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Two Stage Flow shop Scheduling Model with Transportation Time and Job Block

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Abstract

In this paper we proposed the new heuristic algorithm for n job 2 machines $(n \times 2)$ flowshop scheduling problem to find the optimal or near optimal solution when transportation time and job block are given. Transportation Time and Job Block both are momentous criteria in scheduling problems because in most of the manufacturing and service industries, machines are situated distantly and for improving the productivity of production in scheduling, jobs are required to be processed together as block or group. The objective of this paper is to find the optimal or near optimal sequence so as to minimize the total completion time or makespan. With the help of numerical illustration we compared proposed heuristic algorithm to existing heuristic algorithm and the result show that proposed algorithm outperformed. Gantt charts give the complete information of proposed heuristic algorithm as well as existing heuristic algorithm and it is verify the effectiveness of proposed heuristic algorithm.

Key words: Flow shop scheduling, transportation time, job-block, Johnson rule, Makespan, Gantt Chart.

1. INTRODUCTION

Scheduling is one of the most mathematical involved and proposed fields in Industrial Engineering and Operations Management. Sequencing and Scheduling is a form of decision making that play the crucial role in manufacturing and service industries. Scheduling is practical theory and is a term in our daily life. Resources are usually called machines and tasks are called jobs or operations. The environment of the scheduling problem is called shop there are different types of shops using in scheduling problems like job shop, flow shop, mixed shop, open shop etc.. Here we deal with flowshop scheduling environment for two machine *n* jobs. In flowshop environment all the jobs have same route on each machine. Sequencing simply refers to the determination of ORDER in which the jobs are to be processed on various machines. Scheduling refers to the time table that includes the start time and completion time of jobs on machines etc.

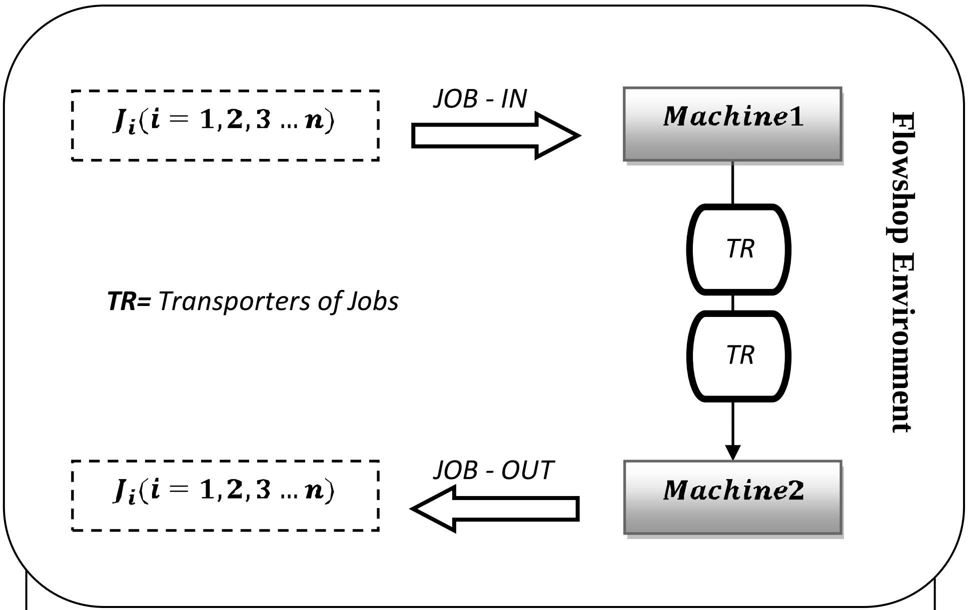

Fig1. Two Stage n jobs Flowshop Scheduling Environment with TR

Fig1. Flowshop Scheduling Environment for n jobs 2 machiens with TR

During last 4 decades many researchers work on scheduling and sequencing. Sequencing is the order of *n* jobs on *m* machine and scheduling is the process in which set of jobs is sequenced. The idea of flowshop sequencing is given by Johonson[9] in 1954. The general flow shop scheduling problem is a production problem where a set of *n* jobs have to be processed with identical flow pattern on *m* machines. Pinedo [15] arranged these parameters into a triplet $\alpha \mid \beta \mid \gamma$ that helps classifying sequencing and scheduling problems. The triplet determines the specific problem with,

α describing the station environment,
β providing details of the processing characteristics and constraints and
γ containing the objective to be minimized.

