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Retrial Queueing System with customer Impatience

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Abstract

This paper presents an analysis of a single server retrial queue with customer impatience in the orbit. Exit of impatient customer leads to loss of customer which has an impact on business. Managing customer impatience plays a vital role in improving the efficiency of queueing systems. The probability generating function has been used to obtain the steady state probabilities of the system. Also we derived the performance measures and system efficiency discussed by graphical illustrations to demonstrate how the various parameters influence the behaviour of the system. Existing results have been deduced.

AMS subject classification: 60K25, 60K30, 90B22.
Keywords: Retrial Queues, Reneging, Retention, Kummer confluent function.

1. Introduction

Retrial queues are characterized by the phenomenon that a customer finding all the servers busy upon arrival is obliged to leave the service area and repeat his request for service after some random time. Between trials, the blocked customer joins a group of unsatisfied customers called orbit. The customers in the orbit repeatedly attempt to access service. On the other hand if the server is free upon the arrival of a customer, service is started immediately and the customer leaves the system after completion of his service. If the waiting time of a customer exceeds the expected waiting time, the customer decides to leave the system such situation is introduced in [14]. There are three behaviours of impatient customers. The first is balking, the hesitation of a customer to join a queue upon arrival, the second is reneging, the reluctance to continue in after joining and waiting in queue. The third is jockeying, where the impatient customer shifts between parallel lines. The detailed overview of the related references with retrial queue can be found in the book of Falin and Templeton [4, 5] and the survey papers, Artalejo [1, 2]. Retrial queues have been widely used to model many problems in telecommunications networks, the local and wide area networks with random access protocols, Carrier Sense Multiple Access Collusion Detection (CSMA/CD), optical fiber communication networks with hybrid fiber-coax, call centers, fast reservation protocols for asynchronous transfer mode (ATM) networks, telephone systems and electronic services of mail on the internet, computer networks and computer systems in daily life. In recent years there has been a rising interest in analysis of the retrial phenomenon in communication system [1, 2, 3, 4, 5, 6] and mobile communications in [7, 8]. In [6, 9, 10, 11, 12, 13], further applications are given. Krishna Kumar and Pavai Madheswari [15] have analyzed the M/G/1 retrial queues with feedback and starting failures. Jeongsim Kim [17] have analyzed retrial queueing system with collision and impatience. Garg and Sanjeev Kumar [16] have studied a single server retrial queue with impatient customers. Senthil Kumar and Arumuganathan [18] have developed performance analysis of single server retrial queue with general retrial time, impatient customers, two phases of service and Bernoulli schedule. Patrick Wchner and JnosSztrik [19] have developed finite-source M/M/S retrial queue with search for balking and impatient customers from the orbit.

In this paper, we consider the M/M/1 retrial queue with impatience in the orbit. The organization of this article is as follows: The investigated model is described in Section 2. In Section 3, the steady state distribution of the server is derived. Expected number of customers in the system and orbit, expected number of customers waiting time in the system and orbit and probability for fraction of time the system is busy are derived in section 4. Some of the existing results have been deduced in section 5. In Section 6, various measures of efficiency of the system is discussed by graph. In Section 7, conclusion is stated.

2. Model Description

We consider $M/M/1$ retrial queue with impatient customer in the orbit. The primary customers who enter from the outside arrive according to a Poisson process with rate $\lambda > 0$. A primary arrival receives service immediately if the server is idle, otherwise it joins a pool of waiting customers called orbit. A secondary customer in the orbit repeatedly request the service with an exponentially distributed retrial time with rate ν. Service time follows exponential distribution with mean μ. The secondary customers, on joining the orbit activate an individual timer exponentially distributed with parameter α. If the server is not available for the customer before the timer expires, the customer abandons the orbit i.e., never to return the system. We denote the state space $\{\xi, n\}$, where $\xi \in \{0, 1\}$ when $\xi = 0$ if the server is idle, $\xi = 1$ if the server is busy n is the number of customers in the orbit. To derive a system of differential equation for the probabilities $p_{\xi n}(t), n = 0, 1, 2, 3, \ldots$

$p_{0n}(t) =$ Pr[server is idle and there are n customers in the orbit at time t]
$p_{1n}(t) =$ Pr[server is busy and there are n customers in the orbit at time t].

