209	1083.1	3.4036	136.1794
β	0.4	3.7343	581.1042	444.1643	133.8939	622.4505	1478.8	3.8179	278.1551
	0.5	3.4651	475.2571	389.0014	144.2960	548.6216	1308.8	3.0243	226.7008
	0.6	3.2571	404.3869	350.9669	153.5108	496.6221	1188.5	3.2073	187.2269
	0.7	3.0903	353.7970	323.3125	161.7966	457.9451	1098.7	3.3719	155.4914
	0.8	2.9527	315.9258	302.3819	169.3365	427.9823	1029	3.5217	129.0987
s	9.8	3.2072	388.6927	333.1813	155.8992	484.7752	1137.8	3.1897	163.7824
	9.9	3.2322	396.4945	341.9970	154.6934	490.6807	1163	3.1985	175.4358
	10.0	3.2571	404.3869	350.9669	153.5108	496.6221	1188.5	3.2073	187.2269
	10.1	3.2821	412.4348	360.1663	152.3415	502.6474	1214.5	3.2159	199.1568

	10.2	3.3072	420.6410	369.6008	151.1853	508.7579	1240.9	3.2244	211.2271
C_0	3.8	3.3111	421.9275	371.0848	151.0072	484.2272	1218.8	3.1576	212.3344
	3.9	3.2841	413.0840	360.9107	152.2487	490.5538	1203.6	3.1822	199.6987
	4.0	3.2571	404.3869	350.9669	153.5108	496.6221	1188.5	3.2073	187.2269
	4.1	3.2303	395.8973	341.3203	154.7844	502.4855	1173.7	3.2326	174.9166
	4.2	3.2035	387.5484	331.8926	156.0793	508.1015	1159	3.2583	162.7658

Table-2: Optimal values of Crisp Economic Order Quantity (CEOQ) models

Parameters		Optimum values						
		T_1	Q_1	Holding cost	Ordering cost	Purchasing cost	Sales revenue	Total profit
θ	0.02	3.2529	403.0471	349.4405	153.7090	495.6157	1186.2	187.3875
	0.04	3.0857	366.6638	306.7194	162.0378	475.3071	1096.3	152.2453
	0.06	2.9462	338.2405	274.3432	169.7101	459.2227	1025.8	122.4801
	0.08	2.8272	315.3472	248.9431	176.8534	446.1618	968.6402	96.6818
	0.10	2.7240	296.4851	228.4931	183.5536	435.3672	921.3334	73.9194
a	23	3.2620	399.2177	348.8204	153.2802	489.5373	1171.2	179.5736
	24	3.2575	401.1511	349.1498	153.4919	492.5877	1178.7	183.4782
	25	3.2529	403.0471	349.4405	153.7090	495.6157	1186.2	187.3875
	26	3.2483	404.9375	349.7288	153.9267	498.6454	1193.6	191.3014
	27	3.2438	406.8541	350.0509	154.1402	501.7006	1201.1	195.2198
b	18	3.2610	394.5377	343.1980	153.3272	483.9469	1158	177.5726
	19	3.2569	398.7890	346.3138	153.5202	489.7774	1172.1	182.4783
	20	3.2529	403.0471	349.4405	153.7090	495.6157	1186.2	187.3875
	21	3.2490	407.3127	352.5790	153.8935	501.4623	1200.2	192.3002
	22	3.2451	411.5544	355.6934	154.0785	507.2933	1214.3	197.2164
c	8	3.2595	380.9387	328.4515	153.3978	467.4811	1119	169.6606
	9	3.2561	391.9983	338.9525	153.5579	481.5556	1152.6	178.5229
	10	3.2529	403.0471	349.4405	153.7090	495.6157	1186.2	187.3875
	11	3.2499	414.0916	359.9230	153.8509	509.6669	1219.7	196.2542
	12	3.2471	425.1385	370.4075	153.9836	523.7147	1253.2	205.1229
d	1	3.2431	341.7533	284.7095	154.1735	421.5144	1010.2	149.7774
	2	3.2486	372.3882	317.0450	153.9125	458.5214	1098.1	168.5801
	3	3.2529	403.0471	349.4405	153.7090	495.6157	1186.2	187.3875
	4	3.2564	433.7400	381.8980	153.5438	532.7847	1274.4	206.1981
	5	3.2592	464.4177	414.3549	153.4119	569.9775	1362.8	225.0111
h	0.3	3.4770	479.5886	372.1049	143.8021	551.7269	1315.9	248.3151
	0.4	3.3615	438.8334	360.3066	148.7431	522.1876	1247.7	216.4479
	0.5	3.2529	403.0471	349.4405	153.7090	495.6157	1186.2	187.3875
	0.6	3.1509	371.5647	339.4855	158.6848	471.6934	1130.6	160.7853
	0.7	3.0553	343.8434	330.4483	163.6501	450.1599	1080.6	136.3396
β	0.4	3.7308	579.6186	442.7283	134.0195	621.4416	1476.5	278.3168
	0.5	3.4612	473.8443	387.5090	144.4586	547.6069	1306.4	226.8620

