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Vague Soft Set Relation Mappings

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Abstract

In this paper an attempt is made to extend some standard results in set theory on the basis of soft set relations. Vague soft set relation mappings and inverse vague soft set relation mappings are proposed, and some related properties are discussed.

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Keywords: Soft set relation relation mapping, Vague soft set relation, Soft set, Vague Soft Set.

1. Introduction

Fuzzy set [1] was introduced as a mathematical tool to solve the problem and vagueness in everyday life. For uncertain data Molodtsov [2] introduced soft set and other researchers continued with fuzzy soft set [3–10], vague soft set [11, 12] and further extended to multi Q-fuzzy [13–18] and genetic algorithms [19, 20]. In this paper we define the concept of vague soft set relation mappings as extension to our earlier studies on vague soft sets [21–31]. We examine vague soft set relation mappings by proving some theorems.

Vague Soft Set Relation Mapping

In this section, we extend the concept of soft set mappings as proposed by Babitha [10] to that of vague soft set mappings followed by examples to illustrate the operations of the newly defined relations. We then propose a novel definition of inverse vague soft set relation mappings and discuss some related properties.

Definition 1.1. Let f be a vague soft set function from (F, A) to (G, B).

1. The vague soft relation mapping induced by f, denoted by the notation $f^{\rightarrow}$ is a mapping from (F, A) to (G, B), that maps $\Re$ to $f^{\rightarrow}(\Re)$, where $f^{\rightarrow}(\Re)$ is defined by $f^{\rightarrow}(\Re) = \{f(F(a_1)) \times f(F(a_2)) | F(a_1) \times F(a_2) \in \Re\}$.
2. The inverse vague soft set relation mapping induced by f , denoted by the notation $f^{\leftarrow}$, is a mapping from (G, B) to (F, A) that maps T to $f^{\leftarrow}(T)$, where $f^{\leftarrow}(T)$ is defined by, $f^{\leftarrow}(T) = \{F(a_1) \times F(a_2) | f(F(a_1)) \times f(F(a_2)) \in T\}$.

Example 1.2. Let

$$U = \left\{ \frac{u_1}{\langle 0 \rangle}, \frac{u_1}{\langle 3 \rangle}, \frac{u_3}{\langle 9 \rangle} \right\},$$

$A = \{a_1, a_2, a_3, a_4\}$, and $B = \{b_1, b_2\}$. Consider the vague soft set (F, A) and (G, B) defined by

$$F(a_1) = \left\{ \frac{u_1}{\langle 0.1, 0.2 \rangle}, \frac{u_2}{\langle 0.2, 0.6 \rangle}, \frac{u_3}{\langle 0.4, 0.6 \rangle} \right\},$$
$$F(a_2) = \left\{ \frac{u_1}{\langle 0.7, 0.3 \rangle}, \frac{u_2}{\langle 0.1, 0.3 \rangle}, \frac{u_3}{\langle 0.3, 0.6 \rangle} \right\},$$
$$F(a_3) = \left\{ \frac{u_1}{\langle 0.1, 0.1 \rangle}, \frac{u_2}{\langle 0.8, 0.9 \rangle}, \frac{u_3}{\langle 0.3, 0.3 \rangle} \right\},$$
$$f(a_4) = \left\{ \frac{u_1}{\langle 0.4, 0.4 \rangle}, \frac{u_2}{\langle 0.5, 0.5 \rangle}, \frac{u_3}{\langle 0.7, 0.7 \rangle} \right\}$$

and

$$G(b_1) = \left\{ \frac{u_1}{\langle 0.6, 0.5 \rangle}, \frac{u_2}{\langle 0.6, 0.8 \rangle}, \frac{u_3}{\langle 0, 0 \rangle} \right\},$$
$$G(b_2) = \left\{ \frac{u_1}{\langle 0.4, 0.5 \rangle}, \frac{u_2}{\langle 0.9, 0.9 \rangle}, \frac{u_3}{\langle 1, 1 \rangle} \right\}.$$

Suppose a vague soft set function f from (F, A) to (G, B) is given by

$$f = \{F(a_1) \times G(b_1), F(a_2) \times G(b_1), F(a_3) \times G(b_2), F(a_4) \times G(b_2)\}.$$

(1) If $\Re$ be a vague soft set relation on (F, A) given by $\Re = \{F(a_1) \times F(a_2), F(a_3) \times F(a_2), F(a_4) \times F(a_1)\}$, then $f^{\rightarrow}(\Re) = \{G(b_1) \times G(b_1), G(b_2) \times G(b_2)\}$.

