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MT-Proximal Contractions and Best Proximity Point Theorems

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Abstract

The purpose of this paper is to provide the new class of proximal contractions, which are more general than a class of proximal contractions of the first and second kinds, with the help of Mizoguchi-Takahashi function and by giving the necessary conditions to have best proximity points and we also provide example of our main result.

Keywords: Best proximity point, MT-proximal contraction mapping, proximal contraction mapping.

AMS Subject Classification: 46T99, 47H10, 54H25

1. Introduction:

The importance of fixed point theory comes from the fact that it provides a unified treatment and is a huge tool for solving equations of the form T x=x where T is self-mapping defined on a subset of a metric space or a normed linear space or some suitable space. If T is nonself-mapping, then it is not possible everywhere that the equation T x=x has solution. In that case best approximation theorems explore the existence of an approximate solution that whereas best proximity point theorems analyze the existence of an approximate solution that is optimal. One of the most impressive results in this direction was introduced by Fan[3] and he gave that if A is nonempty compact convex subset of a Hausdorff locally convex topological vector space B and T: $A \to B$ is a continuous mapping, then there exists an element x in A such that d(x, Tx)=d(Tx, A). Later than, motivating by this result many authors including Prolla[4], Reich[5], Sehgal and Singh[7, 8] derived the extensions of Fan's theorem in many ways.

It is interesting to note that best proximity point theorems appear as a natural generalization of fixed point theorems and best proximity point theorem can be boils down to fixed point theorem when the mapping under consideration is a self-mapping. In this paper, we generalized the definition of proximal contractions of the first and second kinds by using the MT-function β which satisfies Mizoguchi-Takahashi's condition (i.e. $\lim_{s \to t^+} sup\beta(s) < 1$ for all t $\in [0, \infty)$) and provide best proximity point theorems for proximal contractions.

2. Preliminaries:

To establish our results of this section, we consider the following definitions and notations:

Let (X, d) be a metric space. Then for given nonempty subsets A and B, we define A_0 and B_0 as follows

d(A, B)=inf{d(x, y): x ∈ A and y ∈ B},

A_0 ={x∈A: d(x, y)=d(A, B) for some y ∈ B},

B_0={ y ∈ B: d(x, y)=d(A, B) for some x∈A}.

If A ∩ B ≠ ∅, then A_0 and B_0 are nonempty. It is also interesting to note that if A and B are closed subsets of normed linear space such that d(A, B) > 0 then A_0 and B_0 are contained in the boundaries of A and B respectively.

Definition 2.1[6] A mapping T: A → B is called a proximal contraction of first kind if there exists k ∈ [0, 1) such that

$$\left.\begin{matrix} d(u, Tx) = d(A, B) \\ d(v, Ty) = d(A, B) \end{matrix}\right\} \Rightarrow d(u, v) \leq kd(x, y)$$

for all u, v, x, y ∈ A.

Definition 2.2[6] A mapping T: A → B is called a proximal contraction of second kind if there exists k ∈ [0, 1) such that

$$\left.\begin{matrix} d(u, Tx) = d(A, B) \\ d(v, Ty) = d(A, B) \end{matrix}\right\} \Rightarrow d(Tu, Tv) \leq kd(Tx, Ty)$$

for all x, y, u, v ∈ A.

Definition 2.3 [11] Let S: A → B and T: B → A be two mappings. The pair (S, T) is called proximal cyclic contraction pair if there exists k ∈ [0, 1) such that

$$\left.\begin{matrix} d(a, Sx) = d(A, B) \\ d(b, Ty) = d(A, B) \end{matrix}\right\} \Rightarrow d(a, b) \leq k\, d(x, y) + (1-k)d(A, B)$$

for all a, x ∈ A and b, y ∈ B.

Definition 2.4[4] A mapping g: A → A is an isometry if for any x, y ∈ A one has d(gx, gy)=d(x, y).

Definition 2.5[11] Let S: A → B be a mapping and g: A → A be an isometry. The mapping S is said to preserve the isometry distance with respect to g if

d(Sgx, Sgy)=d(Sx, Sy) for all x, y ∈ A.

Definition 2.6[6] A point x ∈ A is called a best proximity point of the mapping S: A → B if it satisfies the condition
d(x, Sx)=d(A, B).
For a ∈ ℝ, we recall that
$\lim_{x\to a} sup f(x) = inf_{\varepsilon>0} sup_{0<|x-a|<\varepsilon}$f(x)
and $\lim_{x\to a^+} sup f(x) = inf_{\varepsilon>0} sup_{0<x-a<\varepsilon}$f(x).

