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An Optimal Policy for Weibull Deteriorating items with Power Demand Pattern and Permissible Delay on Payments

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Abstract

In this paper, we have developed an Inventory model for deteriorating items with Weibull distribution deterioration rate with two parameters. Shortages are not allowed. Supplier provides a definite fixed period to the retailer for settling the account which is define permissible delay in payment. There are two cases are considered first is permissible credit period is less than to Inventory cycle length or second is permissible credit period is greater than and equal to Inventory cycle length. The planned models optimize retailer's order quantity by minimizing his total costs. The retailer can get interest on the revenue generated during this period. The model is solved analytically. In this model items are deteriorating with Weibull distribution deterioration rate and demand is taken to be Price dependent demand. The demand rate follows power demand pattern. The numerical solution of the model is obtained to verify the optimal solution. Comprehensive sensitivity analysis has been carried out for showing the effect of variation in the parameter. The model is solved analytically by minimizing the total cost.

Key words:-Inventory, deteriorating items, Weibull distribution with two parameters, time dependent linear demand, permissible delay, Power demand Pattern

1 Introduction:-

The main purpose of inventory control is to reduce the total inventory cost. Inventory control is vital to find out the optimal stock and optimal time of replenishment of the inventory to meet the next demand. Deterioration of physical goods is one of the important factors in any inventory and production system. The deteriorating items have received much attention of several researches in the recent year because most of the physical goods undergo decay or deterioration over time. Commodities such as fruits, vegetables and food stuffs deplete by direct spoilage while kept in store. Ghare and Schrader (1963) were first who presented an economic order quantity model for exponentially decaying inventory. Economic order quantity model with inflation was developed by Buzacott(1975). Mandal and Phaujdar (1989) suggested an inventory model for deteriorating item and stock dependent consumption rate. This paper is developed for definite time horizon. Goh (1994) considered various functions to describe holding cost. Wee (1995) proposed an optimal deterministic lot-size inventory model for deteriorating with shortages and a declining market. Bose, Goswamiand Chaudhari(1995) recommended an EOQ model for deteriorating items with linear time-dependent demand rate and shortage. In this paper inflation was considered in present time. Chen(1998) presented an optimal inventory model for deteriorating items. In this paper he assumed the time dependent demand rate and shortages under inflation. Shortages are partially backlogged. Giri and Chaudhari (1998) introduced inventory models of perishable inventory with stock dependent rate and non-linear holding cost. Some of the significant recent work in this field of Structural properties of an inventory system with deterioration and trended demand has been done by Jalan and Chaudhuri (1999). Mandal and maiti (1999) presented a policy for inventory of damageable items with variable replenishment rate, stock dependent demand and some units in hand. Aggarwal and Jaggi (1995) extended theGoyal model by considering the point that if the credit period is less than the cycle length, the customer continues to accumulate revenue and earn interest on it for the rest of the period in the cycle, from the stock remaining beyond the credit period. Jamal, Sarkerand Wang (1997) introduced an ordering policy for deteriorating items with allowable shortage and permissible delay in payment. Sarker, Jamal and Wang(2000) suggested anoptimalpayment time for a retailer under permitted delay of payment by the wholesaler. Dye (2002) proposed an optimal policy for deteriorating inventory model with stock dependent demand and partial backlogging under condition of permissible delay in payments. An Integrated inventory model under conditions of order processing cost reduction and permissible delay in payments is developed by Huang (2010). Chandra, Goel and Mittal (2011) developed economic order quantity model for deteriorating items with permissible delay on payment. Chaudhary and Sharma (2013) suggested an inventory Model for deteriorating items with Weibull Deterioration with Time Dependent Demand and shortages. Permissible delay in payment is a condition in which supplier provide a fixed period to the retailer for settling the account.

In most of the research paper of permissible delay in payments the demand rate is considered as time dependent and deterioration rate is assumed to be constant. In this model we have consider the demand of the item is price dependent as well as time dependent also. Demand rate follows the power demand pattern. In permissible delay, a certain credit period is provided by the supplier to the retailer for settling the account. There are two cases are considered first is permissible credit period is less than to inventory cycle length or second is permissible credit period is greater than and equal to inventory cycle length. The planned model optimizes the retailer's total costs.