(2) If T be a vague soft set relation on (G, B) given by $T = \{G(b_1) \times G(b_1), G(b_2) \times G(b_2)\}$, then $f^{\leftarrow}(T) = \{F(a_1) \times F(a_1), F(a_1) \times F(a_2), F(a_2) \times F(a_2), F(a_2) \times F(a_1), F(a_3) \times F(a_3), F(a_3) \times F(a_4), F(a_4) \times F(a_3), F(a_4) \times F(a_4)\}$.

Next, we will discuss some basic properties of vague soft set relation mappings and inverse vague soft set relation mappings.

Theorem 1.3. If f is a vague soft set function from (F, A) to (G, B) and $\Re, Q \in (F, A)$, then

1. $\Re \subset Q \Rightarrow f^{\rightarrow}(\Re) \subset f^{\rightarrow}(Q)$.
2. $f^{\rightarrow}(\Re \cup Q) = f^{\rightarrow}(\Re) \cup f^{\rightarrow}(Q)$.
3. $f^{\rightarrow}(\Re \cap Q) \subset f^{\rightarrow}(\Re) \cap f^{\rightarrow}(Q)$. If f is one-one, then $f^{\rightarrow}(\Re \cap Q) = f^{\rightarrow}(\Re) \cap f^{\rightarrow}(Q)$.

Proof.

(1) $\forall G(b_1) \times G(b_2) \in f^{\rightarrow}(\Re)$, there exists $F(a_1) \times F(a_2) \in \Re$ such that $G(b_1) \times G(b_2) = f(F(a_1)) \times f(F(a_2))$ by Definition 1.1 (1). Since $\Re \subset Q$, we have $F(a_1) \times F(a_2) \in Q$. Thus $G(b_1) \times G(b_2) \in f^{\rightarrow}(Q)$. Hence $f^{\rightarrow}(\Re) \subset f^{\rightarrow}(Q)$.

(2) $\forall G(b_1) \times G(b_2) \in f^{\rightarrow}(\Re \cup Q)$, there exists $F(a_1) \times F(a_2) \in \Re \cup Q$ such that $G(b_1) \times G(b_2) = f(F(a_1)) \times f(F(a_2))$ by Definition 1.1 (1) where $F(a_1) \times F(a_2) \in \Re$ or $F(a_1) \times F(a_2) \in Q$. Thus $G(b_1) \times G(b_2) \in f^{\rightarrow}\Re$ or $G(b_1) \times G(b_2) \in f^{\rightarrow}Q$. Hence $f^{\rightarrow}(\Re \cup Q) \subset f^{\rightarrow}(\Re) \cup f^{\rightarrow}(Q)$.
Conversely, since $\Re \cup Q \supset R$ and $\Re \cup Q \supset Q$, we have $f^{\rightarrow}(\Re \cup Q) \supset f^{\rightarrow}(\Re)$ and $f^{\rightarrow}(\Re \cup Q) \supset f^{\rightarrow}(Q)$ by (1). Thus $f^{\rightarrow}(\Re \cup Q) \supset f^{\rightarrow}(\Re) \cup f^{\rightarrow}(Q)$.
Therefore $f^{\rightarrow}(\Re \cup Q) = f^{\rightarrow}(\Re) \cup f^{\rightarrow}(Q)$.

(3) Since $\Re \cap Q \subset \Re$ and $\Re \cap Q \subset Q$, we have $f^{\rightarrow}(\Re \cap Q) \subset f^{\rightarrow}(\Re)$ and $f^{\rightarrow}(\Re \cap Q) \subset f^{\rightarrow}(Q)$ by (1). Thus $f^{\rightarrow}(\Re \cap Q) \subset f^{\rightarrow}(\Re) \cap f^{\rightarrow}(Q)$.