In this paper we consider *n* job 2 machine flowshop scheduling problem. In the real life situations transportations times should be consider apart from processing time. In our problem we assumed that machines are distantly situated. Therefore, sometime is taken in transferring the job from machine1 to the machine2 in the form of loading time, Moving time and Unloading time of job. So transportation time has remarkable role in production management and if the priority of one job over the other job may be significant due to some urgency or demand of one particular type of job over other. Hence the job block criteria become important.

The first rule for flowshop scheduling problem is given by Johnson on two machines in 1954.

Johnson's rule for two machines: Job i precedes job j in an optimal sequence with regard to minimum total elapsed time if,

$$\min(t_{i1,} t_{j2}) \leq \min(t_{i2,} t_{j1})$$

Many researchers introduced the various concepts in different criteria like transportation time, job- block, breakdown of machine, setup time etc. for flowshop scheduling with two or thress stages. Maggu and Das[12] introduced the concept of equivalent job for $n \times 2$ flowshop problem of equivalent job for job block. Baker [1] published a book of scheduling and sequencing in 1974. The scheduling problem practically depends upon the important factors namely, Transportation time, job block, break down effect, Relative importance of a job over another job etc. These concepts were separately studied by Ignall and Scharge [8], Brown and Lomnicki [3], Heydari [7], Bo Chen et. al [2], Gupta, [6], Maggu and Das [11], Yoshida and Hitomi [19], Singh [17], Gupta Deepak [18], Lomnicki, [10], Chandermouli [4] etc. by considering the various parameters. Palmer []14], Nawaz et al. [13], Sarin and Lefoka [16],, Dannenbring [5], etc.

2. PRACTICAL SCENARIO

In real life situation, flowshop scheduling occurs in so many fields such as hospitals, factories, educational intuitions, banking, etc.. Two machines flowshop scheduling problem occur in the company like fabricate apparel industries in which two machines are used one is for cutting and second one for sewing. If first machine (cutting)is A and second machine is B then processing order of jobs on machines will be AB means first jobs processed on machine A then Machine B. in our problem we assumed that machines are distantly situated. Therefore, sometime is taken in transferring the job from machine A to the machine B in the form of loading time, Moving time and Unloading time of job. So transportation time has remarkable role in production management and if the priority of one job over the other job may be significant due to some urgency or demand of one particular type of job over other. Hence the job block criteria become important. In the real life situations transportations times should be consider apart from processing time. Sometimes the priority of one job over the other is preferred. It may be because of urgency or demand of its relative importance, the job block criteria becomes important. The practical situation may be taken in a production industry; manufacturing industry etc.

3. PROBLEM DESCRIPTION

In this study, two machine flowshop scheduling problem are considered with transportation time and job block. The objective is to minimize total completion time (Processing time) or makespan of jobs.

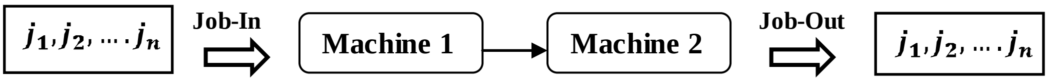

Fig 2. Structure of two stage flowshop scheduling

The symbols used in the formulation are as following:

I. Assumptions

i. The jobs are processed through two machine $M_1$and M_1 in order $M_1 M_2$ with processing time m_{i1} and m_{i2} respectively.
ii. To make job on second machine, it must be completed on the first machine.
iii. No machine processes more than one operation at a time
iv. The machine is constantly in use until all the jobs are completed.
v. All jobs are available simultaneously at time zero.
vi. Processing of job must be completed and cannot be interrupted. (preemption is not allowed)
vii. The machine is assumed to be continuously available and breakdown is not occurring.
viii. Setup times are included in processing times.