3. Steady state probabilities for the system

The differential-difference equations are derived by using the general birth-death arguments. These equations are solved iteratively in the steady-state in order to obtain the steady state solution. The differential-difference equations are

$$p'_{0n}(t) = -(\lambda + n\nu)\ p_{0n}(t) + \mu\ p_{1n}(t), \quad n \geq 0 \tag{3.1}$$

$$\begin{aligned} p'_{1n}(t) = &-(\lambda + \mu + n\alpha)\ p_{1n}(t) + \lambda\ p_{1n-1}(t) + \lambda\ p_{0n}(t) \\ &+ (n+1)\nu\ p_{0n+1}(t) + (n+1)\alpha\ p_{1n+1}(t), \quad n \geq 1 \end{aligned} \tag{3.2}$$

$$p'_{10}(t) = -(\lambda + \mu)\ p_{10}(t) + \lambda\ p_{00}(t) + \nu\ p_{01}(t) + \alpha\ p_{11}(t), \quad n = 0 \tag{3.3}$$

Let $p_{\xi n}$ be the fraction of time that the system is in the state $\{\xi, n\}$, Then the corresponding steady state equations are

$$(\lambda + n\nu)\ p_{0n} = \mu\ p_{1n}, \quad n \geq 0 \tag{3.4}$$

$$(\lambda+\mu+n\alpha)\ p_{1n} = \lambda\ p_{1n-1} + \lambda\ p_{0n} + (n+1)\nu\ p_{0n+1} + (n+1)\alpha\ p_{1n+1}, \quad n \geq 1 \tag{3.5}$$

$$(\lambda + \mu)\ p_{10} = \lambda\ p_{00} + \nu\ p_{01} + \alpha\ p_{11}, \quad n = 0 \tag{3.6}$$

To solve the system of the equations, we follow the approach given in Falin and Templeton (1997) [4]. From (3.4)

$$\begin{aligned} p_{1n} &= \frac{[\lambda + n\nu]}{\mu} p_{0n} \\ p_{1n+1} &= \frac{[\lambda + (n+1)\nu]}{\mu} p_{0n+1} \\ p_{1n-1} &= \frac{[\lambda + (n-1)\nu]}{\mu} p_{0n-1} \end{aligned}$$

and substituting above the results into (3.5), we get

$$\begin{aligned} &(n+1)[\mu\nu + \alpha[\lambda + (n+1)\nu]]\ p_{0n+1} - \lambda(\lambda + n\nu) \\ &p_{0n} = n[\mu\nu + \alpha(\lambda + n\nu)]\ p_{0n} - \lambda(\lambda + (n-1)\nu)\ p_{0n-1} \end{aligned} \tag{3.7}$$

This can be written as

$$x_{n+1}\ p_{0n+1} - y_n\ p_{0n} = x_n\ p_{0n} - y_{n-1}\ p_{0n-1}$$

where $x_n = n[\mu\nu + \alpha(\lambda + n\nu)]$ and $y_n = \lambda(\lambda + n\nu)$.
Let the constant C be

$$x_{n+1}\ p_{0n+1} - y_n\ p_{0n} = C \quad \text{for all} \quad n \geq 0. \tag{3.8}$$

The constant C can be determined from (3.6) as follows:
solve p_{10} and p_{11} in (3.4) and substitute in (3.6), we get

$$[\mu\nu + \alpha(\lambda + \nu)]\ p_{01} - \lambda^2\ p_{00} = 0$$

which can be written as

$$x_1\ p_{01} - y_0\ p_{00} = 0,$$

which is same as equation (3.8) by replacing $n = 0$. Therefore $C = 0$. In general, from (3.8)

$$p_{0n+1} = \frac{y_n}{x_{n+1}}\ p_{0n} \quad \text{for some} \quad n \geq 0,$$

$$\text{for } n \geq 1 \quad p_{0n} = \prod_{i=0}^{n-1} \frac{y_i}{x_{i+1}}\ p_{00}$$

$$= \prod_{i=0}^{n-1} \frac{\lambda(\lambda + i\nu)}{(i+1)[\mu\nu + \alpha(\lambda + (i+1)\nu)]}\ p_{00}$$