Conversely, $\forall G(b_1) \times G(b_2) \in f^{\rightarrow}(\Re) \cap f^{\rightarrow}(Q)$, we have $G(b_1) \times G(b_2) \in f^{\rightarrow}(\Re)$ and $G(b_1) \times G(b_2) \in f^{\rightarrow}(Q)$. By Definition 1.1 (1), there exists $F(a_1) \times F(a_2) \in \Re$ such that $G(b_1) \times G(b_2) = f(F(a_1)) \times f(F(a_2))$ and there exists $F((a_1)^{'}) \times F((a_2)^{'}) \in Q$ such that $G(b_1) \times G(b_2) = f(F((a_1)^{'})) \times f(F((a_2)^{'}))$. Thus $f(F(a_1)) \times f(F(a_2)) = f(F((a_1)^{'})) \times f(F((a_2)^{'})) = G(b_1) \times G(b_2)$, which implies that $f(F(a_1)) = f(F((a_1)^{'}))$ and $f(F(a_2)) = f(F((a_2)^{'}))$. If f is one-one, then $F(a_1) = F((a_1)^{'})$ and $F(a_2) = F((a_2)^{'})$. Thus $F(a_1) \times F(a_2) = F((a_1)^{'}) \times F((a_2)^{'}) \in \Re \cap Q$. By Definition 1.1 (1) $G(b_1) \times G(b_2) \in f^{\rightarrow}(\Re \cap Q)$. Therefore $f^{\rightarrow}(\Re) \cap f^{\rightarrow}(Q) \subset f^{\rightarrow}(\Re \cap Q)$. Hence if f is one-one, then $f^{\rightarrow}(\Re \cap Q) = f^{\rightarrow}(\Re) \cap f^{\rightarrow}(Q)$. ∎

Theorem 1.4. Let f be a vague soft set function from (F, A) to (G, B), $T, S \in (G, B)$. Then

1. $T \subset S \Rightarrow f^{\leftarrow}(T) \subset f^{\leftarrow}(S)$.
2. $f^{\leftarrow}(T \cup S) = f^{\leftarrow}(T) \cup f^{\leftarrow}(S)$.
3. $f^{\leftarrow}(T \cap S) = f^{\leftarrow}(T) \cap f^{\leftarrow}(S)$.

Proof.

(1) $\forall F(a_1) \times F(a_2) \in f^{\leftarrow}(T)$, $f(F(a_1)) \times f(F(a_2)) \in T$ by Definition 1.1 (2). Since $T \subset S$, we have $f(F(a_1)) \times f(F(a_2)) \in S$. Thus $F(a_1) \times F(a_2) \in f^{\leftarrow}(S)$. Hence $f^{\leftarrow}(T) \subset f^{\leftarrow}(S)$.

(2) $\forall F(a_1) \times F(a_2) \in f^{\leftarrow}(T \cup S)$, $f(F(a_1)) \times f(F(a_2)) \in (T \cup S)$. By Definition 1.1 (2) $f(F(a_1)) \times f(F(a_2)) \in (T)$ or $f(F(a_1)) \times f(F(a_2)) \in (S)$. Thus $F(a_1) \times F(a_2) \in f^{\leftarrow}(T)$ or $F(a_1) \times F(a_2) \in f^{\leftarrow}(S)$ implies that $F(a_1) \times F(a_2) \in f^{\leftarrow}(T) \cup f^{\leftarrow}(S)$. Thus $f^{\leftarrow}(T \cup S) \subset f^{\leftarrow}(T) \cup f^{\leftarrow}(S)$.

Conversely, since $T \cup S \supset T$ and $T \cup S \supset S$, we have $f^{\leftarrow}(T \cup S) \supset f^{\leftarrow}(T)$ and $f^{\leftarrow}(T \cup S) \supset f^{\leftarrow}(S)$ by (1). Thus $f^{\leftarrow}(T \cup S) \supset f^{\leftarrow}(T) \cup f^{\leftarrow}(S)$.

Therefore $f^{\leftarrow}(T \cup S) = f^{\leftarrow}(T) \cup f^{\leftarrow}(S)$.

(3) Since $T'\cap S \subset T$ and $T \cap S \subset S$, we have $f^{\leftarrow}(T \cap S) \subset f^{\leftarrow}(T)$ and $f^{\leftarrow}(T \cap S) \subset f^{\leftarrow}(S)$ by (1). So $f^{\leftarrow}(T \cap S) \subset f^{\leftarrow}(T) \cap f^{\leftarrow}(S)$.

