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Cost minimization of Pressure Vessel design problem using PSO, SA, PS, GODLIKE, CUCKOO, FF, FP, ALO, GSA and MVO

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Abstract

The objective functions used in Engineering Optimization are complex in nature with many variables and constraints. Conventional optimization tools sometimes fail to give global optima point. Very popular methods like Genetic Algorithm, Pattern Search, Simulated Annealing, and Gradient Search are useful methods to find global optima related to engineering problems. This paper attempts to use new non-traditional optimization algorithms which are used to find the minimum cost of designing a pressure vessel to obtain global optimum solutions. The cost, number of iterations and the total elapsed time to complete the problems are all compared using these ten non-traditional optimization methods.

Keywords: Pattern search, Simulate annealing, Pattern search, GODLIKE, Cuckoo search, Firefly algorithm, Flower pollination, Ant lion optimizer, Gravitational search algorithm, Multi-verse optimizer

1. Introduction

Large vessels were invented in Great Britain during the industrial revolution for making steam to drive steam engines. Pressure vessels are used to store fluids under pressure. The fluid may undergo a change of state inside the pressure vessel or it may combine with other reagents as in a chemical plant. The material pressure vessel may be brittle such as cast iron, or ductile such as mild steel to avoid explosion. Pressure vessel may be cylinders or tanks. (Khurmi R S)

Pressure vessels may be classified as

i) according to the dimensions: According to the dimensions the pressure vessels are classified as thin shell or thick shell. If the wall thickness of the shell is less than 1/10 of the diameter of the shell then it is called thin shell. On the other hand, if the wall thickness of the shell is greater than 1/10 of the diameter of the shell then it is said to be thick shell. In case of thick shells, the stresses are no longer uniformly distributed and problem becomes complex. Thin shells are used in high pressure cylinders, tanks, gun barrels etc.

ii) According to the end construction: According to the end construction the pressure vessels are classified as open end or closed end, circumferential or hoop stresses are induced by fluid pressure for open end and pressure vessels. Longitudinal stresses are induced with circumferential stresses in closed ends pressure vessels.

iii) Pressure vessels are used in various applications in industry and the private sector. Mainly used as industrial compressed air receivers and domestic hot water storage tanks. Examples of pressure vessels are diving cylinders, recompression chambers, pressure reactor, distillation towers, oil refineries, nuclear reactor vessels, rail vehicle air break reservoirs, submarine and spaceship habitats, road vehicles air brake reservoirs and storage vessels for liquefied gases a such as chlorine, propane, ammonia and LPG.

Nomenclature

R inner radius of the shell
L length of the shell
T_h thickness of the head
T_s thickness of the shell
C_1 cost of longitudinal weld of the cylinder
C_2 welding cost for the spherical shell of the cylindrical shell
C_3 material cost of pressure vessel
ρ density
C_{weld} cost of weld material per Kg
C_s cost of shell per Kg
C_h cost head per Kg

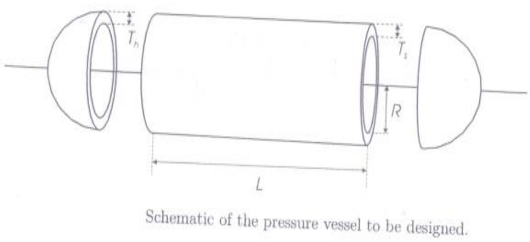


Fig.1Schematic of the pressure vessel to be designed

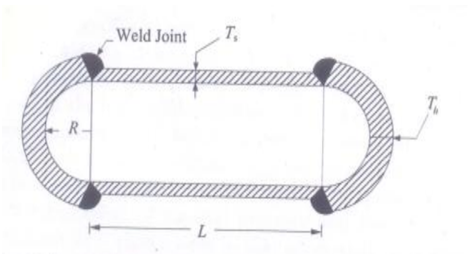


Fig.2 A cross sectional view of a pressure vessel in which two hemi-spherical heads are welded to a cylindrical shell

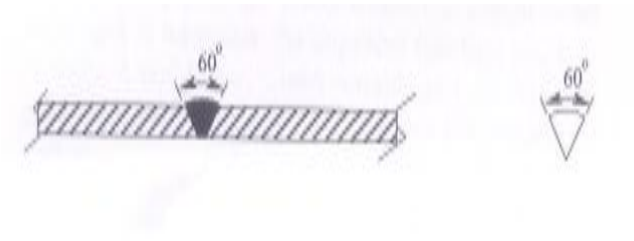


Fig.3 Welding details of 60° single V-groove butt joint (a) actual profile (b) approximate profile.