	0.6	3.2529	403.0471	349.4405	153.7090	495.6157	1186.2	187.3875
	0.7	3.0859	352.5334	321.7745	162.0273	456.9602	1096.4	155.6515
	0.8	2.9480	314.6910	300.7876	169.6065	426.9891	1026.6	129.2584
s	9.8	3.2029	387.3631	331.6840	156.1085	483.7654	1135.5	163.9397
	9.9	3.2279	395.1439	340.4672	154.8995	489.6607	1160.6	175.5949
	10.0	3.2529	403.0471	349.4405	153.7090	495.6157	1186.2	187.3875
	10.1	3.2780	411.1064	358.6442	152.5320	501.6551	1212.2	199.3191
	10.2	3.3031	419.2919	368.0460	151.3730	507.7556	1238.6	211.3912
C_0	3.8	3.3071	420.6081	369.5628	151.1899	483.2967	1216.5	212.4952
	3.9	3.2799	411.7216	359.3489	152.4437	489.5619	1201.2	199.8594
	4.0	3.2529	403.0471	349.4405	153.7090	495.6157	1186.2	187.3875
	4.1	3.2259	394.5170	339.7576	154.9955	501.4165	1171.2	175.0772
	4.2	3.1990	386.1602	330.3308	156.2988	506.9937	1156.5	162.9264

Table-3: Optimal values of Fuzzy Entropic Order Quantity (FEnOQ) models

Parameters		Optimum values							
		T	Q	Holding cost	Ordering cost	Purchasing cost	Sales revenue	Entropy cost	Total profit
θ	0.02	3.2558	403.9718	350.4939	153.5721	496.3104	1187.8	3.2085	186.6192
	0.04	3.0890	367.6661	307.8204	161.8647	476.0973	1098	3.5092	151.3443
	0.06	2.9497	339.2609	275.4294	169.5088	460.0615	1027.5	3.7949	121.4431
	0.08	2.8309	316.3887	250.0215	176.6223	447.0503	970.3743	4.0685	95.5065
	0.10	2.7279	297.5501	229.5690	183.2912	436.3065	923.0842	4.3317	72.6045
a	23	3.2649	400.1415	349.8763	153.1440	490.2343	1172.8	3.2007	178.8126
	24	3.2603	402.0435	350.1681	153.3601	493.2595	1180.3	3.2046	182.7136
	25	3.2558	403.9718	350.4939	153.5721	496.3104	1187.8	3.2085	186.6192
	26	3.2513	405.8946	350.8172	153.7846	499.3628	1195.3	3.2124	190.5293
	27	3.2467	407.7798	351.1017	154.0025	502.3930	1202.7	3.2165	194.4441
b	18	3.2639	395.4470	344.2370	153.1910	484.6312	1159.6	3.2011	176.8179
	19	3.2598	399.7060	347.3599	153.3836	490.4669	1173.7	3.2048	181.7168
	20	3.2558	403.9718	350.4939	153.5721	496.3104	1187.8	3.2085	186.6192
	21	3.2518	408.2131	353.6030	153.7610	502.1380	1201.8	3.2122	191.5251
	22	3.2479	412.4623	356.7244	153.9456	507.9741	1215.9	3.2158	196.4345
c	8	3.2627	381.8909	329.5364	153.2473	468.1900	1120.6	3.2013	168.9265
	9	3.2591	392.9230	340.0059	153.4166	482.2472	1154.2	3.2050	177.7717
	10	3.2558	403.9718	350.4939	153.5721	496.3104	1187.8	3.2085	186.6192
	11	3.2527	415.0143	360.9739	153.7184	510.3629	1221.3	3.2118	195.4688
	12	3.2497	426.0229	371.4148	153.8604	524.3843	1254.8	3.2150	204.3205
d	1	3.2474	342.8038	285.8794	153.9693	422.2501	1011.9	3.2125	149.1062
	2	3.2521	373.3735	318.1564	153.7468	459.2398	1099.7	3.2102	167.8599
	3	3.2558	403.9718	350.4939	153.5721	496.3104	1187.8	3.2085	186.6192
	4	3.2588	434.5953	382.8797	153.4307	533.4421	1275.9	3.2073	205.3822
	5	3.2612	465.2054	415.2647	153.3178	570.5942	1364.2	3.2064	224.1482