2Assumptions and Notations:-

The proposed inventory model is developed under the following assumptions and notations:

2. 1Assumptions:-

The following assumptions are made for development of mathematical model:

- In this model the demand rate $D(t)$ is price dependent and Follows power demand pattern. As $D(p) = b(ap)^{-\varepsilon}$, The annual demand as a function of Price, where $a > 0, b > 0$ and $\varepsilon > 0$
- The deterioration rate is relative to time.
- The deterioration of time is defined by Weibull distribution parameter (two) θ (t) = $\alpha\beta t^{(\beta-1)}$ where 0<α< 1 is the scale parameter and β >0 is the shape parameter.
- Shortages are not considered.
- The supplier provides a fixed credit period *K* to settling the accounts to the retailer.
- The ordering cost is A.

2. 2Notations:-

The following notations are made for development of mathematical model:

- I (t) is the inventory level at time t. $0 \leq \ t \leq T$.
- C is the unit purchase cost.
- T is the Inventory cycle length.
- A is the ordering cost.
- *p* is the unit selling price with $p > C$.
- $\varepsilon$is increase price rate.
- $\mu$is the inventory holding cost per unit item per unit time.
- $\gamma$is the deterioration cost per unit item.
- Q is the ordering quantity.
- *K* is permissible credit or permissible delay in settling the accounts.
- $\theta_\varepsilon$is Interest earned per unit per unit time.
- $\theta_p$is Interest paid per unit item per unit time.

3. Model II: Formulation of the Model for Weibull Deteriorating Items with Power Demand Pattern:-

The length of the cycle is T. Let I(t) be the inventory level at time t (0, T). The demand rate is price dependent. The demand rate follows power demand pattern. $D(p) = b(ap)^{-\varepsilon}$, The differential equation Can be defined when the instantaneous state of I(t) over (0, T) are given by

$$\mathrm{I}'(\mathrm{t}) + \alpha\beta(\mathrm{t})^{(\beta-1)}\mathrm{I}(\mathrm{t}) = -b(ap)^{-\varepsilon} 0 \leq \mathrm{t} \leq \mathrm{T} \ldots. \quad (3.\ 1)$$

Subject to boundary condition t=0, I(t) =Q

I(0) =Q

From equation (3. 1) we get

$$I(t) = Q\left(1 - \alpha t^{\beta}\right) - b(ap)^{-\varepsilon}.t$$

$$0 \leq \mathrm{t} \leq \mathrm{T} \ldots. \quad (3.\ 2)$$

And the order quantity at t=T From equation (3. 2) we get

I(T) = 0

$$Q = \frac{b(ap)^{-\varepsilon}}{(1-\alpha T^{\beta})}.T \ \ldots. \quad (3.\ 3)$$

3. 1Holding Cost:-

The holding cost throughout the time period 0 to T

$$\mathrm{H} = \mu \int_0^T I\ (t)dt$$

$$H = \mu \int_0^T Q\left(1 - \alpha t^{\beta}\right) - b(ap)^{-\varepsilon}.tdt$$

Now holding cost

$$H = \mu\left[Q\left(T - \frac{\alpha T^{(\beta+1)}}{(\beta+!)}\right) - b(ap)^{-\varepsilon}.\frac{T^2}{2}\right] \quad (3.\ 4)$$

3. 2Purchase Cost:-

Purchase cost = CQ .... (3. 5)

3. 3Deterioration Cost:-

Deterioration cost = $\gamma\left[\int_0^{\mathrm{T}} \theta\ (\mathrm{t}).\mathrm{I}(\mathrm{t})\mathrm{dt}\right]$

$$D_T = \gamma\left[\frac{Q\alpha\beta T^{\beta}}{\beta} - \frac{Q\alpha^2\beta T^{2\beta}}{2\beta} - \frac{b(ap)^{-\varepsilon}\alpha\beta T^{(\beta+1)}}{(\beta+1)}\right] \ldots. \quad (3.\ 6)$$

3. 4Interest Payable and Earned, there will be Two Cases i. e. K<T and K≥T:-

3. 4. 1Case Ist:-K<T

In this case, the interest could be earned by retailer on the income produce from the sales the items up to K. Although he has to settle the account at K . To get his remaining stokes, he has to arrange the funds for this he took the funds at some precise rate of interest, the the retailer Pay the interest for the period K to T.