2, PROBLEM DEFINITION

The problem requires designing a pressure vessel consisting of a cylindrical body and two hemispherical heads such that the cost of its manufacturing is minimized subject to certain constraints. The schematic picture of te vessel is presented in Figure.1. There are four variables for which values must be chosen: the thickness of the main cylinder T_s, the thickness of the heads T_h, the inner radius of the main cylinder$R$, and the length of the main cylinder c. While variables R and L are continuous, the thickness of the variables T_s and T_h may be chosen only from a set of allowed values, these being the integer multiples of 0.0625 inch.

2.1. FORMULATION OF PROBLEM:

The problem is to design a compressed air storage tank with pressure of 1000 psi and minimum volume of 750 ft^3. Cylindrical pressure vessel is clapped at both ends by hemispherical heads.

2.2. DESIGN VARIABLES: (Hasancebi.O, 2012)

The four design variables for the problem are defined as

Inner radius of the shell : R

Length of the shell : L

Thickness of the head : T_h

Thickness of the shell : T_s

The main objective is to minimize the overall manufacturing cost of the pressure vessel. Overall manufacturing cost of the pressure vessel includes (1) material cost (2) welding cost and (3) forming cost.
Weld joint joins the hemispherical head and cylindrical shell. This joint is very strong and no failure at the joint.
The standard sizes of sheet thickness are readily available in the market.

2.2.1 Welding cost:
Welding cost of the pressure vessel is calculated by welding the two rolled sheets to make a cylindrical shell using 60^o angle V-groove butt joint. It is approximately as one sixth sector of a circle of radius $\frac{T_s}{\cos 30^\circ}$

Cost of longitudinal weld for a cylinder is

$C_1 = 2\pi \rho L C_{weld} \left(\frac{T_s}{\cos 30^\circ}\right)^2 \left(\frac{60^\circ}{360^\circ}\right) \pi \left(\frac{T_s}{\cos 30^\circ}\right)^2 \left(\frac{60^\circ}{360^\circ}\right)$ represents area of one-sixth sector of a circle, ρ represents density and 2 signifies welding of two rolled sheets longitudinally at two places to make cylindrical shell, C_{weld} is the cost of weld material per kg.
The hemispherical heads are forged. These are welded at the ends of the cylindrical shell using same butt joint. The welding cost for the spherical heads to the cylindrical shell is $C_2 = 4\pi^2 \rho R C_{weld} \left(\frac{T_h}{\cos 30^\circ}\right)^2 \left(\frac{60^\circ}{360^\circ}\right) \pi \left(\frac{T_h}{\cos 30^\circ}\right)^2 \left(\frac{60^\circ}{360^\circ}\right)$ represents area of one-sixth sector of a circle and R is radius of radius of hemispherical head. (Nayan Jyothi Baishya)

2.2.2. MATERIAL COST:
The material cost of the pressure vessel is

$C_3 = 2\pi\rho RLC_s T_s + 4\pi\rho R^2 C_h T_h$

Where C_s represents cost of shell per kg and C_h represents the cost of head per kg.
The total cost of the pressure vessel is $C_1 + C_2 + C_3$
The objective is to minimize the total cost of the pressure vessel by reducing the weight of the vessel which is a non-linear function of four variables under the non-linear constraint of the stresses and yield criteria. The thickness can only take integer multiple of 0.0625 inches.
The objective function is minimize

$f(T_s, T_h, R, L) = 0.6224 T_s RL + 1.7781 T_h R^2 + 3.1611 T_s^2 L + 19.84 T_s^2 R$ The coefficients used in the objective functions are from conversion of units from imperial to metric ones.