Definition 2.7[10] A function β: [0, ∞) → [0, 1) is said to be an Mizoguchi-Takahashi function if it satisfies Mizoguchi-Takahashi's condition $\lim_{s\to t^+} sup\beta(s) < 1$ for all $t \in [0,\infty)$.
Clearly, a non-increasing or a non-decreasing function β: [0,∞) → [0, 1) is MT-function. So the set of MT-function is a rich class. But also note that there exist some functions which are not MT-functions.
For example:
Let β: [0, ∞) → [0, 1) be defined by

$$\beta(t)= \begin{cases} \frac{\sin t}{t}, & if\ t \in [0,\infty) \\ 0, & otherwise. \end{cases}$$

Since $\lim_{s\to 0^+} sup\beta(s) = 1$, β is not an MT-function.

3. Main Results:

Now, we derive a new class of proximal contraction mapping, that is, MT-proximal contraction mappings and then prove best proximity point theorems for this new class.

Definition 3.1 A mapping T: A → B is called MT-proximal contraction of first kind if there exists an MT-function β such that

$$\left.\begin{array}{l} d(u,Tx) = d(A,B) \\ d(v,Ty) = d(A,B) \end{array}\right\} \Rightarrow d(u, v) \leq \beta(d(x, y))d(x, y)$$

for all x, y, u, v ∈ A.

Definition 3.2 A mapping T: A → B is called MT-proximal contraction of second kind if there exists an MT-function β such that

$$\left.\begin{array}{l} d(u,Tx) = d(A,B) \\ d(v,Ty) = d(A,B) \end{array}\right\} \Rightarrow d(Tu, Tv) \leq \beta(d(Tx, Ty))d(Tx, Ty)$$

for all x, y, u, v ∈ A.
Obviously if one can consider β(t)=k where k ∈ $[0, 1)$, then MT-proximal contraction of first kind and MT-proximal contraction of second kind reduce to a proximal contraction of first kind and a proximal contraction of second kind respectively.
We provide a best proximity point theorem for nonself-mappings which are MT-proximal contractions of first kind.

Theorem 3.3 Let (X, d) be a complete metric space and let A, B be two nonempty closed subsets of X such that A_0 and B_0 are nonempty. Let S: A → B, T: B → A and g: A∪B → A∪B satisfy the following conditions:

(1) S and T are MT-proximal contraction of first kind;
(2) g is an isometry;
(3) the pair (S, T) is a proximal cyclic contraction;
(4) S(A_0) ⊆ B_0 and T(B_0)⊆ A_0;
(5) A_0 ⊆ g(A_0) and B_0 ⊆ g(B_0).

Then there exists unique point x ∈ A and there exists unique point y ∈ B such that d(gx, Sx)=d(gy, Ty)=d(x, y)=d(A, B). Moreover, for any fixed $x_0 \in A_0$, the sequence $\{x_n\}$ defined by d(gx_{n+1}, Sx_n)=d(A, B) converges to the element x. For any fixed $y_0 \in B_0$, the sequence $\{y_n\}$ defined by d(gy_{n+1}, Ty_n)=d(A, B) converges to the element y.

On the other hand, a sequence $\{p_n\}$ in A converges to x if there exists a sequence of positive numbers $\{\varepsilon_n\}$ such that $\lim_{n\to\infty}\varepsilon_n$=0, d($p_{n+1}, q_{n+1}$) ≤ ε_n, where q_{n+1} ∈ A satisfies the condition that d(gq_{n+1}, Sp_n)=d(A, B).

Proof: Suppose x_0 is a fixed element in A_0. In view of the fact that S(A_0) ⊆ B_0 and A_0 ⊆ g(A_0), it implies that there exists an element $x_1 \in A_0$ such that d(gx_1, Sx_0)=d(A, B).

Again, by condition (4) and (5), there exists an element $x_2 \in A_0$ such that d(gx_2, Sx_1)=d(A, B). In the similar way, we can find $x_n \in A_0$ such that d(gx_n, Sx_{n-1})=d(A, B).