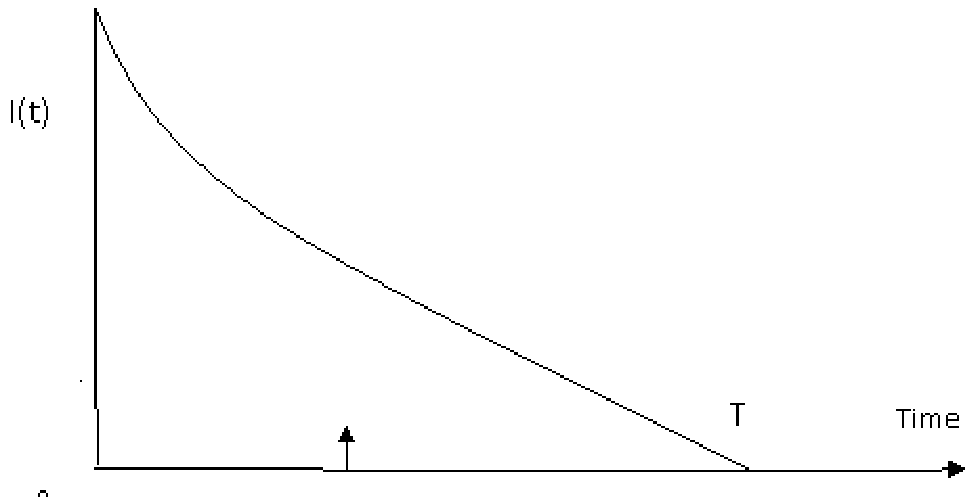

Figure 1: Graphical Representation of the inventory system, When K<T.

(i) Interest earned on the profits by the retailer.

$$IE_1 = p\theta_e \int_0^K D(t).tdt$$

$$= p\theta_e \int_0^K b(ap)^{-\varepsilon}.tdt$$

$$= p\theta_e.\frac{b(ap)^{-\varepsilon}K^2}{2} \quad \ldots. \qquad (3.7)$$

(ii) Interest paid by the retailer throughout the time interval K to T.

$$I.P = C\theta_p \int_k^T I(t)dt$$

$$I.P. = C.\theta_p\left[Q(T-K) - \frac{\alpha}{(\beta+1)}\left(T^{(\beta+1)} - K^{(\beta+1)}\right) - \frac{b(ap)^{-\varepsilon}(T^2-K^2)}{2}\right] \qquad (3.8)$$

3. 4. 1. 1Total Cost when (K<T):-

Total cost per $U_1(T)$ will be

$$U_1(T) = \{\text{ Ordering cost} + \text{ Purchasing cost} + \text{ Holding cost} + + \text{Deterioration cost} + \text{ interest paid} - \text{interest earned}\}$$

$$U_1(T,p) = A + CQ + \mu\left[Q\left(T - \frac{\alpha T^{(\beta+1)}}{(\beta+!)}\right) - b(ap)^{-\varepsilon}.\frac{T^2}{2}\right] + \gamma\left(\frac{Q\alpha\beta T^{\beta}}{\beta} - \frac{Q\alpha^2\beta T^{2\beta}}{2\beta} - \frac{b(ap)^{-\varepsilon}\alpha\beta T^{(\beta+1)}}{(\beta+1)}\right) + C.\theta_p\left[Q(T-K) - \frac{\alpha}{(\beta+1)}\left(T^{(\beta+1)} - K^{(\beta+1)}\right) - \frac{b(ap)^{-\varepsilon}(T^2-K^2)}{2}\right] - p\theta_e.\frac{b(ap)^{-\varepsilon}K^2}{2} \quad \ldots. \qquad (3.9)$$