The symbols used in the formulation are as following:

II. Indices

p: index of jobs$(i = 1, 2, 3 \dots \dots . n)$
k: index of machines$(k = 1, 2)$
l: index of optimal or near optimal sequence$(l = 1, 2, 3 \dots \dots . n)$

III. Parameters

S: sequence of jobs$(i = 1, 2, 3 \dots \dots . n)$
S_l : Sequence of jobs obtain by applying johnson rule, $(l = 1, 2, 3, \dots . . n)$
(e, f): job block of job e and f
α: equivalent or resultant job for job block e and f
M_k : Machine $(k = 1, 2)$
m_{ik} : processing time of job i on machine k
$(t_{i,1\to 2})$: transportation time of job i from machine M_1 to M_2
C_{ik} : Completion time of job i on last stage of machine k

IV. Objective function

minimize the makespan (C_{max}):

$max\{C_{ik}\}$, where C_{ik} is completion time of job i on last stage of machine k

4. MATHEMATICAL TWO STAGE FLOWSHOP SCHEDULING MODEL

Let a set of n jobs $(1, 2 \ldots \ldots n)$ to be processed on two machines M_1 and M_2 and m_{i1} and m_{i2} be the processing time of each job on machine M_1 and M_2 respectively. The jobs are to be processed on the machines in the order M_1 M_2. Let transporting time of each job from machine M_1 to machine M_2 is t_i after its completion on machine M_1. Let α be the equivalent or resultant job for job block (e, f)

Table. 1 Mathematical model of given problem

Jobs (i)	Machine M_1 (m_{i1})	Transportation Time $(t_{i,1\to2})$	Machine M_2 (m_{i2})
1	m_{11}	$t_{1,1\to2}$	m_{12}
2	m_{21}	$t_{2,1\to2}$	m_{22}
3	m_{31}	$t_{3,1\to2}$	m_{32}
α(job block)	$m_{\alpha1}$	$t_{\alpha,1\to2}$	$m_{\alpha2}$
-	-	-	-
-	-	-	-
n	m_{n1}	$t_{n,1\to2}$	m_{n2}

Our aim is to find the optimal or near optimal sequence of jobs so as to minimize the total elapsed time or completion time or makespan.

4.1 Proposed Heuristics Algorithm

Step 1: first create the two fictitious machines R and S with processing time of jobs i on these machines as R_i and S_i.

$R_i = m_{i1} + (t_{i,1\to2})$

$S_i = m_{i2} + (t_{i,1\to2})$.

Step 2: create the equivalent job α for job block (e, f)

$\alpha = (e, f)$

$e = that\ no\ of\ job\ which\ has\ maximum\ processing\ time\ on\ machine\ R$

$f = that\ no\ of\ job\ which\ has\ maximum\ processing\ time\ on\ machine\ S$

$R_\alpha = R_e + R_f - \min(R_f, S_e)$

$S_\alpha = S_e + S_f - \min(R_f, S_e)$

Step 3: Now obtain the optimal or near optimal sequence S_l for jobs $l = 1, 2, 3 \ldots\ldots n$ on machines R and S with the help of Johnson algorithm for two machines.

Step 4: Prepare In- Out table for the optimal sequence obtained in step 3.

Step 5: Evaluate the makespan or total elapsed time for the optimal or near optimal sequence S_l for the original problem.

4.2 Existing Heuristic Algorithm

Step 1: In this heuristic algorithm first create the job block
Step 2: then create two fictitious machines.
Step 3: Now obtain the optimal or near optimal sequence
Step 4: Prepare In- Out table for the optimal sequence obtained in step 3.
Step 5: Evaluate the makespan or total elapsed time for the optimal or near optimal sequence

5. COMPARATATIVE STUDY WITH THE HEPL OF NUMERICAL ILLUSTRATIONS

Consider 5 jobs and 2 machine flowshop problem in which processing time of 5 jobs on 2 different machines are given with transportation time of jobs from machine M_1 to M_2. problem is given in following table.