So, by the principle of mathematical induction, we can determine an element $x_{n+1} \in A_0$ such that d(gx_{n+1}, Sx_n)=d(A, B). (3.1)

Again S(A_0) ⊆ B_0 and A_0 ⊆ g(A_0), S is MT-proximal contraction of first kind, g is an isometry and by using MT-function β, it follows that for each n ≥ 1,

d(x_{n+1}, x_n)=d(gx_{n+1}, gx_n) ≤ $\beta\big(d(x_n,x_{n-1})\big)d(x_n,x_{n-1}) < d(x_n,x_{n-1})$,

which implies that the sequence $\{d(x_{n+1},x_n)\}$ is non-increasing and bounded below. Hence there exists r ≥ 0 such that $\lim_{n\to\infty} d(x_{n+1},x_n)$ =r. Let us suppose that r> 0. We observe that

$$\frac{d(x_{n+1},x_n)}{d(x_n,x_{n-1})} \le \beta(\mathrm{d}(x_n,x_{n-1})).$$

Which implies that when n→∞ then β(d(x_n,x_{n-1})) ≥1 which contradicts that β is an MT-function and hence $\lim_{n\to\infty} d(x_{n+1},x_n)$ =r=0. (3.2)

Now we shall prove that $\{x_n\}$ is a Cauchy sequence. On the contrary, suppose that $\{x_n\}$ is not a Cauchy sequence. Then there exists $\varepsilon > 0$ and subsequences$\{x_{m_k}\}$, $\{x_{n_k}\}$ of $\{x_n\}$ such that for any $n_k > m_k \geq$ k

r_k =d(x_{m_k},x_{n_k}) ≥ ε, d($x_{m_k},x_{n_{k-1}}$) < ε

for any k ∈ {1, 2, 3, ...}. For each n ≥ 1, let $\alpha_n = d(x_{n+1},x_n)$. Then we have

$$\varepsilon \le r_k \le d\big(x_{m_k},x_{n_{k-1}}\big) + d(x_{n_{k-1}},x_{n_k})$$
$$< \varepsilon + \alpha_{n_{k-1}} \quad (3.3)$$

and from equations (3.2) and (3.3), we have

$$\lim_{k\to\infty} r_k = \varepsilon \quad (3.4)$$

Notice that
$\varepsilon \leq r_k$
$\leq d(x_{m_k}, x_{m_{k+1}}) + d(x_{n_{k+1}}, x_{n_k}) + d(x_{m_{k+1}}, x_{n_{k+1}})$
$= \alpha_{m_k} + \alpha_{n_k} + d(x_{m_{k+1}}, x_{n_{k+1}})$
$< \alpha_{m_k} + \alpha_{n_k} + \beta(d(x_{m_k}, x_{n_k}))\, d(x_{m_k}, x_{n_k})$
and so
$\frac{r_k - \alpha_{m_k} - \alpha_{n_k}}{d(x_{m_k}, x_{n_k})} \leq \beta(d(x_{m_k}, x_{n_k}))$.
By taking limit $k \to \infty$ and by using equations (3.2) and (3.4) which provide contradiction since β is an MT function. So, we come to fact that $\{x_n\}$ is a Cauchy sequence. Hence $\{x_n\}$ converges to some element x ∈ A.
Similarly, by using conditions of (4) and (5), that is, $T(B_0) \subseteq A_0$ and $A_0 \subseteq g(A_0)$, we can conclude that there exists a sequence $\{y_n\}$ such that it converges to some element y ∈ B. Since the pair (S, T) is proximal cyclic contraction and g is an isometry, we have
$d(x_{n+1}, y_{n+1}) = d(gx_{n+1}, gy_{n+1}) \leq kd(x_n, y_n) + (1-k)d(A, B)$ (3.5)
Taking $n \to \infty$ in equation (3.5) it follows that d(x, y)=d(A, B) (3.6)
and so $x \in A_0$ and $y \in B_0$. Since $S(A_0) \subseteq B_0$ and $T(B_0) \subseteq A_0$, there exists $u \in A$ and $v \in B$ such that
d(u, Sx)=d(A, B)
d(v, Ty)=d(A, B). (3.7)
From equations (3.1), (3.7) and using S is MT-proximal contraction of first kind, we get
$d(u, gx_{n+1}) \leq \beta(d(x, x_n))\, d(x, x_n)$ (3.8)
Letting $n \to \infty$, we get d(u, gx) ≤ 0 and so u=gx.
Therefore, we have
d(gx, Sx)=d(A, B). (3.9)
In the same way, we can prove that v=gy and so
d(gy, Ty)=d(A, B). (3.10)
From equations (3.6), (3.9) and (3.10), we get d(x, y)=d(gx, Sx)=d(gy, Ty)=d(A, B).
Uniqueness, suppose there exists $x^* \in A$ and $y^* \in B$ with $x \neq x^*$ and $y \neq y^*$ such that
$d(gx^*, Sx^*) = d(A, B)$ and $d(gy^*, Sy^*) = d(A, B)$.
Using isometry of g and S is MT-proximal contraction of first kind, it follows that
$d(x, x^*) = d(gx, gx^*) \leq \beta(d(x, x^*))\, d(x, x^*)$
and hence
$1 = \frac{d(x, x^*)}{d(x, x^*)} \leq \beta(d(x, x^*)) < 1$.
which is contradiction. Thus we have $x = x^*$. Similarly, we can prove that $y = y^*$.
On the other hand, let $\{p_n\}$ be a sequence in A and $\{\varepsilon_n\}$ be a sequence of positive real numbers such that $\lim_{n\to\infty} \varepsilon_n = 0$,
$d(p_{n+1}, q_{n+1}) \leq \varepsilon_n$. (3.11)
where $q_{n+1} \in A$ satisfies the condition that
$d(gq_{n+1}, Sp_n) = d(A, B)$. (3.12)