3. 4. 1. 2 Mathematical Formulation of the Model:-

Our main objective to optimize the total cost $U_1(T)$, the necessary condition for minimize the cost is

$$\frac{\partial U_1(T,p)}{\partial T} = 0, \ldots. \tag{3. 10}$$

$$\frac{\partial U_1(T,p)}{\partial p} = 0 \ldots. \tag{3. 11}$$

With the computer software mathematica- 5.8, we can compute the optimal value of T by equation (3. 10) and optimal value of *p* by the equation (3. 11). The optimal value of cost function $U_1(T,p)$ is determined by equation (3. 9). The optimal value of T and p satisfy the sufficient conditions for minimizing cost function $U_1(T,p)$ is

$$\frac{\partial^2 U_1(T,p)}{\partial T^2} < 0, \frac{\partial^2 U_1(T,p)}{\partial p^2} < 0 \ldots. \tag{3. 12}$$

And $\frac{\partial^2 U_1(T,p)}{\partial T^2} . \frac{\partial^2 U_1(T,p)}{\partial p^2} - \frac{\partial^2 U_1(T,p)}{\partial T \partial p} > 0$ and at $T = T^*$ (optimal value) $p = p^*$(optimal value)

3. 4. 2 Case IInd:-K≥T

The retailer can earn interest on the total income, which is generated from selling the items during the time interval 0 to T. The interest paid by the retailer during the time interval 0 to T is zero.

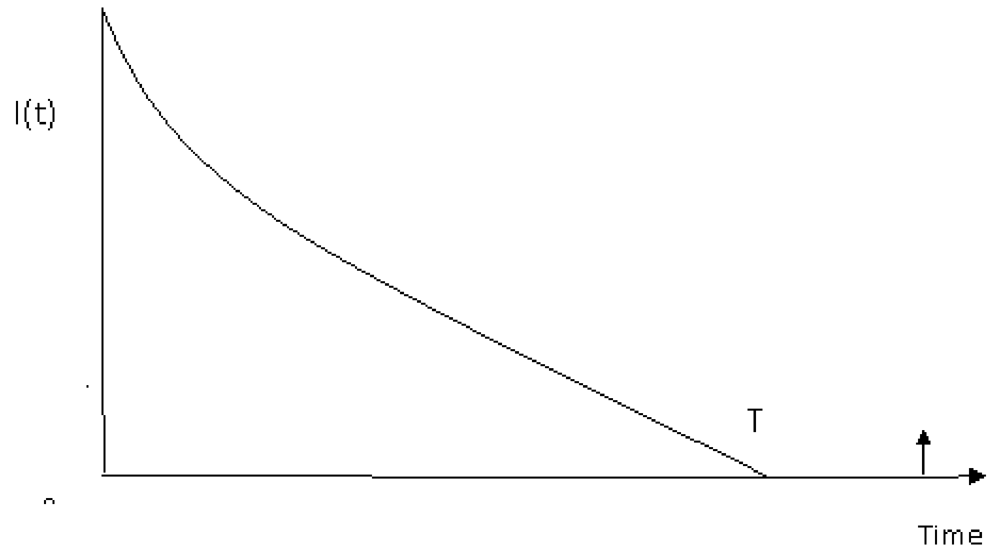

Figure 2: Graphical Representation of the inventory system, When K>T

Interest earned on the profits by the retailer.

$$IE_2 = p\theta_e \int_0^T D(t).tdt$$

$$= p\theta_e \int_0^T b(ap)^{-\varepsilon}.tdt$$

$$= p\theta_e.\frac{b(ap)^{-\varepsilon}T^2}{2} \quad \ldots. \tag{3. 13}$$

3. 4. 2. 1 Expected total cost when (K>T):-

The total cost per unit time $U_2(T,p)$ will be

$$U_2(T,p) = \{\text{ Ordering cost } + \text{ Purchasing cost} + \text{ Holding cost } + \text{ Deterioration cost } - \text{interest earned}\}$$

$$U_2(T,p) = A + CQ + \mu\left(Q\left(T - \frac{\alpha T^{(\beta+1)}}{(\beta+1)}\right) - b(ap)^{-\varepsilon}.\frac{T^2}{2}\right) + \gamma\left(\frac{Q\alpha\beta T^{\beta}}{\beta} - \frac{Q\alpha^2\beta T^{2\beta}}{2\beta} - \frac{b(ap)^{-\varepsilon}\alpha\beta T^{(\beta+1)}}{(\beta+1)}\right) - p\theta_e.\frac{b(ap)^{-\varepsilon}T^2}{2} \tag{3. 14}$$