Table. 2 Five job two machine scheduling problem with Transportation Time

Jobs (i)	Machine M_1 (m_{i1})	Transportation Time $(t_{i,1\to 2})$	Machine M_2 (m_{i2})
1	2	3	6
2	5	1	3
3	5	6	7
4	3	4	6
5	9	2	8

Now we find the optimal or near optimal sequence so as to minimize the C_{max}. We solved this example with the help of proposed heuristic algorithm and existing Johnson algorithm for 2 machine *n* job. In our proposed algorithm first we create two fictitious machines then create the job block. From existing algorithm first we create job block then create two fictitious machines for finding optimal or near optimal scheduling.

5.1 Numerical Solved by proposed heuristic algorithm

Step 1: first create the two fictitious machines R and S with processing time of jobs i on these machines as R_i and S_i.

$R_i = m_{i1} + (t_{i,1\to 2})$

$S_i = m_{i2} + (t_{i,1\to 2})$.

Table. 3 For Fictitious Machines

Jobs (i)	Machine R (R_i)	Machine S (S_i)
1	5	9
2	6	4
3	11	13
4	7	10
5	11	10

Step 2: As per step 2, find the two jobs for job block.
For job block we choose one job from machine R which has maximum processing time in that machine and second one job from machine S which has maximum processing time in that machine. So we select job 3 and job 5 because these job has the maximum processing time Reduced table is,

Table. 4 for job block

Jobs (i)	Machine R (R_i)	Machine S (S_i)
1	5	9
2	6	4
α	R_α	S_α
4	7	10

Job block is (3, 5)
Resultant or equivalent job $\alpha = (3, 5)$
Now we find the value of R_α and S_α
$\alpha = (3,5)$
$R_\alpha = R_e + R_f - \min(R_f, S_e)$
$S_\alpha = S_e + S_f - \min(R_f, S_e)$
$R_\alpha = R_3 + R_5 - \min(R_5, S_3)$
$R_\alpha = 11 + 11 - \min(11, 13)$
$R_\alpha = 11$
$S_\alpha = S_3 + S_5 - \min(R_5, S_3)$
$S_\alpha = 13 + 10 - 11$
$S_\alpha = 12$

Step 3: Now obtain the optimal or near optimal sequence S_l

Table. 5 for job block value

Jobs (i)	Machine R (R_i)	Machine S (S_i)
1	5	9
2	6	4
α	11	12
4	7	10

Apply Johnson rule find the sequence
$S_l = \{1, 4, \alpha, 2\}$
$S_l = \{1, 4, 3, 5\ 2\}$

Step 4: Now obtain in- out table for this sequence of original problem

Table. 6 In – Out table

Jobs (i)	Machine M_1 (m_{i1})	Transportation Time $(t_{i,1\to 2})$	Machine M_2 (m_{i2})
	$In - Out$		$In - Out$
1	$0 - 2$	3	$5 - 11$
4	$2 - 5$	4	$11 - 17$
3	$5 - 10$	6	$17 - 24$
5	$10 - 19$	2	$24 - 32$
2	$19 - 24$	1	$32 - 33$

Step 5: Evaluate the makespan. Makespan is 33.

5.2 Numerical Solved by existing algorithm

We solve this numerical by existing Johnson heuristic algorithm for n job 2 machines with transportation time and job block. From this method first we find the job block value then introduce two fictitious machines, then obtain the optimal or near optimal sequence and for this optimal sequence we proposed $In - Out$ table.

Step 1: in this algorithm we first create the job block of two jobs. We choose those one jobs from machine M_1 which has maximum processing time and another job from machine M_2 which has maximum processing time on this machine.

Table. 7 job block for existing algorithm

Jobs (i)	Machine M_1 (m_{i1})	Transportation Time $(t_{i,1\to 2})$	MachineM_2 (m_{i2})
1	2	3	6
2	5	1	3
α	$m_{\alpha 1}$	0	$m_{\alpha 2}$
4	3	4	6

$\alpha = (3,5)$
$m_{\alpha 1} = 7$
$m_{\alpha 2} = 8$
$t_{\alpha,1\to 2} = 0$

Step 2: Create two fictitious machines R_i and S_i

Table. 8 for fictitious machines

Jobs (i)	Machine R (R_i)	Machine S (S_i)
1	5	9
2	6	4
α	7	8
4	7	10

Apply Johnson rule find the sequence
$S'_l = \{1, \alpha, 4, 2\}$
$S'_l = \{1, 3, 5, 4\ 2\}$
From this heuristics we obtain $\{1, 3, 5, 4\ 2\}$ this sequence.