By using equation (3.1), (3.12), S is MT-proximal contraction of first kind and g is an isometry, we have
d(x_{n+1}, q_{n+1})=d(gx_{n+1}, gq_{n+1}) $\leq \beta(d(x_n, p_n))d(x_n, p_n)$.
For any $\varepsilon > 0$, choose a positive integer n such that $\varepsilon_n \leq \varepsilon$ for all n $\geq N$.
We observe that
d(x_{n+1}, p_{n+1}) $\leq d(x_{n+1}, q_{n+1}) + d(q_{n+1}, p_{n+1})$
$\leq \beta(d(x_n, u_n))d(x_n, u_n) + \varepsilon_n$
$\leq d(x_n, u_n) + \varepsilon.$
Since $\varepsilon > 0$ is arbitrary, we can conclude that for all n $\geq$ N the sequence $\{d(x_n, p_n)\}$ is non-increasing and bounded below and hence converges to some non-negative real number r'. Since the sequence $\{x_n\}$ converges to x, we obtain
$\lim_{n\to\infty} d(p_n, x) = \lim_{n\to\infty} d(p_n, x_n) = r'$ (3.13)
Suppose that r'$>$ 0.Since
d(p_{n+1}, x) $\leq d(p_{n+1}, x_{n+1}) + d(x_{n+1}, x)$
$\leq \beta(d(x_n, p_n))d(x_n, p_n) + \varepsilon_n + d(x_{n+1}, x)$. (3.14)
From inequalities (3.11), (3.13) and (3.14) we get
$\frac{d(p_{n+1}, x) - \varepsilon_n - d(x_{n+1}, x)}{d(x_n, p_n)} \leq \beta(d(x_n, p_n)).$
Since β is MT-function. we observe that
$\lim_{n\to\infty} d(p_n, x) = \lim_{n\to\infty} d(p_n, x_n) = 0.$
which is contradiction. Thus r'=0 and hence $\{p_n\}$ is convergent to the point x.

Corollary 3.4 Let (X, d) be a complete metric space and let A, B be two nonempty closed subsets of X such that A_0 and B_0 are nonempty. Let S: A → B, T: B → A and g: A∪B → A∪B be the mappings satisfying the following conditions:
1. S and T are MT-proximal contraction of first kind;
2. g is an identity map;
3. the pair (S, T) is a proximal cyclic contraction;
4. S(A_0) ⊆ B_0 and T(B_0)⊆ A_0.

Then there exists unique point x ∈A and there exists unique point y ∈ B such that d(x, Sx)=d(y, Ty)=d(x, y)=d(A, B).
If we consider $\beta(t) = k$, where k ∈ [0, 1), we obtain the following corollary:

Corollary 3.5 Let (X, d) be a complete metric space and let A, B be two nonempty closed subsets of X such that A_0 and B_0 are nonempty. Let S: A → B, T: B → A and g: A∪B → A∪B satisfy the following conditions:
1. S and T are proximal contraction of first kind;
2. g is an isometry;
3. the pair (S, T) is a proximal cyclic contraction;
4. S(A_0) ⊆ B_0 and T(B_0)⊆ A_0;
5. A_0 ⊆ g(A_0) and B_0 ⊆ g(B_0).