2.3. DESIGN CONSTRAINTS: (Al-Milli, Nabeel, 2014)
The four important constraints under consideration are

2.3.1. Stress constraints:
There are two constraints about stresses, to satisfy these two conditions, the hoop

stresses $\frac{T_s}{R}$ and $\frac{T_h}{R}$ should be as small as possible or less than allowable stress.

$$Hoop\ stress \leq Allowable\ stress$$

$$R \leq \frac{T_s}{0.0193}$$

$$0.0193\,R \leq T_s \qquad (1)$$

$$Longitudinal\ stress \leq Allowable\ stress$$

$$R \leq \frac{T_h}{0.00954}$$

$$0.00954\,R \leq T_h \qquad (2)$$

2.3.2. Volume constraint:

It gives the minimum capacity or volume of a pressure vessel. Mathematically volume constraint is expressed by

$$Volume \geq 750 \times 1728\ \ inch^3$$

$$\frac{4}{3}\pi R^3 + \pi R^2 L \geq 750 \times 1728$$

$$1296000 \leq 1.3333\pi R^3 + \pi R^2 L \qquad (3)$$

2.3.3. Width constraint:

It represents the limit on the width of a sheet influenced by capacity of rolling equipment.

It is assumed that the width should be less than 200 mm. The mathematical form of width constraint is given as

$$width of\ sheet \leq 240$$

$$L \leq 240 \qquad (4)$$

2.3.4. Variables bounds:

The upper bounds and lower bounds of design variables are

$$10 \leq R \leq 200$$

$$10 \leq L \leq 200$$

$$0.0625 \leq T_s \leq 99 \times 0.0625$$

$$0.0625 \leq T_h \leq 99 \times 0.0625$$

	Thickness ($T=x_1$)		Head Thickness ($T_h=x_2$)		Inner Radius ($R=x_3$)		Length of cylinder ($L=x_4$)	
	inch	mm	inch	mm	inch	mm	inch	mm
Upper Bound	6.1875	157.162	6.1875	157.162	200	5080	200	5080
Lower Bound	0.0625	1.5875	0.0625	1.5875	10	254	10	254
Optimum	0.9405	23.8895	0.4649	11.808	48.732	1237.78	115.74	2939.90

2.4. MATHEMATICAL FORMULATION: (Harish Garg;, 2014) (Xia Jian Sheng, Dou Sha Sha, Yang Zirun, Li Qingzhu, 2014)
The objective is to minimize the total cost including the cost of the material, forming and welding. The four design variables associated with the pressure vessel design are thickness of the vessel $T_s = x_1$, thickness of the head $T_h = x_2$, inner radius of the vessel $R = x_3$, and length of the vessel without heads $L = x_4$. The variables vectors are given (in inches) by $X = (T_s, T_h\ R, L) = (x_1, x_2, x_3, x_4)$
The objective is to minimize the cost of the pressure vessel design problem.
The mathematical model of the problem is summarized as (Xuesong Yan, 2012)
Minimize

$$f(X) = 0.6224 x_1 x_3 x_4 + 1.7781 x_2 x_3^2 + 3.1611 x_1^2 x_4 + 19.84 x_1^2 x_3$$

Subject to

$$0.0193\ x_3 \le x_1$$
$$0.00954\ x_3 \le x_2$$
$$1296000 \le 1.333\pi x_3^3 + \pi x_3^2 x_4$$
$$x_4 \le 240$$