Conversely, $\forall F(a_1) \times F(a_2) \in f^{\leftarrow}(T) \cap f^{\leftarrow}(S)$, thus $F(a_1) \times F(a_2) \in f^{\leftarrow}(T)$ and $F(a_1) \times F(a_2) \in f^{\leftarrow}(S)$. By Definition 1.1 (2), $f(F(a_1)) \times f(F(a_2)) \in T$ and $f(F(a_1)) \times f(F(a_2)) \in S$, i.e. $f(F(a_1)) \times f(F(a_2)) \in (T \cap S)$. Thus $F(a_1) \times F(a_2) \in f^{\leftarrow}(T \cap S)$. Hence $f^{\leftarrow}(T) \cap f^{\leftarrow}(S) \subset f^{\leftarrow}(T \cap S)$. Therefore $f^{\leftarrow}(T \cap S) = f^{\leftarrow}(T) \cap f^{\leftarrow}(S)$. ■

Theorem 1.5. Let f be a vague soft set function from (F, A) to (G, B), $\Re \in (F, A)$ and $T \in (G, B)$. We will have the following.

1. $f^{\leftarrow}(f^{\rightarrow}(\Re)) \supset \Re$. If f is one-one, then $f^{\leftarrow}(f^{\rightarrow}(\Re)) = \Re$.
2. $f^{\rightarrow}(f^{\leftarrow}(T)) \subset T$. If f is surjective, then $f^{\rightarrow}(f^{\leftarrow}(T)) = T$.

Proof.

(1) $\forall F(a_1) \times F(a_2) \in \Re$, we have $f(F(a_1)) \times f(F(a_2)) \in f^{\rightarrow}(\Re)$. By Definition 1.1 (2), $F(a_1) \times F(a_2) \in f^{\leftarrow}(f^{\rightarrow}(\Re))$. Hence $f^{\leftarrow}(f^{\rightarrow}(\Re)) \supset \Re$.

Conversely, $\forall F(a_1) \times F(a_2) \in f^{\leftarrow}(f^{\rightarrow}(\Re))$, we have $f(F(a_1)) \times f(F(a_2)) \in f^{\rightarrow}(\Re)$. By Definition 1.1 (2), there exists $F((a_1)^{'}) \times F(a_2)^{'}) \in \Re$ such that

$f(F(a_1)) \times f(F(a_2)) = f(F((a_1)^{'})) \times f(F((a_2)^{'}))$, which implies that $f(F(a_1)) = f(F((a_1)^{'}))$ and $f(F(a_2)) = f(F((a_2)^{'}))$. If f is one-one, then $F(a_1) = F((a_1)^{'})$ and $F(a_2) = F((a_2)^{'})$. Thus $F(a_1) \times F(a_2) = F((a_1)^{'}) \times F((a_2)^{'}) \in \Re$. Thus if f is one-one, then $f^{\leftarrow}(f^{\rightarrow}(\Re)) = \Re$.

(2) By Definition 1.1, $\forall G(b_1) \times G(b_2) \in f^{\rightarrow}(f^{\leftarrow}(T))$, there exists $F(a_1) \times F(a_2) \in f^{\leftarrow}(T)$ such that $G(b_1) \times G(b_2) = f(F(a_1)) \times f(F(a_2))$. Thus $G(b_1) \times G(b_2) = f(F(a_1)) \times f(F(a_2)) \in T$. Hence $f^{\rightarrow}(f^{\leftarrow}(T)) \subset T$.

Conversely, $\forall G(b_1) \times G(b_2) \in T$, if f is surjective, then there exists $F(a_1), F(a_2) \in (F, A)$ such that $G(b_1) = f(F(a_1))$ and $G(b_2) = f(F(a_2))$. Thus $f(F(a_1)) \times f(F(a_2)) = G(b_1) \times G(b_2) \in T$, which implies that $F(a_1) \times F(a_2) \in f^{\leftarrow}(T))$. By Definition 1.1 (1), $G(b_1) \times G(b_2) = f(F(a_1)) \times f(F(a_2)) \in f^{\rightarrow}(f^{\leftarrow}(T))$. Therefore if f is surjective, then $f^{\rightarrow}(f^{\leftarrow}(T)) = T$. ■

Theorem 1.6. Let f be a vague soft set function from (F, A) to (G, B), $\Re \in (F, A)$.