$$= \ p_{00}\frac{\lambda^n}{n!} \prod_{i=0}^{n-1} \frac{\frac{\lambda}{\nu} + i}{[\mu + \alpha(\frac{\lambda}{\nu} + 1 + i)]}$$

$$= \ p_{00}\frac{\lambda^n}{n!\alpha^n} \prod_{i=0}^{n-1} \frac{a + i}{[\frac{\mu}{\alpha} + (a+1) + i]}$$

therefore

$$p_{0n} = \frac{\lambda^n}{n!\alpha^n}\frac{(a)_n}{(b)_n}\ p_{00} \tag{3.9}$$

where $a = \dfrac{\lambda}{\nu}$, $b = \dfrac{\mu}{\alpha} + a + 1$ and $(a)_n \equiv a(a+1)(a+2)(a+3)\cdots(a+n-1)$ is the rising factorial function.

To obtain probability of n customers when the system is busy

$$p_{1n} = \frac{[\lambda + n\nu]}{\mu}\ p_{0n}$$

$$= \frac{\nu}{\mu}[a+n]\ p_{0n}$$

$$= \frac{\nu}{\mu}[a+n]\frac{\lambda^n}{n!\alpha^n}\frac{(a)_n}{(b)_n}\ p_{00}$$

$$= \frac{\nu}{\mu}[a+n]\frac{\lambda^n}{n!\alpha^n}\frac{a(a+1)(a+2)(a+3)\cdots(a+n-1)}{(b)_n}\ p_{00}$$

therefore

$$p_{1n} = \frac{\lambda}{\mu}\frac{\lambda^n}{n!\alpha^n}\frac{(a+1)_n}{(b)_n}\ p_{00}. \tag{3.10}$$

We defined the probability generating functions as

$$P_0(z) = \sum_{n=0}^{\infty} p_{0n} z^n$$

$$P_1(z) = \sum_{n=0}^{\infty} p_{1n} z^n$$

multiplying equations (3.9) and (3.10) by z^n, summing over n from 0 to ∞ and applying Kummer confluent function we get,

$$P_0(z) = p_{00}\sum_{n=0}^{\infty}\frac{\lambda^n}{n!\alpha^n}\frac{(a)_n}{(b)_n}z^n$$
$$= p_{00}\sum_{n=0}^{\infty}\frac{(a)_n}{n!(b)_n}(\frac{\lambda z}{\alpha})^n$$

$$P_1(z) = p_{00}\sum_{n=0}^{\infty}\frac{\lambda\lambda^n}{\mu n!\alpha^n}\frac{(a)_{n+1}}{(b)_n}$$
$$z^n = p_{00}\frac{\lambda}{\mu}\sum_{n=0}^{\infty}\frac{(a)_{n+1}}{n!(b)_n}\left(\frac{\lambda z}{\alpha}\right)^n$$

This can be written as

$$P_0(z) = \Phi\left(a, b; \frac{\lambda z}{\alpha}\right) p_{00} \tag{3.11}$$

$$P_1(z) = \rho\Phi\left(a+1, b; \frac{\lambda z}{\alpha}\right) p_{00} \tag{3.12}$$

where Φ is the Kummer confluent function is defined as $\Phi(a.b; z) = \sum_{n=0}^{\infty} \frac{(a)_n}{(b)_n}\frac{z^n}{n!}$

here p_{00} can be determined using normalizing condition $P_0(1) + P_1(1) = 1$, implying

$$p_{00} = \frac{1}{\Phi(a, b; \frac{\lambda}{\alpha}) + \rho\Phi(a+1, b; \frac{\lambda}{\alpha})}. \tag{3.13}$$

4. Performance measures

The various performance measures can be derived from the probability generating functions. The probability for fraction of the time the server is busy

$$p_1 \equiv \sum_{n=0}^{\infty} p_{1n} = P_1(1) = \frac{\rho\Phi(a+1, b; \frac{\lambda}{\alpha})}{\Phi(a, b; \frac{\lambda}{\alpha}) + \rho\Phi(a+1, b; \frac{\lambda}{\alpha})} = \frac{\rho^*}{1+\rho^*}$$

$$p_1 = \frac{\rho^*}{1+\rho^*} \tag{4.1}$$

$$\text{where} \qquad \rho^* = \frac{\rho\Phi\left(a+1, b; \frac{\lambda}{\alpha}\right)}{\Phi\left(a, b; \frac{\lambda}{\alpha}\right)}$$