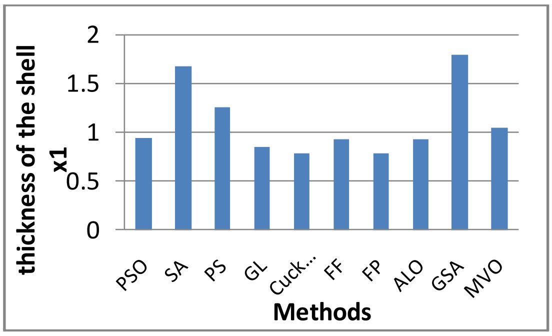
Variable region is

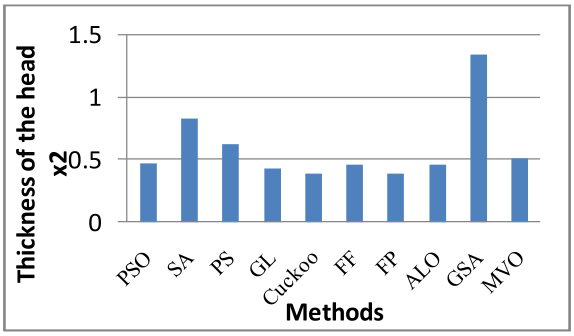
$$0.0625 \le x_1 \le 99 \times 0.0625$$
$$0.0625 \le x_2 \le 99 \times 0.0625$$
$$10 \le x_3 \le 200$$
$$10 \le x_4 \le 200$$

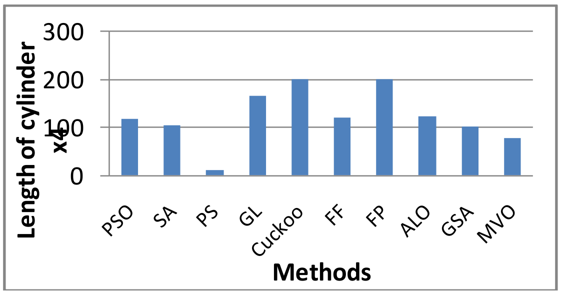
3. Comparative Results

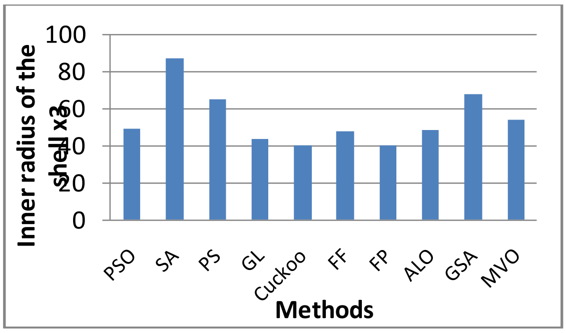
The ten methods are run 20 trails and the average is taken and the results were compared.
Thickness of the shell ($x_{1\ inch}$)
Thickness of head ($x_{2\ inch}$)
Inner radius of the shell ($x_{3\ inch}$)
Length of Cylinder($x_{4\ inch}$)

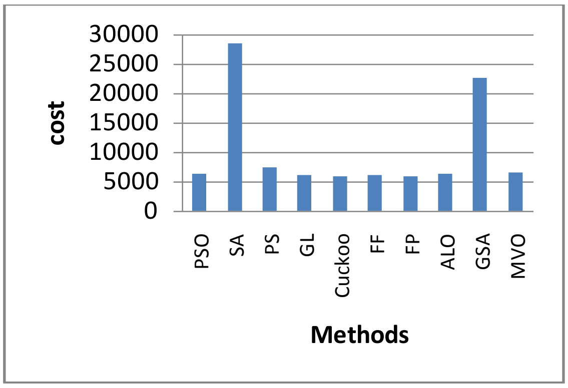
Trial No	**PSO**	**SA**	**PS**	**GL**	**Cuckoo**	**FF**	**FP**	**ALO**	**GSA**	**MVO**
X_1	0.940532	1.67692	1.254501	0.846077	0.778187	0.922941	0.778189	0.930615	1.794241	1.041341
X_2	0.464905	0.828902	0.620102	0.425243	0.384658	0.456301	0.384661	0.460004	1.33396	0.515303
X_3	48.73222	86.88705	65	43.53558	40.32058	47.81747	40.32068	48.2184	67.62963	53.92546
X_4	115.7443	103.8387	10.99561	164.3913	200	119.3964	199.9988	121.3761	102.0901	76.62616
Cost	**6259.574**	28453.46	7300.789	6109.653	5885.047	6194.024	5885.061	6237.925	22623.68	6556.783
Time	1.082004	3.284661	0.451833	3.077241	9.96759	7.327763	5.841572	33.92342	11.64061	5.194229
Iteration	**200**	3398	3	33623	100000	20000	2000	1000	1000	1000