Then there exists unique point x ∈A and there exists unique point y ∈ B such that d(gx, Sx)=d(gy, Ty)=d(x, y)=d(A, B). Moreover, for any fixed $x_0 \in A_0$, the sequence $\{x_n\}$ defined by d(gx_{n+1}, Sx_n)=d(A, B) converges to the element x. For any fixed $x_0 \in A_0$, the sequence $\{x_n\}$ defined by d(gx_{n+1}, Ty_n)=d(A, B) converges to the element y.
If in the above corollary g is an identity mapping, we obtain the following corollary:

Corollary 3.6 Let (X, d) be a complete metric space and let A, B be two nonempty closed subsets of X such that A_0 and B_0 are nonempty. Let S: A → B, T: B → A and g: A∪B → A∪B satisfy the following conditions:
1. S and T are proximal contraction of first kind;
2. the pair (S, T) is a proximal cyclic contraction;
3. S(A_0) ⊆ B_0 and T(B_0)⊆ A_0.

Then there exists unique point x ∈ A and there exists unique point y ∈ B such that d(x, Sx)=d(y, Ty)=d(x, y)=d(A, B).

4. Examples:

Now we give an example to show the difference between the definition of proximal contraction of first kind and MT-proximal contraction of first kind. For this, firstly we use some proposition:

Proposition 4.1[11] Let f: [0, ∞) → [0, ∞) be a function defined by f(x)=$\log(1+x)$. Then we have to prove that
f(x)-f(y) $\leq f(|x-y|)$ for all x, y ∈ [0, ∞).

Proposition 4.2[11]For each x, y ∈ ℝ, we have the following inequality
$$\frac{1}{(1+|x|)(1+|y|)} \leq \frac{1}{1+|x-y|}.$$
Now we presents an example which shows that MT-proximal contraction of first kind need not be proximal contraction of first kind

Example 4.3 Consider the complete metric space $\mathbb{R}^2$ with metric defined by d((a, b), (x, y))=$|x-a| + \frac{|y-b|}{2}$. Let A=$\{(0,x): x \in \mathbb{R}\}$ and B=$\{(3,y): y \in \mathbb{R}\}$. Then d(A, B)=3. Define the mapping S: A → B as follows:
S($(0,x)$)=$(3, \log 1+|x|)$).
Solution: Firstly, we shall prove that S is MT-proximal contraction of first kind with MT-function β defined by
$$\beta(t) = \begin{cases} \frac{1}{2}, t = 0 \\ \frac{\log(1+t)}{2t}, t > 0. \end{cases}$$
Let $(0,x_1),(0,x_2),(0,a_1)$ and $(0,a_2)$ be elements in A satisfying
d($(0,a_1)$S$(0,x_1)$)=d(A, B)=3, d($(0,a_2)$S$(0,x_2)$)=d(A, B)=3.

Then we have $a_1 = \log(1 + |x_1|)$ and $a_2 = \log(1 + |x_2|)$.If $x_1 = x_2$, then nothing to prove. Suppose that $x_1 \neq x_2$ by proposition 4.1 and the fact that $\log(1 + |x|)$ is increasing function, we have

d($(0, a_1), (0, a_2)$)=d($(0, \log(1 + |x_1|)), (0, \log 1 + |x_2|)$)

$= \frac{1}{2}|\log(1 + |x_2|) - \log 1 + |x_1|)| \leq \frac{1}{2}|\log(1 + |x_1| - |x_2|)|$

$= \frac{|\log(1+|x_1|-|x_2|)|}{2|x_1 - x_2|}|x_1 - x_2|$

$= \beta$(d($(0, x_1), (0, x_2)$)) d($(0, x_1), (0, x_2)$)

d($(0, a_1), (0, a_2)$) $\leq$ β(d($(0, x_1), (0, x_2)$)) d($(0, x_1), (0, x_2)$).

This implies that S is MT-proximal contraction of first kind. Now we shall prove that S is not proximal contraction of first kind. For this suppose that S is proximal contraction of first kind, then for each $(0, x_1), (0, y_1), (0, a_1)$ and $(0, b_1) \in A$ satisfying

d($(0, x_1)$S$(0, a_1)$)=d(A, B)=3, d($(0, y_1)$S$(0, b_1)$)=d(A, B)=3. (4.1)

there exists k $\in [0, 1)$ such that

d((0, x_1), $(0, y_1)$) $\leq kd((0, a_1), (0, b_1))$

from equation (4.1), we observe that $x_1 = \log(1 + |a_1|)$ and $x_2 = \log(1 + |b_1|)$.

