Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 11, Number 5 (2015), pp. 3189–3193

http://www.ripublication.com/gjpam.htm

Ordering on Vague Soft Set

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Abstract

The traditional soft set is a mapping from a parameter to the crisp subset of universe. Molodtsov introduced the theory of soft sets as a generalized tool for modeling complex systems involving uncertain or not clearly defined objects. In this paper we introduce the concept of ordering on vague soft set, and some related properties are discussed.

AMS subject classification:
Keywords: Ordering on soft set relation, Vague soft set relation, Soft set, Vague Soft Set.

1. Introduction

The notion of ordering on a soft set is introduced by Babitha and Sunil [1]. One of the most important new mathematical tools is fuzzy set theory defined by Molodtsov [2]. This is extended to soft set relation [3, 4], fuzzy soft set [5, 6] and then to vague soft sets [7-19], multi Q-fuzzy [20-25] and genetic algorithms [26, 27]. In this paper we define the concept of ordering on vague soft set as extension to our earlier studies on vague soft set relation and function [8]. We examine ordering on vague soft set by proving some theorems.

2. Ordering on Vague Soft Set

Definition 2.1. A binary vague soft set relation $\Re$ on (F, A) is antisymmetric if $F(a) \times F(b) \in \Re$ and $F(b) \times F(a) \in \Re$ for every $F(a)$ and $F(b)$ in (F, A) implying $F(a) = F(b)$.

Definition 2.2. A binary vague soft set relation $\Re$ on (F, A) which is reflexive, antisymmetric and transitive is called a partial ordering of (F, A). The triple $(F, A, \Re)$ is called a partially ordered vague soft set.

Definition 2.3. A binary vague soft set relation $\Re$ on (F, A) is asymmetric if for every $F(a)$ and $F(b)$ in (F, A), $F(a)\Re F(b)$ implies that $F(b)\Re F(a)$ does not hold. That is, $F(a)\Re F(b)$ and $F(b)\Re F(a)$ can never both be true simultaneously.

Definition 2.4. A binary vague soft set relation $\Re$ on (F, A) is called a strict ordering if it is asymmetric and transitive. We now establish relationships between orderings and strict orderings.

Theorem 2.5.

(a) If $\Re$ is an ordering of (F, A), then the vague soft set relation S on (F, A) defined by $F(a)SF(b)$ iff $F(a)\Re F(b)$ and $F(a) \neq F(b)$ is a strict ordering of (F, A).

(b) If S is a strict ordering of (F, A), then the vague soft set relation $\Re$ defined by $F(a)\Re F(b)$ iff $F(a)SF(b)$ or $F(a) = F(b)$ is an ordering of (F, A).

Proof.

(a) Let us show that S is asymmetric. Assume that both $F(a)SF(b)$ and $F(b)SF(a)$ for some $F(a)$ and $F(b)$ in (F, A). We will have $F(a)\Re F(b)$ and $F(b)\Re F(a)$ by Definition 2.4. Thus $F(a) = F(b)$ ($\Re$ is antisymmetric). This contradicts the definition of $F(a)SF(b)$. To prove the transitivity of S, suppose that $F(a)SF(b)$ and $F(b)SF(c)$. We will have $F(a)\Re F(b)$ and $F(b)\Re F(c)$. Thus $F(a)\Re F(c)$ by transitivity of $\Re$. Hence $F(a)SF(c)$. We proved the symmetric and transitive of the relation, thus the relation $\Re$ on (F, A) is a strict ordering.

(b) Straightforward by using Definition 2.4. ■

Definition 2.6. Let $\Re$ be a partial ordering on the vague soft set (F, A). $\Re$ is called a total ordering on (F, A) if every element in (F, A) is comparable in the ordering $\Re$.

Definition 2.7. Let $(F, A, \Re)$ be a partially ordered soft set. Then,

(a) $F(a)$ is the least element of (F, A) if $F(a)\Re F(x)$ for every $F(x)$ in (F, A).

(b) $F(a)$ is a minimal element of (F, A) if there exists no $F(x)$ such that $F(x)\Re F(a)$ and $F(x) \neq F(a)$.

(c) $F(a)$ is the greatest element of (F, A) if $F(x)\Re F(a)$ for every $F(x)$ in (F, A).

(d) $F(a)$ is a maximal element of (F, A) if there exists no $F(x)$ such that $F(a)\Re F(x)$ and $F(x) \neq F(a)$.

Theorem 2.8. Let $\Re$ be a reflexive and antisymmetric relation on (F, A). The following are equivalent,

(1) $\Re$ is a total order on (F, A).

(2) $\Re$ and its complimentary vague soft set relation $\Re^c$ are both transitive.

Proof. (1)$\Rightarrow$ (2): Clearly $\Re$ is transitive. Let $F(a)$, $F(b)$ and $F(c)$ be in (F, A). Let $F(a)\Re^c F(b)$ and $F(b)\Re^c F(c)$. Thus neither $F(a)\Re F(b)$ nor $F(b)\Re F(c)$ holds. Therefore $F(a)$ is not $\Re$ related to $F(c)$. Thus $F(a)\Re^c F(c)$ and $\Re^c$ is transitive.
(2)$\Rightarrow$ (1): Suppose $\Re$ and its complementary vague soft set relation $\Re^c$ are both transitive. If $F(a)$ and $F(b)$ are distinct elements of (F, A) then either $F(a)\Re F(b)$ or $F(b)\Re F(a)$ must hold. Otherwise we would have $F(a)\Re^c F(b)$ and $F(b)\Re^c F(a)$. Hence $F(a)\Re^c F(a)$. ($\Re^c$ is transitive.) However this contradicts $F(a)\Re F(b)$. Hence $\Re$ is a total order. ∎

Theorem 2.9. Let (F, A) be a vague soft set defined on the universal set U and $\Re$ be an ordering on A. The induced relation $\Re_A$ is an ordering on (F, A). If $(A, \Re)$ is a lattice then (F, A) is also a lattice with meet $\cap$ and join $\cup$ defined as $F(a) \cap F(b) = F(a \wedge b)$ and $F(a) \cup F(b) = F(a \vee b)$ where $\wedge$ and $\vee$ are the corresponding meet and join on $(A, \Re)$.

Proof. By definition, $F(a)\Re_A F(b) \iff a\Re b$. Clearly $F(a)\Re_A F(a)$ as $a\Re a$. Thus $\Re_A$ is reflexive. If $F(a)\Re_A F(b)$ and $F(b)\Re_A F(a)$, then $a\Re b$ and $b\Re a$, since $\Re$ is antisymmetric, $a = b$ and so $F(a) = F(b)$. Thus $\Re_A$ is antisymmetric. If $F(a)\Re_A F(b)$ and $F(b)\Re_A F(c)$ then $a\Re b$ and $b\Re c$, since $\Re$ is transitive. $a\Re c$ and so $F(a)\Re_A F(c)$. Thus $\Re_A$ is transitive. Hence $\Re_A$ is an ordering on (F, A). Suppose $(A, \Re)$ is a lattice with meet and join represented by $\wedge$ and $\vee$ respectively. Now we define the corresponding meet and join of any two elements $F(a)$ and $F(b)$ of a vague soft set as $F(a) \cap F(b) = F(a \wedge b)$ and $F(a) \cup F(b) = F(a \vee b)$. Hence (F, A) is a lattice. ∎

3. Conclusion

Throughout this paper we have made an attempt to widen the set theoretical aspect of vague soft sets. We define the ordering of a vague soft set and we prove some set theoretical results based on it. To extend this work one could generalize these concepts to fuzzy soft sets so that problems regarding uncertainty could be solved more easily.

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