1. If f is surjective and $\Re$ is reflexive, then $f^{\rightarrow}(\Re)$ is a reflexive vague soft set relation on (G, B).

2. If f is bijective and $\Re$ is anti-reflexive, then $f^{\rightarrow}(\Re)$ is an anti-reflexive vague soft set relation on (G, B).

3. If $\Re$ is symmetric, then $f^{\rightarrow}(\Re)$ is a symmetric vague soft set relation on (G, B).

Proof.

(1) $\forall G(b) \in (G, B)$, if f is surjective, then there exists $F(a) \in (F, A)$ such that $G(b) = f(F(a))$. If $\Re$ is reflexive, then $F(a) \times F(a) \in \Re$, thus $G(b) \times G(b) = f(F(a)) \times f(F(a)) \in f^{\rightarrow}(\Re)$, i.e. $f^{\rightarrow}(\Re)$ is a vague reflexive soft set relation on (G, B).

(2) $\forall G(b) \in (G, B)$, if f is bijective, then there exists $F(a) \in (F, A)$ such that $G(b) = f(F(a))$. If $\Re$ is anti-reflexive, then $F(a) \times F(a) \notin \Re$, thus $G(b) \times G(b) = f(F(a)) \times f(F(a)) \notin f^{\rightarrow}(\Re)$, i.e. $f^{\rightarrow}(\Re)$ is an anti-reflexive relation on (G, B).

(3) If $G(b_1) \times G(b_2) \in f^{\rightarrow}(\Re)$, then there exists $F(a_1) \times F(a_2) \in \Re$ such that $G(b_1) \times G(b_2) = f(F(a_1)) \times f(F(a_2))$, which implies that $G(b_1) = f(F(a_1))$ and $G(b_2) = f(F(a_2))$. If $\Re$ is symmetric, then $F(a_1) \times F(a_2) \in \Re$, thus $G(b_1) \times G(b_2) = f(F(a_1)) \times f(F(a_2)) \in f^{\rightarrow}(\Re)$. Therefore $f^{\rightarrow}(\Re)$ is a symmetric vague soft relation on (G, B). ■

Theorem 1.7. Let f be a vague soft set function from (F, A) to (G, B), $T \in (G, B)$.

1. If T is reflexive, then $f^{\leftarrow}(T)$ is a reflexive vague soft set relation on (F, A).

2. If T is anti-reflexive, then $f^{\leftarrow}(T)$ is an anti-reflexive vague soft set relation on (F, A).
3. If T is symmetric, then $f^{\leftarrow}(\Re)$ is a symmetric vague soft set relation on (F, A).

Proof.

(1) $\forall F(a) \in (F, A)$, $f(F(a)) \in (G, B)$. If T is reflexive, then $f(F(a)) \times f(F(a)) \in T$, which implies that $F(a) \times F(a) \in f^{\leftarrow}(T)$ by Definition 1.1. Thus $f^{\leftarrow}(T)$ is a reflexive vague soft set relation on (F, A).

(2) $\forall F(a) \in (F, A)$, $f(F(a)) \in (G, B)$. If T is anti-reflexive, then $f(F(a)) \times f(F(a)) \notin T$, which implies that $F(a) \times F(a) \notin f^{\leftarrow}(T)$ by Definition 1.1. Thus $f^{\leftarrow}(T)$ is an anti-reflexive vague soft set relation on (F, A).

(3) $\forall F(a_1) \times F(a_2) \in f^{\leftarrow}(T)$, then $f(F(a_1)) \times f(F(a_2)) \in T$. If T is symmetric, then $f(F(a_2)) \times f(F(a_1)) \in T$, which implies that $F(a_2) \times F(a_1) \in f^{\leftarrow}(T)$ by Definition 1.1. Thus $f^{\leftarrow}(T)$ is a symmetric vague soft set relation on (F, A). ■

2. Conclusion

Throughout this paper we have made an attempt to widen the set theoretical aspect of vague soft sets. We define the vague soft set relation mappings and inverse vague soft set relation mappings are proposed, and some related properties are discussed. Ordering of a vague soft set is defined and we prove some set theoretical results based on it. To extend this work one could generalize these concepts to fuzzy soft sets so that problems regarding uncertainty could be solved more easily.

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