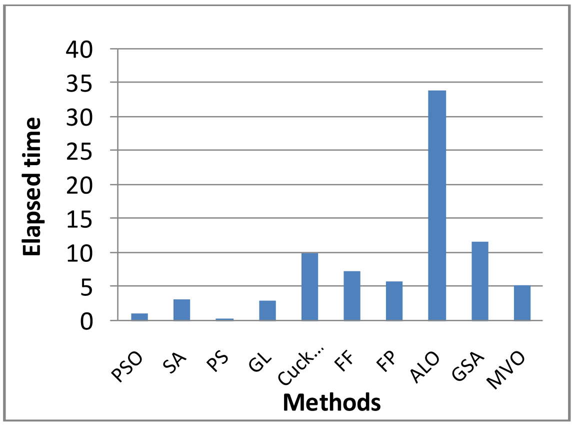


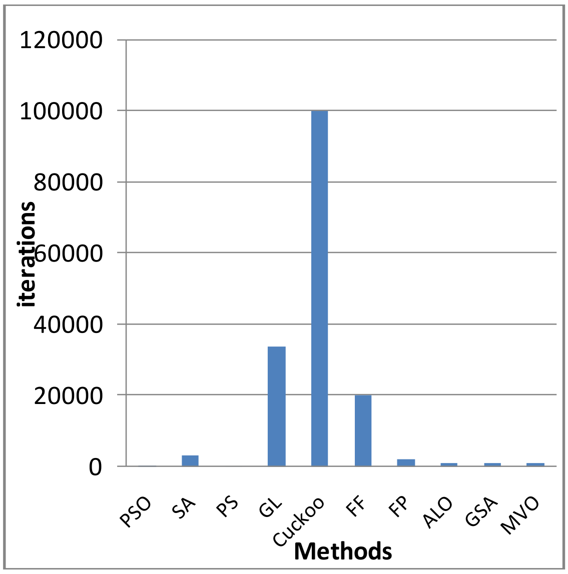












From the above graphs we know that the cost, the number of iteration and the elapsed time is minimum in PSO and PS but PS is high in other four parameters x_1, x_2, x_3, x_4.

4. Results and Discussion:

With the two extreme values of the parameters the optimization is carried out with different solvers. As they are stochastic type the results may vary from trial to trial. So the problem is made to run for 20 trials. (Elbeltagi.E., Tarek Hegazy.I., Grierson D., 2005) And an average of all trials is taken as a final value of the parameter by the solver. The solvers are compared with three different criteria.

4.1. Consistency

The cost is consistent in Pattern Search (7300.789)

4.2. Minimum run time:

For minimum run time of the problem we have PS (0.451833 seconds), PSO (1.082004 seconds).

4.3. Minimum Evaluation:

This Criterion will determine the effectiveness of the algorithm. From the table we see that the PS and PSO algorithm have minimum evaluation of 3 and 200 respectively.

4.4. The Simplicity of Algorithm:

Of all the algorithms, Pattern Search algorithm is the most simplest followed by Particle Swarm Optimization.

Thus it is seen that the PS solver satisfies all the criteria. Even though the pattern search satisfies all the above criteria, the cost becomes maximum whereas the cost in PSO is 6259.574. Therefore the particle swarm optimization has the minimum cost with time 1.082004 seconds and 200 iteration so the appropriate algorithm for pressure vessel design is suggested as Particle Swarm Optimization. It is apparent from the results that PSO algorithm is able to provide promising solutions with less objective function evaluations. This desirable characteristic of PSO algorithm would be more significant in one engineering problems which entail higher computational effort.