h	0.3	3.4792	480.3928	372.9496	143.7112	552.3026	1317.3	3.0127	247.4777
	0.4	3.3641	439.7198	361.2764	148.6282	522.8379	1249.2	3.1104	215.6468
	0.5	3.2558	403.9718	350.4939	153.5721	496.3104	1187.8	3.2085	186.6192
	0.6	3.1542	372.5520	340.6557	158.5188	472.4519	1132.4	3.3068	160.0465
	0.7	3.0589	344.8569	331.6974	163.4575	450.9555	1082.4	3.4049	135.6273
β	0.4	3.7326	580.3823	443.4663	133.9549	621.9603	1477.7	2.8191	277.3934
	0.5	3.4636	474.7133	388.4269	144.3585	548.2311	1307.9	3.0256	226.0296
	0.6	3.2558	403.9718	350.4939	153.5721	496.3104	1187.8	3.2085	186.6192
	0.7	3.0892	353.4807	322.9274	161.8542	457.6987	1098.1	3.3730	154.9309
	0.8	2.9517	315.6628	302.0421	169.3939	427.7708	1028.5	3.5228	128.5749
s	9.8	3.2059	388.2904	332.7280	155.9624	484.4697	1137.1	3.1909	163.1892
	9.9	3.2308	396.0544	341.4983	154.7604	490.3484	1162.2	3.1998	174.8355
	10.0	3.2558	403.9718	350.4939	153.5721	496.3104	1187.8	3.2085	186.6192
	10.1	3.2808	412.0132	359.6831	152.4019	502.3326	1213.7	3.2171	198.5417
	10.2	3.3058	420.1799	369.0693	151.2493	508.4154	1240.1	3.2257	210.6047
C_0	3.8	3.3097	421.4654	370.5515	151.0711	483.9014	1218	3.1589	211.7107
	3.9	3.2827	412.6295	360.3895	152.3136	490.2230	1202.8	3.1835	199.0831
	4.0	3.2558	403.9718	350.4939	153.5721	496.3104	1187.8	3.2085	186.6192
	4.1	3.2289	395.4577	340.8224	154.8515	502.1452	1172.9	3.2339	174.3168
	4.2	3.2022	387.1470	331.4408	156.1427	507.7813	1158.3	3.2596	162.1737