3. 4. 2. 2 Mathematical Formulation of the Model:-

Our main objective to optimize the total cost $U_2(T)$, the necessary condition for minimizing the cost is

$$\frac{\partial U_2(T,p)}{\partial T} = 0, \ldots. \tag{3. 15}$$

$$\frac{\partial U_2(T,p)}{\partial p} = 0 \ldots. \tag{3. 16}$$

With the computer software mathematica-5.8, we can compute the optimal value of T by equation (3. 15) and optimal value of p by the equation (3. 16). The optimal value of cost function $U_2(T,p)$ is determined by equation (3. 14). The optimal value of T and p satisfy the sufficient conditions for minimizing cost function $U_2(T,p)$ is

$$\frac{\partial^2 U_2(T,p)}{\partial T^2} < 0, \quad \frac{\partial^2 U_2(T,p)}{\partial p^2} < 0 \ldots. \tag{3. 17}$$

And $\frac{\partial^2 U_2(T,p)}{\partial T^2}.\frac{\partial^2 U_2(T,p)}{\partial p^2} - \frac{\partial^2 U_2(T,p)}{\partial T\partial p} > 0$ and at $T = T^*$ (optimal value) $p = p^*$(optimal value)

3.5 Numerical Illustration:-

The model has been solved numerically. We have considered the following data for the study which based on previous study.

Example:

Let us consider D =1200 units/year, A = 50/cycle, μ = 1. 5, C = 6. 2, γ =. 02, α =. 14, β =0. 35, K =. 6 year, θ_e = 5/year, $\varepsilon = 0.2$ θ_p = 7/year

Case Ist:-K<T

Solving the problem numerically using the software mathematica-5.8, we compute the optimal value of T, p and $U_1(T,p)$simultaneously by equation (3. 10), equation (3. 11) and equation (3. 9).

T^* = 12. 0273 year, p = 15. 43 and $U_2^*(T,p)$ = 1650. 1083 approximate.

Case IInd:-K≥T
Based on above input data with the help of software mathematica-5.8, we calculate the optimal value of T, p and $U_2(T,p)$simultaneously by equation (3. 15), equation (3. 16) and equation (3. 14).
T^* = 10. 9273 year, p^* = 14. 83 and $U_2^*(T,p)$ = 1350. 1083 approximate.

3. 6 Sensitivity Analysis and Observations:
We have studied the effects of changes of the parameters on the optimal values of $U(T,p)$ and T has been derived by proposed method. The sensitivity analysis is performed in view of the numerical example given above. We have executed sensitivity analysis by changing the parameters α, β, a, b and p as +20%, +50%,-20% and-50%. All remaining parameters have original values with respect to these changes. The corresponding changes in $U(T,p)$, and T* are showed in the table given below.

Table 3. 6. 1: Sensitivity Analysis of Optimal Solution {*U (T, p)*} w. r. t various Parameters,

parameters	% change	T^*	$U^*(T,p)$
α	-50	12. 9662	1499. 7257
	-20	12. 1471	1550. 3897
	+20	11. 8739	1702. 8789
	+50	10. 4344	1987. 5435
β	-50	11. 9272	1393. 6804
	-20	10. 2000	1408. 3341
	+20	9. 2040	1689. 3247
	+50	7. 1087	1786. 5702
a	-50	12. 9962	1999. 5976
	-20	12. 0010	1798. 8802
	+20	9. 9085	1567. 1480
	+50	6. 6930	1299. 3532
b	-50	9. 1232	1899. 3443
	-20	8. 8010	1898. 0243
	+20	6. 9085	1376. 8970
	+50	4. 6930	1259. 2313
p	-50	7. 6930	1099. 7865
	-20	9. 0085	1398. 0276
	+20	11. 0010	1789. 5976
	+50	12. 6962	1899. 3554

We study above table brings out the following.