<u>Tables for option set and Stopping criteria for the ten methods</u>

methods	PSO	SA	PS	GL	CUCKOO	FF	FP	ALO	GSA	MVO
Option set	Max.Generation =200 Max.Time Limit=∞ Average change in fitness value=10^{-6} Function Tolerance:10^{-6} Cognitive Attraction=0.5 Population Size=40 Social Attraction=1.25	Initial Temperature:100 Annealing Function: Fast Annealing Reannealing Interval :100 Time limit:∞ Max.Function Evaluation:3000* No.of variables. Max.Iteration:∞ Function Tolerance:10^{-6} Objective Limit:10^{-6}	Poll Method:GPS positive basis 2N Initial Mesh Size:1 Expansion Function:2 Contraction Factor:0.5 Mesh Tolerence:10^{-6} Max.Function Evaluation:2000*No.of variables. Max.Iteration:100* No.of variables. Max.Time Limit:∞ Function Tolerance:10^{-6}	Max.Fun Evaluations= 10^{-5} Max.Iterations =20 Min.Iterations =2 Total Iterations=15 Functions Tolerance= 10^{-4}	Max.Fun. Evaluations=10^{-5} Max.Iterations =20 Functions Tolerance= 10^{-6} Max.Time Limit=∞	Max.Fun. Evaluations=10^{-5} Max.Iterations =20 Functions Tolerance= 10^{-6} Max.Time Limit=∞	Max.Fun Evaluations=10^{-5} Max.Iterations =20 Functions Tolerance= 10^{-6} Max.Time Limit=∞	Max.Fun Evaluations=10^{-5} Max.Iterations =20 Functions Tolerance=10^{-6} Max.Time Limit=∞	Max.Fun Evaluations=10^{-5} Max.Iterations =20 Functions Tolerance=10^{-6} Max.Time Limit=∞	Max.Fun Evaluations=10^{-5} Max.Iterations =20 Functions Tolerance= 10^{-6} Max.Time Limit=∞

Stopping criteria	Max.Generation =200 Max.Time Limit=∞ Average change in fitness value=10^{-6} Function Tolerance:10^{-6}	Max.Time reached The average change in value of the objective function is < 10^{-6} max.iterations are reached if the number of functions evaluations reached. If the best objective function value is less than or equal to the value of objective limit.	Mesh Tolerance:10^{-6} Max.Iteration: 100*No.of variables. Evaluation:2000*No.of variables Max.Time Limit:∞ Function Tolerance:10^{-6}	Max.Fun Evaluations= 10^{-5} Max.Iterations =20 Min.Iterations =2 Total Iterations=15 Functions Tolerance= 10^{-4}	Max.Fun Evaluations=10^{-6} Max.number of Iterations= 100000 Functions Tolerance= 10^{-6} Max.Time Limit=∞	Max.Fun Evaluations=10^{-6} Max.number of Iterations= 100000 Functions Tolerance= 10^{-6} Max.Time Limit=∞	Max.Fun Evaluations=10^{-6} Max.number of Iterations= 100000 Functions Tolerance= 10^{-6} Max.Time Limit=∞	Max.Fun Evaluations= 10^{-6} Max.number of Iterations= 100000 Functions Tolerance=10^{-6} Max.Time Limit=∞	Max.Fun Evaluations=10^{-6} Max.number of Iterations= 100000 Functions Tolerance=10^{-6} Max.Time Limit=∞	Max.Fun Evaluations=10^{-6} Max.number of Iterations= 100000 Functions Tolerance= 10^{-6} Max.Time Limit=∞

5. Conclusion:

In the present study the PSO algorithm is proposed as a simple and efficient optimization technique for handling pressure vessel design problem. PSO algorithm is a population based technique which follows a stochastic iterative procedure to locate the optimum or a reasonably near-optimum solution for the pressure vessel design optimization. Performance evaluation of the PSO algorithm through pressure vessel design optimization reveals the efficiency of this technique in solving practical optimization problems. Although in the present study the PSO algorithm is utilized only for solving pressure vessel design optimization problem, it can be easily employed for solving other types of optimization problems as well.

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