Table-4: Optimal values of Fuzzy Economic Order Quantity (FEOQ) models

Parameters		Optimum values						
		T_1	Q_1	Holding cost	Ordering cost	Purchasing cost	Sales revenue	Total profit
θ	0.02	3.2516	402.6331	348.9691	153.7705	495.3046	1185.4	186.7798
	0.04	3.0845	366.2999	306.3198	162.1008	475.0201	1095.7	151.6624
	0.06	2.9449	337.8621	273.9406	169.7851	458.9115	1025.1	121.9169
	0.08	2.8260	315.0100	248.5942	176.9285	445.8740	968.0783	96.1346
	0.10	2.7228	296.1580	228.1630	183.6345	435.0786	920.7953	73.3854
a	23	3.2607	398.8041	348.3479	153.3413	489.2251	1170.5	178.9733
	24	3.2561	400.7055	348.6416	153.5579	492.2521	1177.9	182.8743
	25	3.2516	402.6331	348.9691	153.7705	495.3046	1185.4	186.7798
	26	3.2470	404.5233	349.2580	153.9883	498.3348	1192.9	190.6899
	27	3.2425	406.4397	349.5807	154.2020	501.3905	1200.4	194.6047
b	18	3.2597	394.1306	342.7330	153.3883	483.6404	1157.3	176.9793
	19	3.2556	398.3785	345.8456	153.5815	489.4686	1171.4	181.8777
	20	3.2516	402.6331	348.9691	153.7705	495.3046	1185.4	186.7798
	21	3.2476	406.8631	352.0680	153.9598	501.1247	1199.5	191.6854
	22	3.2437	411.1010	355.1789	154.1450	506.9532	1213.5	196.5945
c	8	3.2582	380.5524	328.0117	153.4590	467.1934	1118.3	169.0874
	9	3.2548	391.5982	338.4969	153.6193	481.2562	1151.9	177.9324
	10	3.2516	402.6331	348.9691	153.7705	495.3046	1185.4	186.7798

	11	3.2486	413.6638	359.4359	153.9125	509.3441	1218.9	195.6293
	12	3.2457	424.6628	369.8661	154.0500	523.3544	1252.4	204.4808
d	1	3.2419	341.4606	284.3837	154.2305	421.3092	1009.7	149.2604
	2	3.2473	372.0227	316.6330	153.9741	458.2548	1097.4	168.0179
	3	3.2516	402.6331	348.9691	153.7705	495.3046	1185.4	186.7798
	4	3.2550	433.2418	381.3263	153.6098	532.4016	1273.5	205.5449
	5	3.2578	463.8669	413.7190	153.4778	569.5462	1361.8	224.3124
h	0.3	3.4756	479.0774	371.5681	143.8601	551.3608	1315.1	247.6387
	0.4	3.3601	438.3567	359.7853	148.8051	521.8377	1246.9	215.8077
	0.5	3.2516	402.6331	348.9691	153.7705	495.3046	1185.4	186.7798
	0.6	3.1497	371.2062	339.0608	158.7453	471.4178	1130	160.2069
	0.7	3.0541	343.5061	330.0328	163.7144	449.8950	1080	135.7875
β	0.4	3.7291	578.8981	442.0321	134.0806	620.9521	1475.4	277.5551
	0.5	3.4597	473.3018	386.9363	144.5212	547.2171	1305.5	226.1907
	0.6	3.2516	402.6331	348.9691	153.7705	495.3046	1185.4	186.7798
	0.7	3.0847	352.1894	321.3561	162.0903	456.6919	1095.8	155.0911
	0.8	2.9470	314.4287	300.4493	169.6641	426.7780	1026.1	128.7346
s	9.8	3.2016	386.9618	331.2324	156.1719	483.4605	1134.8	163.3465
	9.9	3.2266	394.7363	340.0058	154.9619	489.3526	1159.9	174.9945
	10.0	3.2516	402.6331	348.9691	153.7705	495.3046	1185.4	186.7798
	10.1	3.2766	410.6536	358.1256	152.5972	501.3167	1211.4	198.7040
	10.2	3.3017	418.8320	367.5163	151.4371	507.4137	1237.8	210.7686
C_0	3.8	3.3057	420.1470	369.0314	151.2539	482.9714	1215.8	211.8714
	3.9	3.2786	411.3006	358.8666	152.5041	489.2552	1200.5	199.2438
	4.0	3.2516	402.6331	348.9691	153.7705	495.3046	1185.4	186.7798
	4.1	3.2246	394.1099	339.2970	155.0580	501.1010	1170.5	174.4774
	4.2	3.1977	385.7599	329.8807	156.3624	506.6740	1155.8	162.3343