We observed that

- As parameter α increases, optimal value of T^* decreases and the average total profit $U^*(T,p)$ of an inventory system increases whereas parameter α decreases, optimal value of T^* increases while the average total profit $U^*(T,p)$ of an inventory system decreases.
- As parameter β increases, optimal value of T^* slightly decreases and the average total cost $U^*(T,p)$ of an inventory system slightly increases whereas parameter β decreases, optimal value of T^* decreases while the average total cost $U^*(T,p)$ of an inventory system slightly decreases.
- As parameter a increases, optimal value of T^* decreases and the average total cost $U^*(T,p)$ of an inventory system highly decreases whereas parameter a decreases, optimal value of T^* increases while the average total cost $U^*(T,p)$ of an inventory system highly increases.
- It is interesting to observe that as parameter b increases, optimal value of T^* slightly decreases and the average total cost $U^*(T,p)$ of an inventory system slightly decreases whereas parameter b decreases optimal value of T^* slightly decreases while the average total cost $U^*(T,p)$ of an inventory system highly increases.
- As parameter p increases, optimal value of T^* highly increases and the average total cost $U^*(T,p)$ of an inventory system highly increases whereas parameter p decreases, optimal value of T^* highly decreases while the average total cost $U^*(T,p)$ of an inventory system highly decreases.

5. 8 Graphical Representation of Convexity of the Cost Function:

We find a three dimensional convex graph of total cost given by the following figures.

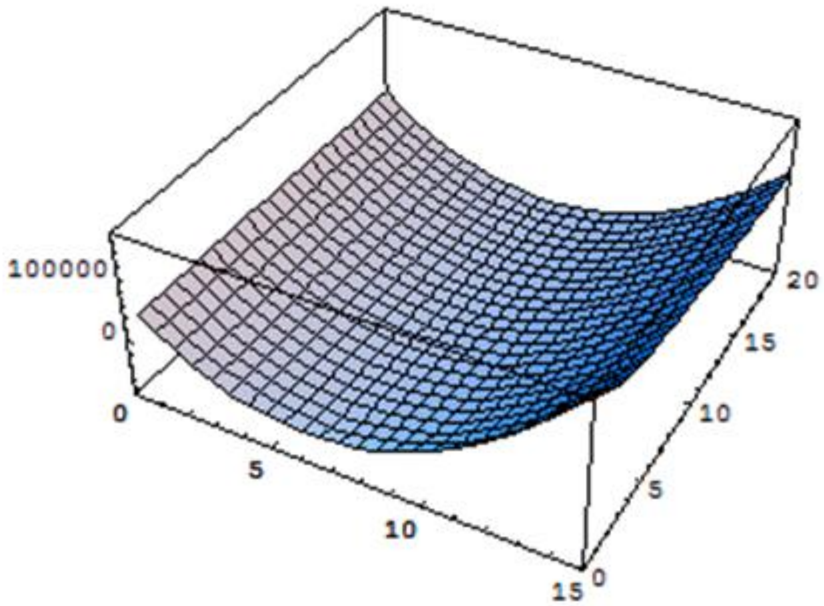

Figure. 3: Graphical representation of convexity of the cost function in 3D.

5. 9 Conclusion:-

This model presented a cost minimizing inventory model for Weibull deteriorating item when demand rate is time dependent and price dependent also. The credit time is given by the supplier to the retailer for paying the remaining balance. The retailer should extra order recurrently to take the maximum benefit on the total income. In this chapter two models are derived in first model demand is considered as time dependent. The second model is explored with power demand pattern. Demand rate is price dependent with increasing price rate parameter. In case of highly deteriorating item retailer wants to more order frequently to decreased his loss due to deterioration. The increasing price rate parameter and deteriorate parameter are always controlled by the manager to reduce the total inventory cost.

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