Now we derive the mean number of customers in the orbit L_0, using probability generating function

$$L_0 = p_0^{'}(1) + p_1^{'}(1) \tag{4.2}$$

$$P_0^{'}(1) = \lambda a[\Phi(a+1, b; \lambda) - \Phi(a, b; \lambda)]\ p_{00} \tag{4.3}$$

$$P_1^{'}(1) = \frac{\nu}{\mu}[(\lambda + a - b + 1)\Phi(a+1, b; \lambda) - (a - b + 1)\Phi(a, b; \lambda)]\ p_{00} \tag{4.4}$$

combining (4.3) and (4.4) we get

$$L_0 = \frac{p_{00}}{\nu\mu + \alpha\lambda + \alpha\nu}\left\{\lambda^2 + \rho\lambda\left(\frac{\lambda}{\alpha} + \frac{\lambda}{\nu} - \frac{\mu}{\alpha} - 1\right)\right\}\Phi\left(a+1.b+1; \frac{\lambda}{\alpha}\right)$$
$$+\left(\frac{\mu}{\alpha}+1\right)\Phi\left(a.b+1; \frac{\lambda}{\alpha}\right) \tag{4.5}$$

The mean number of the customer in the system is

$$L = L_0 + p_1$$

The performance metrices like mean number of customers waiting time in the orbit W_0 and mean number of customers waiting time in the system W can be determined from L_0 and L using Little's formula

$$W_0 = \frac{L_0}{\lambda} \qquad \text{and} \qquad W = \frac{L}{\lambda}$$

5. Particular Cases

Case 1. If impatient rate $\alpha = 0$, our model becomes M/M/1 retrial queueing studied by Donald Gross and John F. Shortle [12].
Case 2. If impatient rate $\alpha = 0$, $\nu = 1$, our model becomes M/M/1 queueing studied by Donald Gross and John F. Shortle [12].

6. Numerical illustrations

In this section, some numerical results are presented to illustrate the effect of various parameters on the performance measures of the system. By choosing arbitrary values for the parameters $\lambda = 2, \mu = 16, \alpha = 1, 2, 3, \ldots 15$, $\nu = 1, 2$ and 3 two dimensional graphs are drawn in Figure 1-6. Figure 1 shows that the busy probability p_1 of the system decreases if the impatient rate α increases with its varied retrial rate ν. The idle probability p_{00} of the system increases if the impatient rate α increases with its varied retrial rate ν as shown in Figure 2. Figure 3 shows that the mean system size L decreases if the impatient rate α increases with its varied retrial rate ν. Figure 4 shows that the mean waiting time of the system W decreases if the impatient rate α increases with its varied retrial rate ν. Figure 5 and Figure 6 shows that the mean orbit size L_0 decreases if the impatient rate α increases with its varied retrial rate ν and the mean waiting time of the orbit W_0 decreases if the impatient rate α increases with its varied retrial rate ν.

Choosing arbitrary values for the parameters $\lambda = 2, \mu = 32, \alpha = 1, 2$ and 3, $\nu = 1, 2, 3.$, 15 two dimensional graphs are drawn in Figure 7–12. Figure 7 shows that the busy probability p_1 of the system increases if the retrial rate ν increases with its varied impatient rate α. The idle probability p_{00} decreases if the retrial rate ν increases with its varied impatient rate α as shown in Figure 8. Figure 9 shows that the mean system size L decreases if the retrial rate ν increases with its varied impatient rate α. Figure 10 shows that the mean waiting time of the system W decreases for increasing retrial rate ν with varying the impatient rate α. The mean orbit size L_0 decreases if the retrial rate ν increases with its varied impatient rate α and the mean waiting time of the orbit W_0 decreases if the retrial rate ν increases with its varied impatient rate α are shown in Figure 11 and 12.