The three dimensional graphs are shown in the following Fig. 2 and Fig.3

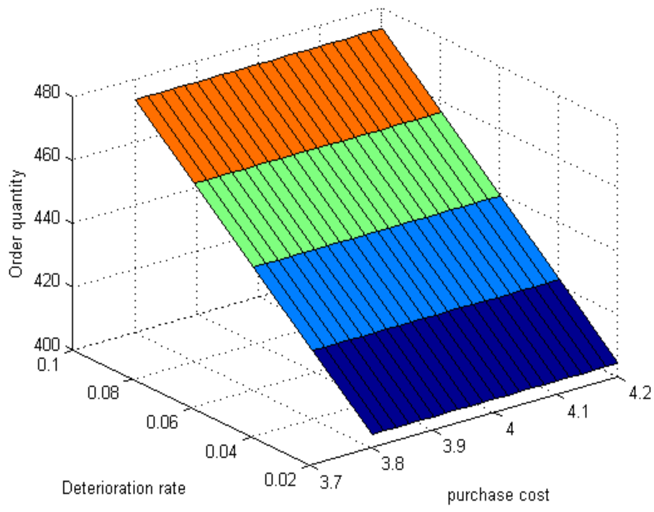

Fig.2: Effect of changing parameters θ and C_0 in Order quantity

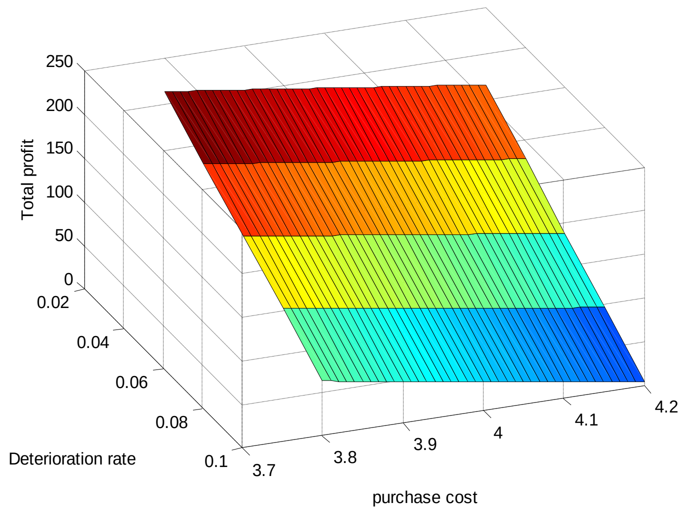

Fig.3: Effect of changing parameters θ and C_0 in Profit

8. Observations

I. The comparative analysis of the two models which are Crisp Entropic Order Quantity (CEnOQ) model and Crisp Economic Order Quantity (CEOQ) model are given below by using the Tables 1 and 2.

(i) When $\theta, a, b, c, d, h, \beta, s$ *and* C_0 increases, the cycle time (T_1), the order quantity (Q_1), the holding cost (HC), the purchasing cost (PC) and the sales revenue (SR) are decreasing more in CEOQ model, than the CnEOQ model whereas Ordering cost and Total profit are more decreasing in CnEOQ than the CEOQ model.

II. The comparative analysis of the two models which are Fuzzy Entropic Order Quantity (FEnOQ) model and Fuzzy Economic Order Quantity (FEOQ) model are given below by using the Tables 3 and 4.

(i) When $\theta, a, b, c, d, h, \beta, s$ *and* C_0 increases, the cycle time (T_1), the order quantity (Q_1), the holding cost (HC), the purchasing cost (PC) and the sales revenue (SR) are decreasing more in FEOQ model, than the FnEOQ model whereas Ordering cost and Total profit are more decreasing in FnEOQ than the FEOQ model.

Figure 2 shows that in CEnOQ model, the Total Profit (TP) decreases for increasing values of the deterioration rate (θ) and the purchasing cost (C_0).

Figure 3 shows that in CEOQ model, the Order quantity (Q_1) is decreasing for increasing values of the deterioration rate (θ) and the purchasing cost (C_0).