$\frac{1}{2}|\log(1 + |a_1|) - \log(1 + |b_1|)|$=d ((0, x_1), $(0, x_2)$)

$\leq kd((0, a_1), (0, b_1))$

$= k\frac{|a_1 - b_1|}{2}$.

Taking $b_1 = 0$

$$\frac{\log(1 + |a_1|)}{2} \leq k\frac{a_1}{2}$$

$1 = \lim_{a_1 \to \infty} \frac{\log(1+|a_1|)}{|a_1|} \leq k < 1$

which is contradiction.

Hence S is not proximal contraction of first kind.

Now we give an example which satisfying our main result.

Example 4.4 Consider the complete metric space $\mathbb{R}^2$ with metric defined by d($(x_1, x_2), (y_1, y_2)) = |x_1 - y_1| + |x_2 - y_2|$ for all $(x_1, x_2), (y_1, y_2) \in \mathbb{R}^2$. Let A=$\{(0, x): x \in \mathbb{R}\}$ and B=$\{(3, y): y \in \mathbb{R}\}$. Define two mappings S: A → B, T: B → A and g: A∪B → A∪B as follows:

s((0, x))=(3, $\frac{|x|}{2(1+|x|)}$), T((3, y))=(0, $\frac{|y|}{2(1+|y|)}$) and g((x, y))=(x,-y).

Solution: Clearly d(A, B)=3, $A_0 = A$, $B_0 = B$ and mapping g is an isometry.

Next, we shall prove that S and T are MT-proximal contraction of first kind with MT-function β defined by $\beta(t) = \frac{1}{2(1+t)}$ for all t≥ 0.

Let $(0, x_1), (0, x_2), (0, a_1)$ and $(0, a_2)$ be elements in A satisfying

d($(0, a_1)$S$(0, x_1)$)=d(A, B)=3, d($(0, a_2)$S$(0, x_2)$)=d(A, B)=3.

Then we have $a_1 = \frac{|x_1|}{2(1+|x_1|)}$ and $a_2 = \frac{|x_2|}{2(1+|x_2|)}$.If $x_1 = x_2$, then nothing to prove. Suppose that $x_1 \neq x_2$ then by proposition (4.2), we have d((0, a_1), (0, a_2)) $= d((0, \frac{|x_1|}{2(1+|x_1|)}), (0, \frac{|x_2|}{2(1+|x_2|)}))$

$= \left| \frac{|x_1|}{2(1+|x_1|)} - \frac{|x_2|}{2(1+|x_2|)} \right|$

$= \left| \frac{|x_1| - |x_2|}{2(1+x_1)(1+|x_2|)} \right|$

$\leq \frac{|x_1 - x_2|}{2(1+|x_1 - x_2|)}$

$=\beta(\mathrm{d}((0, x_1), (0, x_2)))\ \mathrm{d}((0, x_1), (0, x_2))$.

Thus S is MT-proximal contraction of the first kind.

Similarly, we can prove that T is MT-proximal contraction of the first kind. Now, we show that the pair (S, T) is a proximal cyclic contraction. let (0, u), (0, x) ∈ A and (3, v), (3, y) ∈ B be such that

d((0, u)S(0, x))=d(A, B)=3, d((3, v), T(3, y))=d(A, B)=3.

Then we get

u=$\frac{|x|}{2(1+|x|)}$, v=$\frac{|y|}{2(1+|y|)}$.

The result is true for x=y. So suppose that x ≠ y, then we observe

d((0, u), (3, v))=$|u - v| + 3$

=$\left| \frac{|x|}{2(1+|x|)} - \frac{|y|}{2(1+|y|)} \right| + 3$

=$\left| \frac{|x|-|y|}{2(1+|x|)(1+|y|)} \right|$ +3 ≤ $\frac{|x-y|}{2(1+|x|)(1+|y|)}$ +3 ≤ $\frac{1}{2}|x - y|$ +3

≤ k($|x - y|$ +3) +(1-k)3

=k d((0, x)(2, y)) +(1-k)d(A, B)

where k ∈ [$\frac{1}{2}$, 1). This implies that (S, T) is a proximal cyclic contraction. Therefore all the hypothesis of theorem 3.3 are satisfied. Next, it is clear to see that (0, 0) ∈ A and (3, 0) ∈ B are the unique elements such that

d(g(0, 0), S(0, 0))=d(g(3, 0), T(3, 0))=d((0, 0)(3, 0))=d(A, B).

Competing Interests:

The authors declare that they have no competing interests.

Authors' Contributions:

Both authors contributed equally and significantly in writing this paper.

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