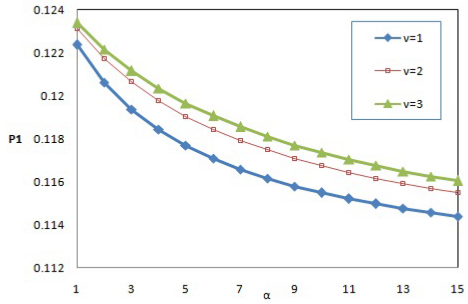

Figure 1: p_1 versus $\alpha = 1, 2, 3, \ldots, 15$ for $\nu = 1, 2, 3$.

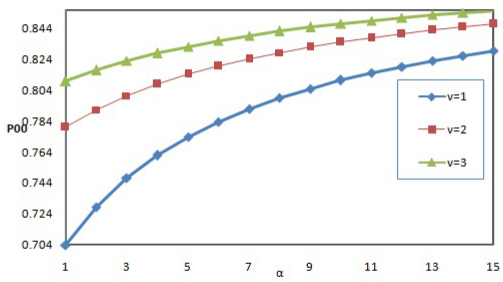

Figure 2: p_{00} versus $\alpha = 1, 2, 3, \ldots, 15$ for $\nu = 1, 2, 3$.

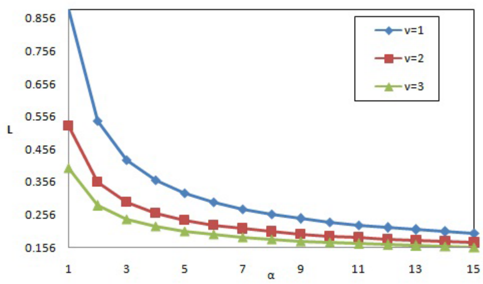

Figure 3: L versus $\alpha = 1, 2, 3, \ldots, 15$ for $\nu = 1, 2, 3$.

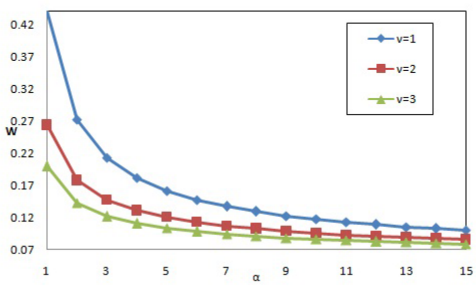

Figure 4: W versus $\alpha = 1, 2, 3, \ldots, 15$ for $\nu = 1, 2, 3$.

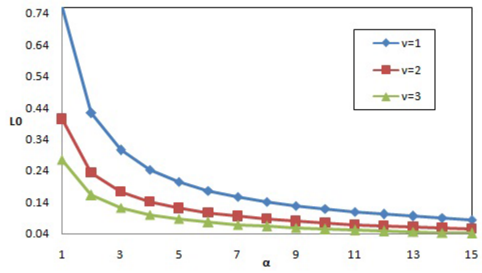

Figure 5: L_0 versus $\alpha = 1, 2, 3, \ldots, 15$ for $\nu = 1, 2, 3$.

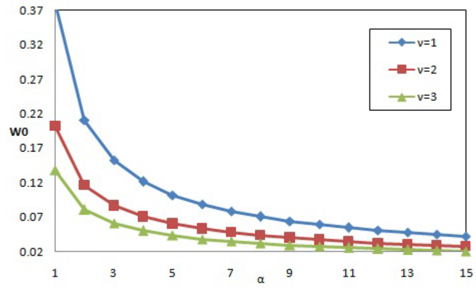

Figure 6: W_0 versus $\alpha = 1, 2, 3, \ldots, 15$ for $\nu = 1, 2, 3$.

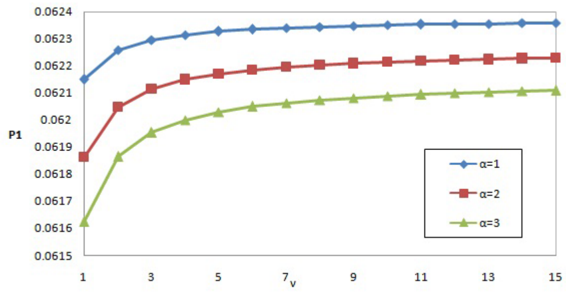

Figure 7: p_1 versus $\nu = 1, 2, 3, \ldots, 15$ for $\alpha = 1, 2, 3$.