9. Conclusion

In this paper, we presented an entropic economic order quantity model for perishable items with cubic demand rate and time dependent holding cost.To capture the real life situation we have considered that the orderingcostand purchasing cost parameters are uncertain. Optimal results of fuzzy model are defuzzified by signed distance method. Numerical experiments of the solution from the entropic model computed and compared to the solutions of other different traditional EOQ model. The sensitivity analysis in this model for different parameter values establishes that the optimal value of the cycle time T_1, the order quantity Q_1 are decreasing in CEOQ model, where as it is increasing in CEnOQ model and consequently the total profit is decreasing in CEnOQ model, where as it is increasing in CEOQ model.

References

[1] S. K. Goyal., Economic order quantity under conditions of permissible delay in payments. Journal of Operational Research Society. 36,335-338(1985).

[2] E. Raafat., Survey of Literature on Continuously Deteriorating Inventory Model. Journal of Operational Research Society. 42, pp.27-37,(1991).

[3] K. Richter., The EOQ and waste disposal model with variable setup numbers. European Journal of Operational Research. 95(2), pp.313-324(1996a).

[4] K. Richter., The extended EOQ repair and waste disposal model. International Journal of Production Economics. 45(1-3),pp.443-447.(1996b)

[5] R. Whewell., Turning up the heat under the supply-chain. Logistics Focus 5(4), pp.18-22,(1997).

[6] S. C. Chang., J.S. Yao and H.M. Lee, Economic reorder point for fuzzy backorder quantity, European Journal of Operational Research, 109, pp. 183-202.(1998).

[7] S.K. Goyal., B. C. Giri., Recent trends in modelling of deteriorating inventory. European Journal of Operational Research.134, pp. 1-16.(2001).

[8] C. T. Chang., An EOQ model with deteriorating items under inflation when supplier credits linked to order quantity. International Journal of Production Economics. 88, pp.307-316,(2004).

[9] M.Y. Jaber., R.Y. Nuwayhid and R.Y. Rosen., Price-driven economic order systems from a thermodynamic point of view. International Journal of Production Research 42, pp.5167-5184(2004).

[10] J. K. Dye., S. Kar and M. Maiti., An interactive method for inventory control with fuzzy lead-time and dynamic demand, European Journal of Operational Research, 167(2005) 381-397.

[11] T.L. Urban., Inventory model with inventory level dependent demand a comprehensive review and unifying theory. European Journal of Operational Research 162, pp.792-804,(2005).

[12] M.Y. Jaber., Lot sizing for an imperfect production process with quality corrective interruptions and improvemants, and reduction in setups. Computers and Industrial Engineering 51, pp.781-790.(2006).

[13] M.Y. Jaber., R.Y. Nuwayhid and M.A. Rosen., A thermodynamic approach to modelling the economic order quantity. Applied Mathematical Modelling 30, pp.867-88, (2006).

[14] P.L. Abad., Optimal price and order size under partial backordering incorporating shortage, backorder and lost sale costs. International Journal of Production Economics,Vol. 114,no.1, pp.179-186(2008).

[15] M.Y. Jaber., Lot sizing with permissible delay in payments and entropy cost. Computers and Industrial Engineering, 52, pp.78-88,(2007).

[16] Y.D.Chung., P.H. Tsu and Y.Liang., Determining optimal selling price and lot-size with a varying rate of deterioration and exponential partial backlogging. European Journal of Operational Research,181,pp.668-678,(2007).

[17] A. Eroglu., A. Ozdemir., An economic order quantity model with defective items and shortages. International Journal of Production Economics, 106, pp.544-549,(2007).

[18] M.Y. Jaber., S.K.Goyal and M.Imran., Economic production quantity model for items with imperfect quality subject to learning effects, International Journal of Production Economics,115, pp.143-150,(2008).

[19] M.Pattnaik., A Note on Non-linear Profit-Maximization Entropic Order Quantity (EnOQ) Model for Deteriorating Items with Stock Dependent Demand Rate, vol.5,no.2,pp.97-102,(2012).