Step 3:
Now obtain in- out table for this sequence of original problem

Table. 9 In – Out table for existing algorithm

Jobs (i)	Machine M_1 (m_{i1})	Transportation Time $(t_{i,1\to 2})$	Machine M_2 (m_{i2})
	$In - Out$		$In - Out$
1	$0 - 2$	3	$5 - 11$
3	$2 - 7$	6	$13 - 20$
5	$7 - 16$	2	$20 - 28$
4	$16 - 19$	4	$28 - 34$
2	$19 - 24$	1	$34 - 37$

Makespan is 37.

6. GANTT CHART

Here Gantt Chart is shown in fig 3 and fig 4 according to table 6 and 9 respectively. Gantt Chart is generated to verify the effectiveness of proposed heuristic algorithm as compare to existing heuristic algorithm. Fig. 3 show the Gantt chart of proposed algorithm. According to this Total Completion Time (Makespan) of all the jobs on machine1 (M/c1) and machine2 (M/c2) is 33 units. On the other hand Fig. 4 show gantt chart of existing algorithm. The value of makespan is 37 units for existing heuristic algorithm.

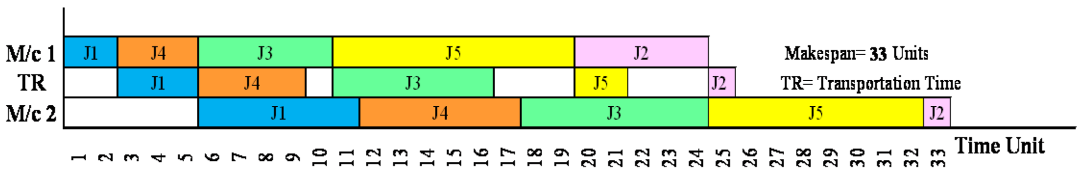

Fig3. Gantt Chart for Table 6

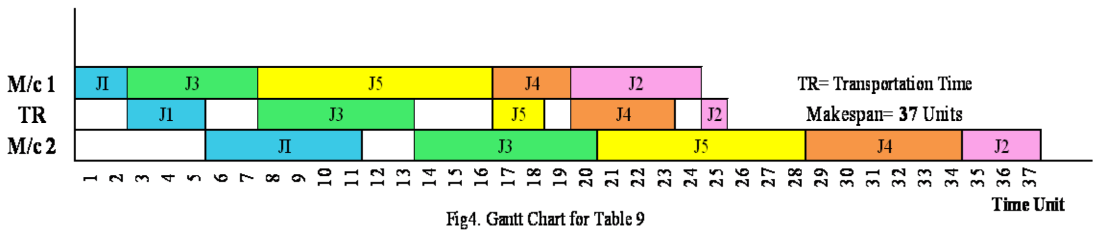

Fig4. Gantt Chart for Table 9

7. CONCLUSION AND FUTURE RESEARCH

In propose Algorithm we find the optimal or near optimal sequence for minimizing total elapsed time or makespan. In this proposed algorithm we find two fictitious machines after that find the value of equivalent job for job block. We select those jobs for job block that have maximum processing times. In existing algorithm first find the equivalent job then create two fictitious machines. We compare both the algorithm with the help of numerical example and result show that proposed algorithm outperformed to existing algorithm. Because the value of makespan from proposed algorithm is 33 and from existing algorithm is 37. So proposed algorithm minimize the makespan. We also generate the Gantt chart to verify the effectiveness of the proposed heuristic algorithm.

For future research different parameters can be used such as setup times, breakdown of machines, rental cost of machines etc for two or three machines. Metaheuristic approach is also used for solving this type of flowshop scheduling problem. Waiting time of jobs, ideal time of machines are also optimize with different parameters using heuristics or metaheuristics algorithms.

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