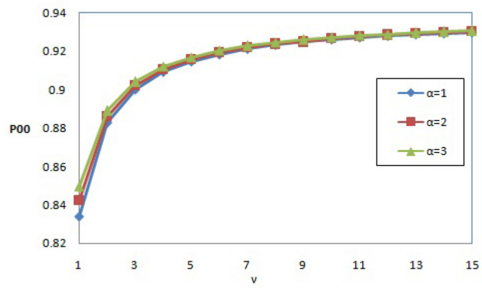

Figure 8: p_{00} versus $\nu = 1, 2, 3, \ldots, 15$ for $\alpha = 1, 2, 3$.

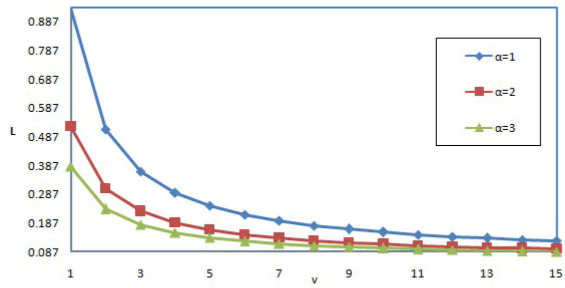

Figure 9: L versus $\nu = 1, 2, 3, \ldots, 15$ for $\alpha = 1, 2, 3$.

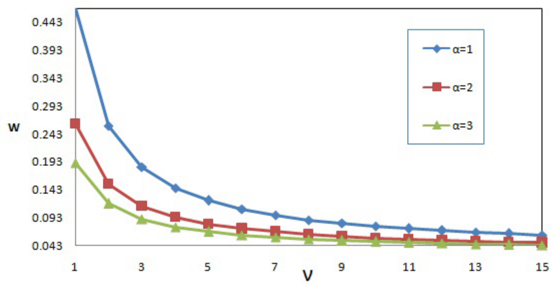

Figure 10: W versus $\nu = 1, 2, 3, \ldots, 15$ for $\alpha = 1, 2, 3$.

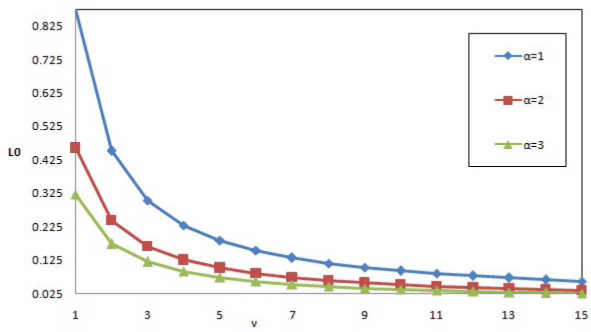

Figure 11: L_0 versus $\nu = 1, 2, 3, \ldots, 15$ for $\alpha = 1, 2, 3$.

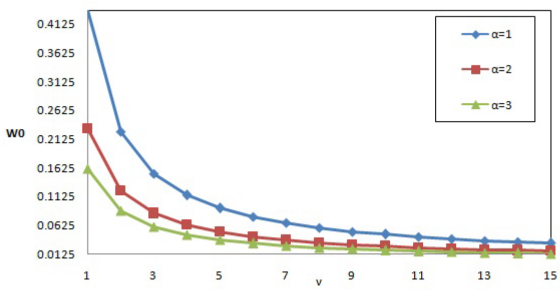

Figure 12: W_0 versus $\nu = 1, 2, 3, \ldots, 15$ for $\alpha = 1, 2, 3$.

7. Conclusion

In this paper, a single server retrial queueing system with impatient customer in the orbit is studied. For this model, closed form of solution is obtained by using probability generating function. The various performance measures such as the probability for fraction of time the server is busy, mean number of customers in the orbit and the system, mean number of customers waiting in the orbit and in the system have been derived. The above derived system can be represented by graph with the help of numerical values by using MATLAB.

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